Graph-Based Interference Models for Ad Hoc Networks

- Arbitrary wireless network $\Rightarrow$ Graph $G = (V, E)$
  - $V$: set of vertices $\rightarrow$ wireless terminals
  - $E$: set of edges joining two vertices $\rightarrow$ wireless links between two terminals or interference connection

- **Adjacent vertices**: two vertices connected by an edge
- **Complete graph**: a graph in which there is an edge connecting every pair of vertices
- **Induced subgraph**: a subset of vertices of $G$, together with all edges connecting the vertices of that subset.
- **Clique**: an induced subgraph that is complete
- **A maximal clique** of a graph $G$ is a clique that is not contained in any other clique of $G$
- **Independent set**: set of vertices of a graph $G$ that are not connected by edges
Graph-Based Interference Models for Ad Hoc Networks

Preliminary Concepts of Graph Theory

Cliquets

Examples

Graph

Clique (BCD)

Clique (ABE)

Clique (BDE)

Independent Sets

Examples

Graph

Independent set (CDG)

Independent set (ADF)

Wireless Networks: Connectivity Graph $G = (V, E)$

- Given the transmission range $r(u)$ of node $u \Rightarrow$ Disk $D(u, r(u))$
- Edge $e = (u, v)$ (undirected):
  $$ e = \{(u, v) \in E \text{ if } v \text{ is covered by } D(u, r(u)) \text{ and } u \text{ is covered by } D(v, r(v))\}$$
- Coverage of node $u$
  $$\text{cov}(u) := \{w \in V | w \text{ is covered by } D(u, r(u))\}.$$  
- Coverage of edge $e$
  $$\text{cov}(e) := \{w \in V | w \text{ is covered by } D(u, r(u))\} \cup \{w \in V | w \text{ is covered by } D(v, r(v))\}.$$
Interference Modeling

- **Node-based Model**: describes interference among nodes
  - One-way transmission case
  - Which nodes are disturbed by transmission of a given node

- **Link-based Model**: describes interference among links
  - Two-way or four-way transmission cases (e.g. IEEE 802.11 MAC)
  - Which links are disturbed by transmissions in a given link

### Incoming interference $I_{in}$
- at a given node $u$: number of nodes $v_j$ whose disks $D(v_j, r(v_j))$ cover node $u$.

### Outgoing interference $I_{out}$
- caused by a given node $v$: number of nodes covered by $D(v, r(v))$.

### Interference Graph
- Representation of interference using graphs
- **Interference range** $r_i$: maximum distance from the transmitter that causes interference.
  - Not enough to connect, but enough to disturb
  - $r_i \geq r_t$
- Distance $r_t$: additive noise and receiver sensibility
- Distance $r_i$: minimum acceptable SIR
Interference Graph (cont’d)

- Connectivity Graph \( G = (V, E) \)
- Interference Graph \( G_i = (V_i, E_i) \)
- Consider one-way transmission (directed edges)

Constructing the interference graph
- Edges (links) of \( G \) ⇒ vertices of \( G_i \)
- \( V_i = \{ l_{uv} \} \), s.t. \( l_{uv} \in E \) and \( u, v \in V \)
- Link \( l_{uv} \) interferers with link \( l_{wx} \) if \( d_{ux} < r^i_u \)
- Edge \( (l_{uv} \rightarrow l_{wx}) \in E_i \) if \( d_{ux} < r^i_i \)

▷ Links \( l_{uv} \) and \( l_{wx} \) cannot be simultaneously active.

Interference among terminals

- A graph can also be used to explicitly model interference among terminals
- Interference graph \( G_i = (V_i, E_i) \)
  - Vertices represent terminals
  - There is a directed edge from terminal \( u \) to terminal \( v \) if:
    - Signal from \( u \) is strong enough to disturb reception at \( v \) of any other signal,
    - But it cannot be correctly decoded by \( v \).
Graph-Based Interference Models for Ad Hoc Networks  
Modeling interference among terminals

Example of a graph modeling interference among terminals

Connectivity
Graph $G = (V,E)$

Interference
Graph $G_i = (V,E_i)$

Example of matching on $G = (V,E \cup E_i)$

Matching on Interference Graphs

- A matching (or independent edge) on a graph $G = (V,E)$ is a subset $E_m$ of $E$, such that no two edges in $E_m$ have a terminal in common.

