On the Performance of CDMA Systems Employing Multiuser Decorrelating Detector and Antenna Array

Renato Baldini Filho, Celso de Almeida, and Gustavo Fraidenraich, Member, IEEE

Abstract—This paper presents analytical expressions to evaluate the capacity of code division multiple access (CDMA) cellular radio systems employing a joint antenna array and a multiuser decorrelating detector. The performance is evaluated for chip-synchronous and asynchronous baseband and bandpass CDMA systems on additive white Gaussian noise and flat-fading Rayleigh channels. Simulation results have shown that the analytical model developed to evaluate the performance of those systems is a very tight approximation.

Index Terms—Antenna array, code division multiple access (CDMA), multiuser detectors (MUD).

I. INTRODUCTION

CODE DIVISION multiple access (CDMA) techniques have been highly regarded for future generation wireless communication systems, not only because they have high bandwidth efficiency but also because they are suitable for handling multimedia and multirate services [1].

Multipath propagation and time variance present on mobile channels produce multiple access interference (MAI) and intersymbol interference (ISI) at the receiver. Conventional receivers based on the matched filter and the rake detector treat MAI and ISI as noise, which can degrade the system performance severely, mainly if the power of interfering users is high.

To improve the performance of such receivers, multiuser detectors (MUDs) have proposed by [2], which cancel the interferences from other users, before the desired user symbol is estimated. Thus, MUDs eliminate the MAI, at the expense of an undesired increase in the noise power. The optimum MUD, e.g.,

\[ \hat{s}_{\text{opt}}(t) = \sum_{k=1}^{N} \beta_{k}(t) s_{k}(t) \]

where \( s_{k}(t) \) is the random spreading sequence,

\[ \beta_{k}(t) = \frac{r(t) - \sum_{j=1}^{k-1} \beta_{j}(t) s_{j}(t)}{\sum_{j=1}^{N} \beta_{j}(t) s_{j}(t)} \]

can be improved by deploying antenna array receivers at the base station (BS). The antenna array is employed to spatially eliminate the other user interferences. It is well known that if the interferers have their angular positions in the uniformly distributed cell, the average interference can be significantly reduced by a factor of \( M \): the number of antennas in the array [4].

Several papers have been published on CDMA systems employing multiuser decorrelating detection joint with antenna arrays, e.g., [1], [5]–[19]. However, the system performance evaluated in the majority of those papers has been obtained by computer simulation, which is time consuming and difficult to validate and does not present an explicit interdependence on all system parameters. On the other hand, papers that present analytical performance evaluation do it for very specific cases, e.g., [18] and [19].

The aim of this paper is to present analytical expressions for the bit error probability (BER) for CDMA systems employing a multiuser decorrelating detector and an antenna array. The reverse link is considered, and it is assumed that we have binary phase-shift keying (BPSK) modulation with perfect power control and random spreading sequences on both additive white Gaussian noise (AWGN) and flat-fading Rayleigh channels. Chip-synchronous and asynchronous, as well as baseband and bandpass systems, are analyzed. Moreover, expressions for the signal-to-noise-plus-interference ratio (SNIR) gains are developed and evaluated from such systems.

This paper is organized as follows. The system model with an antenna array and a multiuser decorrelating detector is presented in Section II. Section III devises performance expressions of chip-synchronous, asynchronous, baseband, and bandpass systems on an AWGN channel. In Section IV, analytical expressions for the SNIR gain are evaluated for all cases considered in the previous section. The performance analysis on a flat-fading Rayleigh channel is treated in Section V. Final remarks and conclusions are made in Section VI.

II. BANDPASS SYSTEM MODEL

This section gives a brief description of CDMA systems employing a jointly decorrelating detector and an antenna array. The following analysis is made for the reverse link and BPSK modulation.

The complex envelope of the transmitted signal for the \( k \)th user can be expressed as

\[ p_{k}(t) = a_{k} b_{k}(t) s_{k}(t) \]

where \( a_{k} \) denotes the signal amplitude, \( b_{k}(t) \) is the transmitted bit of duration \( T_{b} \), and \( s_{k}(t) \) is the random spreading sequence,
whose chips have duration $T_c$ and magnitude $\pm 1$, are pairwise statistically independent, and have equal probabilities. The ratio $T_c/T_s = G_p$ is the processing gain.

