A network traffic prediction approach based on multifractal modeling

Flávio Henrique Teles Vieira a,*, Gabriel Rocon Bianchi b and Luan Ling Lee b

a School of Electrical and Computer Engineering, Federal University of Goiás, Setor Leste Universitário-Goiânia, GO, Brazil
E-mail: flavio@eeec.ufg.br
b Department of Communications, State University of Campinas, Campinas, SP, Brazil
E-mails: {bianchi, lee}@decom.fee.unicamp.br

Abstract. This work extends the notion of the widely mentioned and used fractional Brownian traffic model in the literature. Extensive experimental investigations indicate that the proposed traffic model, named extended fractional Brownian traffic, can capture not only the self-similar properties, but also the inherent multifractal characteristics of those traffic flows found in modern communication networks. Additionally, the structure of this traffic model is taken into account in a traffic prediction algorithm that benefits from the more accurate traffic modeling. The experimental results clearly point out the advantages of using the proposed model in traffic modeling as well as in traffic prediction.

Keywords: Network traffic, multifractal model, traffic prediction

1. Introduction

The seminal paper published by Leland et al. triggered a new direction on traffic modeling research, by unveiling the fractal nature of wideband traffic signals [20]. These authors experimentally demonstrated that the LAN Ethernet traffic collected in Bellcore Morristown Research and Engineering Center exhibits self-similar properties and burstiness in a wide range of time scales. After that, many research papers have shown that such self-similar behavior, unable to be faithfully represented through most classic Markovian stochastic models, is not restricted solely to the Ethernet LAN environment [33], and in fact, strongly impacts on the network performance [27]. Among many different fractal modeling approaches, the fractional Brownian motion (fBm) or its incremental process, called fractional Gaussian noise (fGn), has become popular for modern traffic modeling because of its simplicity. However, such a self-similar traffic model is capable of capturing the traffic correlation property only over large time scales.

A more sophisticated description of network traffic behavior based on multifractal analysis was introduced by Riedi et al. [30], and latter by Feldmann et al. [10]. The multifractal analysis generalizes the self-similar behavior observed in the network traffic in a natural way.

Self-similar processes, or more generically speaking, monofractal processes are characterized by a single time-invariant parameter, the Hurst parameter [13]. Contrary to monofractal processes, multifractal processes allow such characteristics to vary in time, therefore, increasing the flexibility in describing irregular phenomena that arise in real signals.

Statistical models derived from multifractal processes are capable of representing the actual network traffic behavior in a more complete and accurate manner. Although the research carried out on multifractals is not quite

*Corresponding author: Flávio Henrique Teles Vieira, School of Electrical and Computer Engineering, Federal University of Goiás, Av. Universitária, 1488, Qd. 86, Bloco A 74605 010, Setor Leste Universitário, Goiânia, GO, Brazil. E-mails: flavio@eeec.ufg.br, flavio@decom.fee.unicamp.br.
recent, few multifractal models in fact have been developed up to now. Among them, we cite the following: the Multi-
fractional Wavelet Model (MWM) proposed by Riedi et al. [31], the Variable Variance Gaussian Multiplier Model
(VVGMM) proposed by Krishna et al. [19], and the multifractional Brownian model (mBm) in [28]. The definition
of the mBm process generalizes the fractional Brownian motion, a monofractal process with a constant exponent \( H \)
(global scaling parameter) to the case where \( H \) is no longer a constant, but a time-varying function.

In this work, we propose a network traffic model that is capable of capturing not only the self-similar properties,
but also the inherent multifractal characteristics of the traffic flows in modern communication networks. The pro-
posed model can also be viewed as an extension of the classical and broadly used fractional Brownian traffic model
(fBm) [26]. Moreover, under the extended fractional Brownian traffic modeling, another important contribution of
this work is an adaptive traffic predictor that considers multifractal properties.