$G = (V,E)$

Matching (AB,DE)

Matching on Interference Graphs

Two edges $(x, y)$ and $(x', y')$ of a graph $G = (V,E)$ are said to be strongly independent if none of the edges $(x, x'), (x, y'), (y, x'), (y, y')$ belongs to $E$.

$G = (V,E)$

Strong independent edges

The transmission scheduling problem can be solved by finding subsets of $E$ of edges that are strongly independent with respect to $G = (V,E \cup E_i)$.

For instance, edges $(u,v)$ and $(t,s)$ form a maximal matching, since $\{(u,s), (u,t), (t,v), (s,v)\} \notin E \cup E_i$.

Therefore, links $(u,v)$ and $(t,s)$ can be scheduled to transmit simultaneously.

The resulting scheduling may lead to communication links with degraded performance: the interference graph is constructed based on a pairwise interference model, while communication links are degraded by the aggregate interference.
Aggregate Interference

- Previous graph-based models: graphs model interference between two links (or terminals) - pairwise interference models.
- Real world: two links that, individually, do not interfere with a given link may disturb that link when they are active simultaneously.
- Jain, Padhye, Padmanabhan and Qiu [Jain, 2005] proposed the weighted conflict graph that takes into account the aggregate interference.

Weighted conflict graph

- Let us consider a set of transmitter-receiver pairs \( \{(X_i, X_{R(i)}): i \in \mathcal{N}\} \).
- Successful communication in the pair \((X_i, X_{R(i)})\) is guaranteed if

\[
\frac{P_i \left| X_i - X_{R(i)} \right|^\eta}{\sigma^2 + \sum_{k \in \mathcal{N}, k \neq i} P_k \left| X_k - X_{R(i)} \right|^\eta} \geq \beta.
\]

- \( \sigma^2 \) is the noise power,
- \( \beta \) is the minimum required SINR.

Construction of the weighted conflict graph

- \( l_i \) denotes the communication link \((X_i, X_{R(i)})\) and is represented by a vertex in the weighted conflict graph.
- There is a weighted directed edge from vertex \( l_k \) to vertex \( l_i \) with weight \( w_{ki} = \frac{1}{l_{i,\text{max}}} \frac{P_k}{\left| X_k - X_{R(i)} \right|^\eta} \).

Clearly, a necessary condition for \( \text{SINR} \geq \beta \) at \( X_{R(i)} \) is \( w_{ji} \leq 1 \).
Construction of the weighted conflict graph

- Using the weighted conflict graph, one can determine sets of communication links $S_m$, $m = 1, 2, \ldots$, such that:

\[ \sum_{l_k \in S_m} w_{l_k} \leq 1 \]

for every link $l_i \in S_m$.

- This means that, all links $l_i$ in $S_m$ can be active simultaneously active, since $I_{i,max}$ is not exceeded for all $l_i$.

- Therefore, links $l_i \in S_m$ can be scheduled to transmit at the same time-slot.

Optimization of Wireless Ad Hoc Networks

Input parameters and constraints
- Set of $N$ terminals (nodes) $n_i$, $i = 1, 2, \ldots, N$ with their positions
- Transmission ranges $R_i$ and interference ranges $R'_i$, $i = 1, 2, \ldots, N$
- Maximum capacity $F_{ij}$ of each $l_{ij}$ connecting nodes $n_i$ and $n_j$
- Source - Destination pair $(n_s, n_d)$ (single flow problem)
- Multipath routing

Output parameters
- Flow $f_{ij}$ of each link $l_{ij}$ that maximizes the flow between source $n_s$ and destination $n_d$

Optimization of a wireless ad hoc network
- Solution proposed by Jain et al. in [Jain, 2005]
- Nodes located at arbitrary positions
- Objective: Maximum flow in a given source-destination link
Constructing the Connectivity Graph

- Graph $G = (V_C, E_C)$
- Vertices: $V_C = \{n_i\}_{i \in I}$
- Directed link $l_{ij} = (n_i, n_j) \Rightarrow l_{ij} \in E_C$ if $d_{ij} < R_i$

Maximization Problem

**Linear Programming Formulation**

$$\text{max} \sum_{l_{si} \in E_C} f_{si}$$

Subject to:

$$\sum_{l_{ij} \in E_C} f_{ij} = \sum_{l_{ji} \in E_C} f_{ji}, \text{ for } n_j \in V_C \setminus \{n_s, n_d\}$$  