The complex envelope of the received signal at the $m$th antenna of the BS can be represented as the summation of $K$ received envelope signals

$$r_m(t) = \sum_{k=1}^{K} a_k \alpha_k b_k(t-t_k) s_k(t-t_k) e^{-j\phi_k} + \eta_m(t)$$

for $1 \leq m \leq M$ (2)

where $\alpha_k$, $\phi_k$, and $t_k$ denote the attenuation, phase, and time delay, introduced by the channel, respectively, for the $k$th user. The noise at the receiver is modeled as a Gaussian random variable $\eta_m(t)$ with zero mean and two-sided power spectral density equal to $N_0/2$ (W/Hz), and $M$ is the number of antennas in the array. Notice that for the baseband case analysis, the received signal can also be represented by (2), with the term $e^{-j\phi_k}$ suppressed.

A uniformly spaced linear array with antenna spacing $\Delta$ is assumed. For a flat-fading channel, the direction of arrival $\theta_k$ of the $k$th user is a random variable uniformly distributed in the interval $[0, 2\pi]$. Then, $a(\theta_k)$ is the steering vector, which is defined as

$$a(\theta_k) = [a_1(\theta_k) ~ a_2(\theta_k) ~ \cdots ~ a_M(\theta_k)]^T$$

(3)

where $[\cdot]^T$ denotes transposition, and

$$a_m(\theta_k) = \frac{e^{j2\pi(m-1)\frac{\Delta}{\lambda}\sin(\theta_k)}}{\sqrt{M}}$$

(4)

for $1 \leq m \leq M$ where $\lambda$ is the wavelength.

Therefore, the received signal at the base-station antenna array can be expressed as

$$r(t) = \sum_{k=1}^{K} A_k b_k(t-t_k) s_k(t-t_k) a(\theta_k) e^{-j\phi_k} + \eta(t)$$

(5)

where $A_k = a_k \alpha_k$, and

$$r(t) = [r_1(t) ~ r_2(t) ~ \cdots ~ r_M(t)]^T$$

(6)

and $\eta(t)$ represents the additive noise vector, which is given by

$$\eta(t) = [\eta_1(t) ~ \eta_2(t) ~ \cdots ~ \eta_M(t)]^T$$

(7)

Fig. 1 shows the block diagram of the receiver that uses a multiuser decorrelating detector with an antenna array at the BS. For the block of the user $i$, $r(t)$ is detected by the antennas and weighted by the $i$th user steering vector $a(\theta_i)$. Then, it is passed through a filter matched to its random sequence. It is assumed that the steering vector and phases can be perfectly estimated. For the $i$th user’s receiver, the signal after the matched filter is sampled at the instant $t_i + T_b$ and it is given by

$$y_i = \int_{t_i}^{t_i+T_b} a^H(\theta_i) e^{j\phi_i} s_i(t-t_i) r(t) dt$$

(8)

where $(\cdot)^H$ is the Hermitian operation.

Substituting (5) into (8)

$$y_i = \sum_{k=1}^{K} \rho_{i,k} a^H(\theta_i) a(\theta_k) A_k b_k(t-t_k) + n$$

(9)

where $\rho_{i,k}$ is the cross correlation between the spreading sequences of the $i$th and the $k$th users given by

$$\rho_{i,k} = \int_{t_i}^{t_i+T_b} s_i(t-t_i) s_k(t-t_k) dt$$

(10)

and the filtered noise is given by

$$n = \int_{t_i}^{t_i+T_b} s_i(t-t_i) a^H(\theta_i) \eta(t) dt$$

(11)

The signal processed by the antenna array and the matched-filter bank can be written in a matrix form as

$$y = RA_b + n$$

(12)
where $y$ is a $K$-dimensional vector given by

$$y = [y_1 \ y_2 \ \cdots \ y_K]^T$$

whose entries are given by (9). The $K \times K$ cross correlation matrix $R$ includes the spreading-sequence cross correlations and the steering vectors, whose entries are expressed by

$$R_{jl} = a^H(\theta_j)a(\theta_l)\rho_{jl}$$

and $A$ is a diagonal amplitude matrix of dimension $K \times K$, $b$ is the $K$-dimensional column vector of the transmitted bits, and $n$ represents the filtered background noise column vector.