The remaining part of this work is organized as follows. Section 2 presents some concepts related to multifractal
processes and some methods of multifractal parameter estimation. Section 3 introduces the extended fractional
Brownian traffic model. Section 4 shows how the correlation structure of the extended fractional Brownian traffic
model is applied to the design of a novel traffic predictor. Section 5 presents the results of the experimental investi-
gations that intend to evaluate the proposed extended fractional Brownian traffic model. Section 6 summarizes the
evaluation results of the proposed traffic predictor applied to real traffic traces. Finally, in Section 7 the conclu-
sions and some plausible ideas of future work are presented.

2. The multifractal characteristics

In order to mathematically represent the self-similarity detected in the network traffic, a variety of stochastic
models were proposed [20,25]. Particularly, the fractional Brownian motion is pointed as possibly the simplest
mathematical model capable of taking account of the monofractal characteristics observed in traffic data. However,
investigations into WAN TCP/IP traffic traces [10,30] concluded that there are properties observed in short time
scales that are better described through multifractal analysis. Such multifractal traffic properties are consequences
of the action of some network layer protocols that impose certain end-to-end traffic control mechanisms in order
to control the information flow behavior between different layers in, for example, the TCP/IP protocol stack [11].

We can define multifractals through the scaling properties of process moments over different time increments.

Definition 1. A stochastic process \( Z(t) \) is called multifractal if it satisfies the following condition:

\[
E(|Z(t)|^q) = c(q)t^{\tau(q)+1} \quad \forall t \in T, q \in Q,
\]

where \( T \) and \( Q \) are intervals on the real line, and \( \tau(q) \) and \( c(q) \) are functions with domain \( Q \). The function \( \tau(q) \)
is called the scaling function of multifractal processes or the partition function [22,23].

The multifractal analysis is regarded as a generalization of the monofractal analysis, allowing different traffic
behavior to be observed in various time scales, and therefore, providing a better description of irregularities (or
singularities) for the traffic process.

Mathematically speaking, singularities are points (for example, in an equation, on a curve, on a surface, etc.) at
which abrupt transitions occur or the object becomes degenerated (lack of convergence). To characterize singular
structures, the local regularity of a signal needs to be precisely quantified. Lipschitz exponents provide regularity
measurement over time intervals, as well as for isolated signal points. If a function \( f(t) \) has a singularity at \( v \),
which means that this function is not differentiable at \( v \), then the Lipschitz exponent (also known as the Hölder
exponent) characterizes this singular behavior [21].

The presented definition of multifractals is based on the scaling properties of process moments over different
time increments. In that way, the ‘multifractality’ is defined in terms of the global properties of processes and it
was not taken into account their local variability at specific time instances. A second definition of multifractals can
be stated using the concept of Hölder exponents as we will show [22].
Definition 2. A function $Z$ is said to belong to a $C^a_x$ set, if there exists a polynomial $P$ whose order is less than $a$, and a constant $C$ such that for $x, y \in [0, \infty)$:

$$|Z(y) - P(y)| < C|y - x|^a.$$  

(2)

Then, the local Hölder exponent $\alpha(x)$ of function $Z$ at point $x$ is given by

$$\alpha(x) = \sup\{a: Z \in C^a_x\}.$$  

(3)

Local Hölder exponents characterize function behavior on a neighborhood of a given point $x$. From Eqs (2) and (3), we can verify that for a process sample path $Z$, its infinitesimal variations on the neighborhood of $t$ can be described by:

$$|Z(t + \Delta t) - Z(t)| \sim C_1(t)\alpha(t),$$  

(4)

where $C_1(t)$ is called prefactor.

After simple manipulation of Eq. (4), the Hölder exponents at given time $t$ can be expressed as

$$\alpha(t) = \sup\{\alpha : \alpha = \lim_{\Delta t \to 0} \frac{\ln |Z(t + \Delta t) - Z(t)|}{\ln \Delta t}\}.$$  

(5)

The Hölder exponent of a signal at a particular point $t_0$ characterizes the signal variability locally. In the context of network traffic, such local variations are measured from traffic rate processes, expressed in terms of the number of bits, bytes, or packets crossing the network in a time interval, say $(t_0; t_0 + \Delta t]$. A network traffic is said to have a local multiscaling behavior with Hölder exponent $\alpha(t_0)$ at time $t_0$ if the process rate behaves according to $(\Delta t)^{\alpha(t_0)}$ for $\Delta t \to 0$.