(1)

$$\sum_{l_{is} \in E_C} f_{is} = 0$$  

(2)

$$\sum_{l_{di} \in E_C} f_{di} = 0$$  

(3)

$$f_{ij} \leq F_{ij} \text{ for } \forall i, j \mid l_{ij} \in E_C$$  

(4)

$$f_{ij} \geq 0 \text{ for } \forall i, j \mid l_{ij} \in E_C$$  

(5)

Interpretation of the constraints (I)

**Objective function**

$$\text{max} \sum_{l_{si} \in E_C} f_{si}$$

Maximization of all outgoing flows from source node $n_s$
Interpretation of the constraints (II)

Constraint 1

\[ \sum_{l_{ij} \in EC} f_{ij} = \sum_{l_{ji} \in EC} f_{ji}, \quad \text{for} \quad n_i \in V_C \setminus \{n_s, n_d\} \]

Flow conservation constraints: the sum of all incoming flows to a given node must be equal to the sum of all outgoing flows from that node.

Interpretation of the constraints (III)

Constraints 2 and 3

\[ \sum_{l_{is} \in EC} f_{is} = 0 \]
\[ \sum_{l_{di} \in EC} f_{di} = 0 \]

Incoming flows to the source node are zero and outgoing flows from the destination node are zero.

Interpretation of the constraints (IV)

Constraints 4 and 5

\[ f_{ij} \leq F_{ij} \quad \text{for} \quad \forall i, j \mid l_{ij} \in EC \]
\[ f_{ij} \geq 0 \quad \text{for} \quad \forall i, j \mid l_{ij} \in EC \]

Flows must be non-negative and cannot exceed the link capacity.

Adding interference effects in the constraints set

- Constraints (1)-(5) do not take interference into account
- Two forms of interference [Krumke, 1990]
- **Primary Interference**: when a terminal is involved in more than one communication task at the same time (sending and receiving, receiving from two different transmitters, etc.).
- **Secondary Interference**: when a terminal \( n_i \) receives from a particular transmitter \( n_j \), but also receives unwanted signals from another transmitter, such that the communication between \( n_i \) and \( n_j \) deteriorates.
Example: Interference caused by link $l_{31}$

- Links $l_{31}$ and $l_{54}$ (and others) are in conflict.
- Links $l_{31}$ and $l_{46}$ (and others) can be active simultaneously.
- We need to determine the sets of links that can be active simultaneously.

**Procedure:**

Step 1: Determine the Interference (or Conflict) Graph;
Step 2: Determine the maximal independent sets of the Interference Graph.

- Links of a given independent set can only transmit during a given fraction of time $\lambda_k (\sum_k \lambda_k = 1)$
- Therefore: In a feasible solution of the LP formulation, the flow of a link is no longer upper bounded by link capacity.
**Interference sets**

Interference set of each link (1 = interference; 0 = no interference)

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**Conflict Matrix**

1 = Conflict (links cannot be active simultaneously); 0 = No conflict

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**Interference Graph**

Representing edges of the connectivity graph as vertices in the interference graph...
Step 2: Determining the Maximal Independent Sets of $G_i$

Based on the Interference Graph $G_i = (V_i, E_i)$, construct sets of nodes in $V_i$ (i.e., communication links) that are not connected by edges in $E_i$

<table>
<thead>
<tr>
<th>Set $k$</th>
<th>Links</th>
<th>$\lambda_k$</th>
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<tbody>
<tr>
<td>$S_1$</td>
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<td>$S_5$</td>
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<td>$S_6$</td>
<td>$l_{14}, l_{46}$</td>
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<td>$S_7$</td>
<td>$l_{34}$</td>
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<td>$S_8$</td>
<td>$l_{43}$</td>
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Links of the independent sets

- Set $S_k$ can be active during $\lambda_k$ seconds
- $\tau_{ij}$ = sum of the activity periods of the independent sets of which link $l_{ij}$ is a member.

$$\tau_{ij} = \sum_{\forall k \mid l_{ij} \in S_k} \lambda_k$$

- New constraints:

$$f_{ij} \leq \sum_{\forall k \mid l_{ij} \in S_k} \lambda_k$$ for $\forall i, j \mid l_{ij} \in E_C$

$$\sum_k \lambda_k = 1$$
Example of Application of Interference Models

References I


References II


References III


References IV


M.C. Necker, “Integrated Scheduling and Interference Coordination in Cellular OFDMA Networks”, *IEEE Broadnets 2007*.