The decorrelating detector eliminates the MAI by multiplying the matched-filter bank outputs $y$ by $R^{-1}$, resulting in

$$v = R^{-1}y = Ab + R^{-1}n$$

as presented in Fig. 1. Finally, $v$ is passed through a comparator bank to produce estimates $b$ of the transmitted bits. Notice from (15) that the interference can be completely eliminated at an expense of increasing the noise power.

III. PERFORMANCE ON AWGN CHANNEL

In this section, the performance of the multiuser decorrelator detector with antenna array is analytically evaluated for AWGN channel. Very tight approximations for the BER expressions, for chip-synchronous and chip-asynchronous, baseband, and bandpass CDMA systems are obtained.

First, we are going to obtain the BER for the chip-synchronous bandpass system for two users, because the cross correlation inverse matrix $R^{-1}$ can be easily evaluated. The BER is then generalized for systems with $K > 2$ users.

A. Chip-Synchronous Bandpass Case

The mean BER for the $i$th user, employing a decorrelating MUD and antenna array, is given by [2]

$$P_b = Q \left( \sqrt{\frac{2 E_b}{N_0} \beta_i} \right)$$

where $E_b$ is the energy per bit, $N_0/2$ is the two-sided power spectral density of the noise, $Q(x)$ is the area under the Gaussian probability density function (pdf), and $\beta_i$ represents the asymptotic multiuser efficiency (AME) for the target user $i$, and it is given by [2]

$$\beta_i = \frac{1}{(R^{-1})_{i,i}}$$

where $(R^{-1})_{j,i}$ is the $j$th row and $l$th column entry of the matrix $R^{-1}$.

Appendix A shows that the mean BER, given by (16), for CDMA systems that use random spreading sequences can be tightly approximated by

$$P_b \approx Q \left( \sqrt{\frac{2 E_b}{N_0} \beta_i} \right)$$

1) Two-Users Case: For the sake of simplicity, let us start the analysis of $\beta$ first for two users and then generalize it for $K > 2$ users. It is supposed that all users have the same average AME, that is, $\beta_i = \bar{\beta}$ for all $i$. The cross correlation matrix $R$, whose entries are given by (14), can be evaluated for two users as

$$R = \begin{bmatrix} a^H(\theta_2)a(\theta_1)\rho \cos(\phi_2 - \phi_1) \\ a^H(\theta_1)a(\theta_2)\rho \cos(\phi_1 - \phi_2) \end{bmatrix}$$

where $\rho$ is the cross correlation of the spreading sequences of user 1 and user 2, $\phi_1$ and $\phi_2$ are the direction of arrival of users 1 and 2, respectively, and $\phi_1$ and $\phi_2$ are the received phases of users 1 and 2, respectively. Thus, $R^{-1}$ can be evaluated as (20), shown at the bottom of the page, where $D = 1 - \rho^2 \cos^2(\phi_1 - \phi_2)a^H(\theta_1)a(\theta_2)\rho \cos(\phi_2 - \phi_1)$.

Assuming that user 1 is the target user, and taking the average of the first entry of $(R^{-1})_{k,k}$, we can evaluate $\bar{\beta}$ by

$$\bar{\beta} = 1 - \rho^2 \cos^2(\phi_1 - \phi_2) \frac{a^H(\theta_1)a(\theta_2)}{[a^H(\theta_1)a(\theta_2)]^2}.$$

Notice that the expectation operation can be split into three terms, as a consequence of the fact that the instantaneous cross correlation, the received phase, and the instantaneous direction of arrival are pairwise independent. It is considered that the directions of arrival are uniformly distributed in the interval $[0, 2\pi]$. Then, the mean-squared terms are given by [2, pp. 33] and [4]

$$\bar{\rho}^2 = \frac{1}{G_{p, 2 \text{ users}}},$$

$$\cos^2(\phi_1 - \phi_2) = \frac{1}{2},$$

$$\frac{|a^H(\theta_i)a(\theta_j)|^2}{[a^H(\theta_1)a(\theta_2)]^2} = \frac{1}{M} + \frac{2}{M^2} \sum_{m=1}^{M-1} (M - m) \times J_0^2 \left( \frac{2\pi m \Delta}{\lambda} \right)$$

where $J_0(\cdot)$ is the zero-order Bessel function of first kind. It is shown in [4] that (24) can be well approximated to

$$\frac{|a^H(\theta_i)a(\theta_j)|^2}{[a^H(\theta_1)a(\theta_2)]^2} \approx \frac{1}{M}.$$
Substituting (22)–(24) into (21) results in the following expression to the average AME for the target user 1:

$$\beta \approx 1 - \frac{1}{2G_{p,2\text{ users}}} \left[ \frac{1}{M} + \frac{2}{M^2} \sum_{m=1}^{M-1} (M-m)J_0^2 \left( \frac{2\pi m \Delta}{\lambda} \right) \right].$$  \hfill (26)