Concerning the traffic behavior, the value of the exponent $\alpha(t_0)$ indicates the degree of traffic’s burstiness in the neighborhood of $t_0$. In other words, $\alpha(t_0)$ measures the degree of variation in the observed traffic’s intensity. Equation (5) gives rise to an estimator of Hölder exponents as defined below.

Definition 3. Let process $Z$ be a function with the $[0, T]$ support interval. Iteratively the interval $[0, T]$ is divided into $b^k$ equally sized pieces where $(b \in \mathbb{N}^*)$ and $k$ denotes the stage number of this sequence division process. Coarse Hölder exponents are defined through evaluating the local measurement of $|Z(t_i + b^{-k}T) - Z(t_i)|$ over the corresponding subinterval piece, as follows

$$\alpha_k(t_i) \equiv \frac{\ln |Z(t_i + b^{-k}T) - Z(t_i)|}{\ln b^{-k}}.$$  

(6)

Based on the Hölder exponent estimates from process sample paths, it is possible to judge whether a process is monofractal or multifractal. The second definition of multifractals based on the spectrum of Hölder exponents is the consequence of the next definition.

Definition 4. Let $N_k(\alpha)$ be the number of coarse Hölder exponents $\alpha_k(t_i)$ valued at the interval $(\bar{\alpha}_j, \bar{\alpha}_j + \Delta\alpha]$. The coarse graining spectrum, denoted by $f(\alpha)$, is given by

$$f(\alpha) \equiv \lim_{k \to \infty} \left\{ \frac{\ln N_k(\alpha)}{\ln b^k} \right\}.$$  

(7)

If $f(\alpha)$ is well defined (i.e., the above limit exists), positive on a support set larger than a single point, we say that the process $Z(t)$ is multifractal. On the other hand, if $f(\alpha)$ is only defined on a single point support set (i.e., a single valued Hölder exponent) the process is classified as monofractal [16].
The coarse graining spectrum is also called the large deviation spectrum. Besides the coarse graining spectrum, there are two other important multifractal spectrum types: the Hausdorff spectrum and the Legendre spectrum [9]. The Legendre spectrum, a concave approximation of the coarse graining spectrum is attractive because it estimates the multifractal spectrum in a robust and easy way. Such good quality is the consequence of its simple and direct definition given below.

**Definition 5.** Let $\tau(q)$ be the partition function of the process $Z(t)$. The Legendre spectrum of $Z(t)$ is defined by

$$f_L(\alpha) := \tau^*(\alpha),$$

where $\tau^*(q)$ is the Legendre transform of the partition function $\tau(q)$, i.e., $\tau^*(\alpha) = \inf_{q}(q\alpha - \tau(q))$.

### 3. Multifractal traffic modeling

The complex variations of regularity verified in some real data traffic flows have motivated the development of more sophisticated stochastic process models. Peltier and Véhel [28] extended the formulation of fBm by proposing a process named multifractional Brownian motion (mBm) with much richer regularity properties. In order to take advantage of the regularity information offered by the mBm in traffic modeling, in this section we present an extension of the fractional Brownian traffic, i.e., the extended fractional Brownian traffic model.

#### 3.1. The multifractional Brownian motion (mBm)

The fBm (fractional Brownian motion) is a self-similar model convenient for characterizing irregular signals with bursts and long-range dependence properties through its Hurst parameter $H$. Under such a modeling structure, $H$ is assumed constant in time; therefore, certain process regularities are maintained unchanged through the time. Different from the fBm, the regularity in mBm is allowed to vary along their sample paths. This additional characteristic of mBm is extremely useful in modeling of Internet traffic that presents varying regularities at different instants.

**Definition 6.** Let $(X, d_X)$ and $(Y, d_Y)$ be two metric spaces. A function $f : X \rightarrow Y$ is called Hölder function with exponent $\beta > 0$, if for each pair of input elements $x, y \in X$ such that $d_X(x, y) < 1$, we have

$$d_Y(f(x), f(y)) \leq c \cdot d_X(x, y)^\beta$$

for some constant $c > 0$.

**Definition 7.** Let $H : [0, \infty) \rightarrow [a, b] \subset (0, 1)$ be a Hölder function with exponent $\beta > 0$. For $t \geq 0$, the following random function $W$ is a multifractional Brownian motion with function parameter $H(t)$:

$$W_{H(t)}(t) = \int_{-\infty}^{0} [t-s]^{H(t)-1/2} - (-s)^{H(t)-1/2} \, dB(s) + \int_{0}^{t} [t-s]^{H(t)-1/2} \, dB(s),$$

where $B$ is a Brownian motion.

**Definition 8.** The mBm process $W(t)$, $t \geq 0$, is called standard multifractional Brownian motion if the following property is valid:

$$\text{var} \left( \frac{W_{t+h} - W_t}{h^{H(t)}} \right) \underset{h \to 0}{\longrightarrow} 1,$$

where $H(t)$ is the Hölder function of process $W(t)$. 
Although the mBm is a Gaussian process, generally its incremental process is not stationary as for the fBm. In the mBm modeling, the Hölder exponent $\alpha(t_0)$ at each point $t_0 \geq 0$ is given by the Hölder function at that point, or equivalently, $\alpha(t_0) = H(t_0)$. In order to illustrate the influence of the Hölder exponent on the behavior (regularity) of a mBm process, Fig. 1 exhibits the synthesized samples of a mBm process with only two distinct Hölder exponents (0.2 and 0.8); that is, a step Hölder function. Notice that we adopt the estimation method proposed by Benassi et al. [3] for the Hölder exponent estimation.

3.2. The extended fractional Brownian traffic model

As mentioned before, the fBm possesses unchangeable regularity index $H$ which limits its capacity in representing data sets with more complex regularity properties. In order to overcome this drawback, in this work we introduce a novel network traffic model, namely the extended fractional Brownian traffic model. In a previous symposium paper [4], we start to study the modeling capabilities of the proposed model. In this paper, we present simulation tests to verify other modeling characteristics of the proposed model such as the queueing behavior and second-order characteristics of its synthetic traces, leading to new conclusions. Besides, we enhance the analysis of the proposed traffic predictor and the comparison to other adaptive predictors.

Precisely, the extended fractional Brownian traffic (efBt) model is defined as follows.

**Proposition 1.** The extended fractional Brownian traffic $E_t$ is a process that describes the accumulated traffic volume up to time $t$ by the following equation:

$$E_t = mt + \sigma W_t, \quad t \in IR,$$

where $m$ is the mean traffic rate, $\sigma$ is the standard deviation of traffic rate, observed in a unit time interval, and $(W_t)$ is a multifractional Brownian motion with Hölder function $H(t) \in (0, 1)$. The extended fBt incremental process is given by $\Delta E_t = E_{t+1} - E_t$.

The traditional fractional Brownian traffic (fBt) model describes the accumulated traffic volume using the fBm process [4,26]. The extended fractional Brownian traffic model is a generalization of the fractional Brownian traffic model...
model once the mBm is a generalized version of fBm. Clearly the extended fractional Brownian traffic model is less parsimonious than the old one due to a more complex time dependent Hölder function. However, the proposed traffic model has the advantage of being able to match not only the second-order statistics (long time correlation) but also the multifractal characteristics of the traffic process.

Notice that in this paper we evaluate the modeling performance of the extended fBt in describing the incremental traffic process (byte arrivals), instead of the accumulated one (envelope process). More precisely, we synthesize incremental traffic processes under the extended fBt modeling through Eq. (12).

4. Traffic prediction based on the extended fractional Brownian traffic model

Self-similarity in network traffic and its impacts on network performance have triggered great interest in new research direction in traffic prediction involving this fractal information [13,33]. Gripenberg and Norros [14] presented an optimum continuous time predictor for fBm’s and stated that including any self-similarity information does not improve the quality of prediction, but, it only guarantees that the prediction performance is similar at any time resolution. In this paper, we demonstrate that a prediction algorithm can be enhanced by incorporating multifractal traffic properties information.