Using (25), (26) reduces to

$$\beta \approx 1 - \frac{1}{2G_{p,2\text{ users}}M}. \hfill (27)$$

Thus, substituting (27) into (18), the BER for two chip-synchronous users can be well approximated to

$$P_b \approx Q \left( \sqrt{\frac{2E_b}{N_0} \left( 1 - \frac{1}{2G_{p,2\text{ users}}M} \right)} \right). \hfill (28)$$

2) $K$-Users Case: For CDMA systems, the load factor $\mu$ is defined as the ratio of the number of interferers to the processing gain

$$\mu = \frac{K-1}{G_p}. \hfill (29)$$

Notice that the BER remains unchanged if the load factor is kept constant. This means that even if the number of users $K$ in the system, or the processing gain $G_p$, is changed but the load factor is kept unchanged, the BER of the CDMA system remains the same. Therefore, the processing gain for two users can be expressed as the processing gain $G_p$ evaluated for $K$ users divided by the number of interferers, that is

$$G_{p,2\text{ users}} = \frac{G_p}{K-1}. \hfill (30)$$

This approximation is valid, since the BER for CDMA systems depends uniquely on the load factor but not individually on the number of users or the processing gain.

Taking (30) into (28), it is possible to rewrite the BER for $K$ chip-synchronous users of a bandpass system on an AWGN channel as

$$P_b \approx Q \left( \sqrt{\frac{2E_b}{N_0} \left( 1 - \frac{(K-1)}{3G_pM} \right)} \right). \hfill (31)$$

Then, the average AME for $K$ users can be obtained from (31), resulting in

$$\beta \approx 1 - \frac{K-1}{3G_pM}. \hfill (32)$$

In order to validate the approximation (30), Fig. 2 presents the average AME for $K$ users in a single antenna system versus the load factor for the synchronous case. Notice that Monte Carlo simulation and the analytical curves are in perfect agreement. Moreover, (25) is also a very good approximation for $M$ antennas, and therefore, (31) is a very good approximation for the mean BER for the chip-synchronous case.

B. Chip-Asynchronous Bandpass Case

The average AME for $K$ users and $M$ antennas can be obtained from (84) presented in Appendix B, that is reproduced here for clarity

$$\beta \approx 1 - \frac{K-1}{3G_pM}. \hfill (33)$$

Therefore, the BER can be written as

$$P_b \approx Q \left( \sqrt{\frac{2E_b}{N_0} \left( 1 - \frac{(K-1)}{3G_pM} \right)} \right). \hfill (34)$$

Fig. 2 also shows the average AME for $K$ users and a single antenna ($M = 1$) versus the load factor for the chip-asynchronous case. Notice that the error between Monte Carlo simulation and the analytical curves are below 10% for load factor under $\mu = 1.5$. As load factor $\mu > 1.5$ is nonpractical for real CDMA systems, (34) is also a good approximation for the BER for the chip-asynchronous case.

C. Chip-Synchronous Baseband Case

For the baseband cases, the lack of carrier only eliminates the factor of 1/2 presented in (23) in the evaluation of the BER. Therefore, the mean-squared cross correlation for a chip-synchronous system is given by $\rho^2 = 1/G_p$. Then, the mean BER for a chip-synchronous baseband system can be written as

$$P_b \approx Q \left( \sqrt{\frac{2E_b}{N_0} \left( 1 - \frac{(K-1)}{G_pM} \right)} \right). \hfill (35)$$
D. Chip-Asynchronous Baseband Case

Similar performance analysis of the previous sections can be made for a chip-asynchronous baseband system. In this case, the time delays of the received signals from the users are distinct. As a consequence of the development presented in Appendix B, the mean BER for chip-asynchronous baseband systems can be written as

\[ P_b \approx Q \left( \sqrt{\frac{2E_b}{N_0}} \left( 1 - \frac{2(K-1)}{3G_pM} \right) \right). \]  \hspace{1cm} (36)

IV. SNIR GAIN

The SNIR gain of the decorrelating detector plus antenna array scheme in relation to the matched-filter detector can be easily obtained by comparing their mean BER expressions.