The design of a discrete filter predictor consists in finding the relation between the future sample \(x(n + \Delta - 1)\) and the past input signal samples \(x(n - 1), x(n - 2), \ldots, x(n - M)\), where \(M\) is the number of considered input elements and \(\Delta\) is the size of forward steps. The input–output predictor relationship can be described by the following convolution sum [15]:

\[
\hat{x}(n + \Delta - 1) = \sum_{k=1}^{M} h_o(k)x(n - k),
\]

where \(\{h_o(k), k = 1, 2, 3, \ldots, M\}\) is the filter coefficient vector, \(x(n - k)\) is an \(k\)-step backward sample, and \(\hat{x}(n + \Delta - 1)\) denotes the desired output.

4.1. A linear predictor based on multifractal characteristics

The traffic prediction algorithm proposed in this work assumes that network traffic can be modeled through the extended fractional Brownian traffic. In this regard, we use the Wiener theory and the correlation structure of the traffic model to obtain the predictor coefficients.

Let \(\text{cor}_X(t, s) = E[X(t), X(s)]\) be the autocorrelation function of a process \(X\) for different time instants \(t\) and \(s\). The autocorrelation function of the normalized extended fractional Brownian traffic process \(E(t)/\sigma\), denoted as \(\text{cor}_{E/\sigma}(t, s)\), is identical to the correlation \(\text{cor}_W(t, s)\) of the multifractional Brownian motion model, that is

\[
\text{cor}_{E/\sigma}(t, s) = \text{cor}_W(t, s).
\]

Ayache and Véhel in [2] explicitly presented the correlation structure of the mBm. Let \(W(t)\) be a standard mBm process with Hölder function \(H(t)\). The autocorrelation of the mBm process is given by

\[
\text{cor}_W(t, s) = D(H(t), H(s))(t^{H(t)+H(s)} + s^{H(t)+H(s)} - |t - s|^{H(t)+H(s)}),
\]

where

\[
D(x, y) = \frac{\sqrt{\Gamma(2x + 1)\Gamma(2y + 1)\sin(\pi x)\sin(\pi y)}}{2\Gamma(x + y + 1)\sin(\pi(x + y)/2)}
\]
with \( \Gamma \) denoting the gamma function.

Since our goal is to predict the traffic volume observed in time intervals \([t_i, t_i + 1], i \in Z\), it is convenient that we focus on the incremental process of the extended fractional Brownian traffic model.

From the autocorrelation of the mBm, we can derive the autocorrelation of its incremental process, \( Y(t) = W(t+1) - W(t) \). After some mathematical manipulations, one can show that the autocorrelation of the incremental process \( Y(t) \) is related to the accumulated process \( W(t) \) by the following equation:

\[
\text{cor}_Y(t, s) = \text{cor}_W(t + 1, s + 1) - \text{cor}_W(t + 1, s) - \text{cor}_W(t, s + 1) + \text{cor}_W(t, s).
\]  (17)

Except for the cases of traffic processes with constant \( H(t) \) functions, both the mBm incremental process and consequently the extended fBm incremental process are nonstationary. In order to match the statistical variations of nonstationary processes, adaptive filter versions are most recommended for random process prediction. The proposed multifractal-behavior-based linear predictor assumes that the traffic volume in time interval \([t_i, t_i + 1]\) can be modeled by the incremental process of the extended fractional Brownian traffic. In this case, the filter predictor coefficients are estimated from a discrete time Wiener–Hopf equation system that combines the process’s Hölder function \( H(t) \) with the correlation structure of mBm. That is, the optimum filter coefficient vector can be obtained by solving the Wiener–Hopf equation given by [15]:

\[
\sum_{m=1}^{M} h_o(m) r_x(k - m) = r_{dx}(k), \quad k = 1, 2, 3, \ldots, M,
\]  (18)

where \( r_x(k - m) \) is the autocorrelation of the filter input sequence defined by

\[
r_x(k - m) = E[x(n - m)x(n - k)], \quad k, m = 1, 2, 3, \ldots, M,
\]  (19)

and \( r_{dx}(k) \) is the cross correlation between the desired value \( x(n + \Delta - 1) \) and the filter input, as given below:

\[
r_{dx}(k) = E[x(n + \Delta - 1)x(n - k)], \quad k = 1, 2, 3, \ldots, M.
\]  (20)

The proposed prediction algorithm consists of calculating the autocorrelations (Eqs (19) and (20)) through the correlation structure of the extended fractional Brownian traffic model (Eq. (17)). Once the coefficient vector of the filter is computed, the algorithm output is given by Eq. (13).

5. Experimental evaluation of the extended fractional Brownian traffic model

In this section we present the results of our experimental investigation. The main purpose of this experimental investigation is to evaluate the capacity of the extended fractional Brownian traffic model in representing the behavior of real network traffic.

For the simulation purposes, we selected some real TCP/IP traffic traces from the Digital Equipment Corporation (DEC) that were extensively used in other previously reported research works [7,8,30], as well as traffic traces collected at the Petrobrás network between the years 2000 and 2003 [29]. The TCP/IP traffic traces collected at the DEC originally consist of IP packets transmitted between the DEC and the rest of the world [18].

The fBm lower cut-off timescale can be interpreted as a time scale boundary between the applicability of self-similar and multifractal modeling, although some studies have shown that the multifractal characteristics also appear in time scales far above the fBm lower cut-off timescale [24]. The fBm lower cut-off timescale is in fact empirically observed which is related to the order of RTT (round-trip-time). In this work, the chosen time scales were 512 and 100 ms for the DEC and Petrobrás traffic traces, respectively. The reason for these choices is due to the fact that for the DEC traces, the observed RTT is approximately of 512 ms [8]. Moreover, the Petrobrás network traffic traces present multifractal characteristics in the 100 ms time scale [29].
5.1. Hurst parameter estimation

The second-order statistics of a process can be globally described through the Hurst parameter. The experimental investigation reported in this section aims at evaluating the capacity of the extended fractional Brownian traffic model in capturing the second-order characteristics of the WAN TCP/IP traffic, described by the Hurst parameter [5].

There are several estimation methods of the Hurst parameter, among them including Whittle estimator [6], Higuchi method [16], method of the variance [7], statistics R/S [23], and multiresolution analysis in the wavelet domain [1]. The multiresolution analysis based estimator proposed by Abry and Veitch [1] was used in this work to evaluate the efficiency of the extended fractional Brownian model in capturing the Hurst parameter of the considered process series.

Table 1 exhibits the estimated values and the 95% confidence intervals for the Hurst parameters of the TCP/IP traffic traces as well as those of the synthetic traffic traces generated by the proposed traffic model (Eq. (12)). For all considered traffic traces, the 95% confidence intervals of the Hurst parameter estimation for the real and synthetic traffic traces overlap.

5.2. The multifractal characteristic

The multifractal characteristic of a given trace can be verified through either the curvature analysis of the partition function $\tau(q)$ or its multifractal spectrum. In this work, the Legendre multifractal spectrum is used to estimate the multifractal characteristic of traffic traces. Figure 2 shows the multifractal spectra obtained from the TCP/IP traffic traces (dec-pkt-1.tcp, dec-pkt-2.tcp, respectively) as well as the corresponding spectra obtained from their

<table>
<thead>
<tr>
<th>Estimated value</th>
<th>95% confidence interval</th>
<th>Estimated values</th>
<th>95% confidence interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>dec-pkt-1.tcp</td>
<td>0.835</td>
<td>(0.798–0.872)</td>
<td>0.880</td>
</tr>
<tr>
<td>dec-pkt-2.tcp</td>
<td>0.704</td>
<td>(0.667–0.741)</td>
<td>0.786</td>
</tr>
<tr>
<td>dec-pkt-3.tcp</td>
<td>0.806</td>
<td>(0.769–0.843)</td>
<td>0.852</td>
</tr>
</tbody>
</table>