A. SNIR Gain for the Chip-Synchronous Bandpass Case

For the chip-synchronous bandpass case, the BER for the matched filter is given by [20]

\[ P_b = Q \left( \sqrt{\frac{(K-1)}{2G_p} + \frac{1}{2 \frac{E_b}{N_0}}} \right)^{-1}. \]  \hspace{1cm} (37)

Therefore, using (37) and (31), the SNIR gain can be evaluated by

\[ G = \left( \frac{E_b}{N_0} \mu + 1 \right) \left( 1 - \frac{1}{2} \frac{\mu}{M} \right). \]  \hspace{1cm} (38)

Notice that the maximum SNIR gain is evaluated by the derivative of (38) with respect to \( \mu \) and equaling zero. Therefore, the maximum SNIR gain and its corresponding load factor are given by

\[ G_{\text{max}} = \frac{M \frac{E_b}{N_0}}{2} + \frac{1}{2} M \frac{E_b}{N_0} \]  \hspace{1cm} (39)

\[ \mu_{G_{\text{max}}} = M - \frac{1}{2} \frac{E_b}{N_0} \]  \hspace{1cm} (40)

B. SNIR Gain for the Chip-Asynchronous Bandpass Case

For the chip-asynchronous bandpass case, the mean BER for the matched filter is given by [20]

\[ P_b = Q \left( \sqrt{\frac{(K-1)}{3G_p} + \frac{1}{2 \frac{E_b}{N_0}}} \right)^{-1}. \]  \hspace{1cm} (41)

Therefore, using (41) and (34), the SNIR gain can be evaluated by

\[ G = \left( \frac{2E_b}{3N_0} \mu + 1 \right) \left( 1 - \frac{1}{3} \frac{\mu}{M} \right). \]  \hspace{1cm} (42)

The maximum SNIR gain and its corresponding load factor are given by

\[ G_{\text{max}} = \frac{M \frac{E_b}{N_0}}{2} + \frac{1}{2} M \frac{E_b}{N_0} \]  \hspace{1cm} (43)

\[ \mu_{G_{\text{max}}} = M - \frac{1}{2} \frac{E_b}{N_0} \]  \hspace{1cm} (44)

Fig. 3 shows the SNIR gain as a function of the load factor for chip-synchronous and chip-asynchronous bandpass systems. Parameters \( M = 4 \) and \( E_b/N_0 = 10 \).

C. SNIR Gain for the Chip-Synchronous Baseband Case

For the chip-synchronous baseband case, the mean BER for the matched filter is given by [20]

\[ P_b = Q \left( \sqrt{\frac{K-1}{G_p} + \frac{1}{2 \frac{E_b}{N_0}}} \right)^{-1}. \]  \hspace{1cm} (45)

Therefore, using (45) and (35), the SNIR gain can be evaluated by

\[ G = \left( \frac{2E_b}{N_0} \mu + 1 \right) \left( 1 - \frac{1}{3} \frac{\mu}{M} \right). \]  \hspace{1cm} (46)
The maximum SNIR gain and its corresponding load factor are given by

\[ G_{\text{max}} \approx \frac{E_b}{N_0} M + 1 \quad \text{for } \frac{E_b}{N_0} \gg 1 \] (47)

\[ \mu G_{\text{max}} = \frac{M}{2} - \frac{1}{4 \frac{E_b}{N_0}} \]

\[ \approx \frac{M}{2} \quad \text{for } \frac{E_b}{N_0} \gg 1. \] (48)

### D. SNIR Gain for the Chip-Asynchronous Baseband Case

For the chip-asynchronous baseband case, the mean BER for the matched filter is given by [20]

\[ P_b = Q \left( \sqrt{\frac{2(K - 1)}{3G_p} + \frac{1}{2 \frac{E_b}{N_0}}} \right). \] (49)

Therefore, using (49) and (36), the SNIR gain can be evaluated by

\[ G = \left( \frac{4}{3} \frac{E_b}{N_0} \mu + 1 \right) \left( 1 - \frac{2}{3} \frac{\mu}{M} \right). \] (50)

The maximum SNIR gain and its corresponding load factor are given by

\[ G_{\text{max}} \approx \frac{E_b}{N_0} M + 1 \]

\[ \approx \frac{1}{2} M \frac{E_b}{N_0} \quad \text{for } \frac{E_b}{N_0} \gg 1 \] (51)

\[ \mu G_{\text{max}} = \frac{3}{4} M - \frac{3}{8} \frac{E_b}{N_0} \]

\[ \approx \frac{3}{4} M \quad \text{for } \frac{E_b}{N_0} \gg 1. \] (52)

Fig. 4 shows the SNIR gain as a function of the load factor for synchronous and asynchronous baseband systems. The number of antennas is set to \( M = 4 \), and the signal-to-noise ratio is \( E_b/N_0 = 10 \). Notice that bandpass and baseband systems (synchronous and asynchronous) present the same value for maximum gain but different load factors for the respective maximum gain. The load factor corresponding to the maximum gain is 50% lower for baseband systems than for bandpass systems.

### V. PERFORMANCE ON FLAT-FADING RAYLEIGH CHANNEL

#### A. Chip-Synchronous Bandpass Case

The instantaneous BER for the matched-filter detector and BPSK modulation is given by

\[ P_b = Q \left( \sqrt{2\gamma_b} \right) \] (53)

where the SNIR for flat-fading Rayleigh channel is

\[ \gamma_b = \frac{\alpha^2}{\alpha^2 K^{-1} G_p + \frac{E_b}{N_0}} \] (54)

and \( \alpha \) is the instantaneous fading amplitude.

The mean BER can be expressed as [21]

\[ \overline{P}_b = \frac{1}{2} \left( 1 - \sqrt{\frac{1}{\overline{\gamma}_b}} \right) \] (55)

where the average SNIR is given by

\[ \overline{\gamma}_b = \frac{1}{K^{-1} G_p + \left( \frac{E_b}{N_0} \right)^{-1}}. \] (56)

For the system combining antenna array and multiuser decorrelating detection, the instantaneous BER is also given by (53), but with \( \gamma_b \) evaluated by

\[ \gamma_b = \frac{\alpha^2 E_b}{N_0}. \] (57)

The mean BER for the system combining antenna array and multiuser decorrelating detection, on a flat-fading Rayleigh channel, is also given by (55), with

\[ \overline{\gamma}_b = \frac{1}{K^{-1} G_p + \left( \frac{\gamma_b}{N_0} \right)^{-1}}. \] (58)

Defining the SNIR gain for the flat-fading Rayleigh channel as the ratio between (56) and (58), it can be concluded that the SNIR gain for the flat-fading channel is identical to the AWGN case.

Therefore, all cases presented previously for the AWGN channel can be evaluated for the flat-fading channel, where the
mean BER is given by (55), with $\gamma_b$ assuming the following values:

\[
\gamma_b = \frac{1}{G_p^* + \left(\alpha^2 \frac{E_b}{N_0}\right)^{-1}}
\]

Matched-Filter Chip-Synchronous Bandpass

\[
\gamma_b = \alpha^2 \frac{E_b}{N_0} \left(1 - \frac{K - 1}{2G_pM}\right)
\]

MUD Antenna Array Chip-Synchronous Bandpass

\[
\gamma_b = \frac{1}{2 \left(\frac{K - 1}{G_p^*} + \left(\alpha^2 \frac{E_b}{N_0}\right)^{-1}\right)}
\]

Matched-Filter Chip-Asynchronous Bandpass

\[
\gamma_b = \alpha^2 \frac{E_b}{N_0} \left(1 - \frac{K - 1}{3G_pM}\right)
\]

MUD Antenna Array Chip-Asynchronous Bandpass

\[
\gamma_b = \frac{1}{4 \left(\frac{K - 1}{G_p^*} + \left(\alpha^2 \frac{E_b}{N_0}\right)^{-1}\right)}
\]

Matched-Filter Chip-Synchronous Baseband

\[
\gamma_b = \alpha^2 \frac{E_b}{N_0} \left(1 - \frac{K - 1}{G_pM}\right)
\]

MUD Antenna Array Chip-Synchronous Baseband

\[
\gamma_b = \frac{1}{4 \left(\frac{K - 1}{G_p^*} + \left(\alpha^2 \frac{E_b}{N_0}\right)^{-1}\right)}
\]

Matched-Filter Chip-Asynchronous Baseband

\[
\gamma_b = \alpha^2 \frac{E_b}{N_0} \left(1 - \frac{2(K - 1)}{3G_pM}\right)
\]

MUD Antenna Array Chip-Asynchronous Baseband.

The SNIR gain can be obtained for flat-fading Rayleigh channel by comparing $\gamma_b$ for the MUD antenna array scheme with the respective $\gamma_b$ for the matched-filter, for all cases. From those comparisons, we can observe that the SNIR gain for flat-fading Rayleigh channel is identical to the SNIR gain for the AWGN channel. Moreover, those results can also be extended for selective-frequency Rayleigh fading channels.

Figs. 5 and 6 show the BER versus the load factor, for chip-synchronous bandpass on AWGN and flat-fading channels, respectively. The number of antennas in the array is set to 1, 2, and 4, and $E_b/N_0 = 10$. Notice that the analytical results are in close agreement with simulations for all curves in both figures.

VI. CONCLUSION

This paper has analyzed CDMA systems using antenna array and multiuser decorrelating detector. First, expressions for the BER have been obtained for synchronous and asynchronous and bandpass and baseband CDMA systems for the two-users case. Then, those expressions were generalized for any number of users. Simple and accurate SNIR expressions concerning all schemes on AWGN and fading Rayleigh channels were obtained for synchronous or asynchronous and baseband or bandpass systems. The SNIR gain was initially defined for AWGN channel for comparison purposes among different receiver schemes. Moreover, the SNIR gain expressions has shown to be also valid for flat and selective-frequency fading channels. Finally, according to (47), the maximum SNIR gain depends only on the number of antennas $M$ and on the $E_b/N_0$.

APPENDIX A

In order to show that (18) is a good approximation to (16), we are going to evaluate the error $\epsilon$ between those expressions for a known case, that is, for a baseband chip-synchronous CDMA two-users system. In this case, the exact mean BER is evaluated by

\[
P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}(1 - \rho^2)\right)
\]

where $\rho = \rho_{i,k} = (1/T_b) \int_0^{T_b} s_i(t)s_k(t)dt$ is the normalized synchronous cross correlation between the spreading sequences $s_i(t)$ and $s_k(t)$. Notice that, in fact, this is the worst case, because simulation results show that the error decreases with the increase of the number of users.
For a convex function $f(x)$, Jensen’s inequality states that $\frac{f(x)}{\sqrt{x}} \geq f(\overline{x})$. The inequality error is proportional to the second derivative of the function. As $Q(x)$ is a convex function for $x \geq 0$, we can use the Jensen’s inequality in order to evaluate (67), that is

$$P_b \geq Q\left(\sqrt{\frac{2E_b}{N_0} (1 - \rho^2)}\right).$$

In the following, we show that (68) is a good approximation for (67) by evaluating the error between both equations.

For random spreading sequences with equiprobable $\pm 1$ chips, it is easy to show that the normalized synchronous cross correlation function can assume the values $\rho = 2i/G_p$, where $i$ is an integer for $-G_p/2 \leq i \leq G_p/2$. The discrete cross correlation pdf is given by

$$p(\rho) = \frac{1}{2G_p} \left(1 - \left(\frac{G_p}{2} + i\right)^2\right) \quad \text{for} \quad -\frac{G_p}{2} \leq i \leq \frac{G_p}{2}.$$ (69)

Therefore, the exact and the approximated mean BER can be calculated using (67)–(69). The difference between the exact and the approximated mean BER is named exact error. The exact mean BER and the exact error are shown in Table I as a function of $G_p$ and $E_b/N_0$.

Moreover, it is also possible to obtain an approximation for the error. Let $f(x)$ be a function of a random variable $x$ that can be expanded in Taylor series about $x = x_0$

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2!} f''(x_0)(x - x_0)^2 + \cdots.$$ (70)

Therefore, the mean value of $f(x)$ is given by

$$\overline{f(x)} = f(x_0) + f'(x_0)(\overline{x} - x_0) + \frac{1}{2!} f''(x_0)(\overline{x} - x_0)^2 + \cdots.$$ (71)

For high values of processing gain, we are going to show that (71) can be tightly approximated to

$$f(\overline{x}) = f(x_0) + f'(x_0)(\overline{x} - x_0) + \frac{1}{2!} f''(x_0)(\overline{x} - x_0)^2 + \cdots.$$ (72)

Let us consider only the three first terms of the Taylor series, then the approximated error $\epsilon$ between (71) and (72) is given by

$$\epsilon \approx \frac{1}{2!} f''(x_0) (x_0^2 - \overline{x}^2).$$ (73)

According to (67), let us define the random variable $x = (2E_b/N_0) (1 - \rho^2)$ and the function $f(x) = Q(\sqrt{x})$. Expanding $f(x)$ about $x_0 = 2E_b/N_0$ is a consequence of expanding (67) about $\rho = 0$.

Performing the second derivative $f''(x = x_0)$ yields

$$\frac{\partial^2 Q(\sqrt{x})}{\partial x^2}_{x=x_0} = \frac{1}{4\sqrt{2\pi} \sqrt{\frac{2E_b}{N_0}}} \left(\frac{1}{\sqrt{\frac{2E_b}{N_0}}} + 1\right) \exp\left(-\frac{E_b}{N_0}\right).$$ (74)

Then, evaluating $\overline{x^2}$, we have that

$$\overline{x^2} = \frac{4E_b^2}{N_0} (1 - \rho^2)^2$$

$$= \frac{4E_b^2}{N_0} \left(1 - \frac{2}{G_p} + \frac{3}{G_p^2}\right).$$ (75)

Similarly, $\overline{x}^2$ is evaluated by

$$\overline{x}^2 = \frac{4E_b^2}{N_0} (1 - \overline{\rho}^2)^2$$

$$= \frac{4E_b^2}{N_0} \left(1 - \frac{2}{G_p} + \frac{1}{G_p^2}\right).$$ (76)

Finally, substituting (74)–(76) into (73), the approximated error is given by

$$\epsilon \approx \frac{1}{2\sqrt{\pi} G_p^2} \frac{1}{2E_b} \left(\frac{E_b}{N_0}\right)^{3/2} \left(\frac{1}{\sqrt{\frac{2E_b}{N_0}}} + 1\right) \exp\left(-\frac{E_b}{N_0}\right).$$ (77)

Table I presents the exact BER and the exact and the approximated errors as a function of $E_b/N_0$ and $G_p$. The exact BER is obtained from (67), the exact error is obtained by simulation, and the approximated error is obtained from (77).

Table I shows that the exact and approximated errors are very close to each other and that they are very small when compared to the exact BER. Both errors become even smaller when the number of users increase. These conclusions are valid for all cases considered in this paper.

**APPENDIX B**

The AME for two users (the asynchronous baseband CDMA system case with a single antenna) is given by [2]

$$\beta = \sqrt{1 - (\rho_{12} + \rho_{21})^2} \left[1 - (\rho_{12} - \rho_{21})^2\right]$$ (78)

where $\rho_{12}$ and $\rho_{21}$ are the asynchronous cross correlation between sequences $s_1(t)$ and $s_2(t)$.

In order to facilitate the evaluation of $\beta$, the previous equation can also be expressed as

$$\beta = \sqrt{1 - (\rho_{12}^2 + \rho_{21}^2) - 2\rho_{12}\rho_{21}}. $$ (79)
The mean value of $\beta$ can be well approximated by

$$\beta \simeq 1 - \frac{\rho_{21}^2}{\rho_{21}^2}$$

(80)

because computer simulations show that the term $4\rho_{21}^2\rho_{22}^2$ is negligible in relation to $[1 - (\rho_{21}^2 + \rho_{22}^2)]^2$ for small loads, but it is not so negligible for loads higher than 1.5. However, (80) is still a good approximation, because loads higher than 1.5 have no practical application.

For the chip-asynchronous bandpass case, we have that

$$\rho_{22}^2 = \rho_{21}^2 = \frac{1}{6G_{p, \text{2 users}}}$$

(81)

where $G_{p, \text{2 users}}$ is the processing gain for the two-user case. Therefore

$$\beta \simeq 1 - \frac{1}{3G_{p, \text{2 users}}}.$$  (82)

Expanding the average AME for $K$ users by using $G_{p, \text{2 users}} = G_p/(K - 1)$ yields

$$\beta \simeq 1 - \frac{K - 1}{3G_p}.$$  (83)

For $M$ antennas, the average AME can be obtained from

$$\beta \simeq 1 - \frac{K - 1}{3G_1M}.$$  (84)

This AME is used in (34) to evaluate the BER for the chip-asynchronous bandpass case. Notice that all those considerations are also valid for the baseband case.

REFERENCES