![Fig. 2. Legendre spectrum for real and synthetic traffic traces; (a) dec-pkt-1 traffic trace; (b) dec-pkt-2 traffic trace.](image-url)
respective extended fractional Brownian traffic synthesis samples. Through these figures we observe that the synthetic traffic traces tend to possess a slightly larger range of Hölder exponents values. This fact is more clearly seen through the multifractal spectrum of the deck-pkt-1 traffic trace that presents a higher level of burstiness than the dec-pkt-2 traffic trace. This observation indicates that the proposed model can match complex traffic behavior such as the multifractal characteristics of real TCP/IP traffic traces.

5.3. Queueing behavior analysis

In this section we evaluate the buffer occupation of a single-server queueing system fed by synthetic traffic traces generated by the Extended Fractional Brownian Traffic model. In order to evaluate the advantage of using more accurate traffic modeling, we also verified the buffer occupation and byte loss rate for the fractional Brownian traffic model (fBt) as traffic input.

We considered as performance measures for the queueing behavior analysis, the mean queue length and the byte loss rate. These performance measures are related to the queueing server utilization defined as

$$\rho = \frac{C_R}{C_T}, \quad (21)$$

where $C_T$ is the maximum capacity (in bytes) possibly served during the whole simulation time and $C_R$ is the traffic volume indeed served. Figure 3 shows the mean queueing length (or mean buffer occupation) versus the server utilization for real TCP/IP and synthetic traces. Although for high queueing server utilization, the mean queueing lengths for the fBt traffic traces tend to be close to those of the efBt traffic, most of the mean queueing length values for the real traffic traces are closer to those given by our approach (Fig. 3(a)). It is also noticed that for server utilization rate above 50% computed using Eq. (21), the mean queueing length grows quite fast as can be seen by Fig. 3(b) for the dec-pkt-3 traffic trace. A reason for such a behavior is the high degree of burstiness (multifractal characteristic) of the traffic traces considered in this work.

The second considered performance parameter is the queueing system’s byte loss behavior. Figures 4 and 5 exhibit the byte loss percentile versus the buffer size for the traffic types (real traffic, synthetic fBt and synthetic extended fBt). It can be observed that the synthetic traffic traces produces a closer queueing behavior for the 10-7-S-10 traffic trace than for the dec-pkt-2. A better match of the real traffic traces by the proposed model is achieved when the traffic process tends to be Gaussian as well as when its Hölder exponent set is precisely captured. Mostly important, the simulation results confirm our finding that a multifractal model can better describe real TCP/IP traffic behavior than a monofractal approach.

![Fig. 3. Mean queueing length; (a) (square) dec-pkt-2.tcp, (circle) proposed model synthesis, (cross) fBt model synthesis; (b) (square) dec-pkt-3.tcp, (circle) proposed model synthesis, (cross) fBt model synthesis.](image-url)
6. Analysis of the proposed traffic predictor

In this section we present comparative evaluations between the proposed predictor and other important predictors existing in the literature. The performance of predicting TCP/IP traffic traces is evaluated through the commonly used mean square error concept.

**Definition 9.** Let \( \hat{x} \) denote the predicted value of process \( X \), and \( x \) the desired sample value. The mean square error (MSE) is the mean value of predicted value deviations with respect to its desired value, defined as

\[
MSE = E[(\hat{x} - x)^2].
\] (22)
The mean square error is an absolute error measure; therefore, it is highly influenced by the amplitude of the predicted trace. In order to eliminate this undesirable effect without losing its discriminating capability, a relative error measure is highly recommended. More precisely, the predictors were evaluated by two relative error measures, both belonging to the normalized mean square error (NMSE) type. The first NMSE type consists in normalizing MSE with respect to the predicted trace variance, while the second one measures the normalized MSE in relation to the mean square error optimal predictor of random walk, i.e., the simplified predictor described in Definition 11. The definitions of these NMSE are given below.

**Definition 10.** Let $\sigma^2_x$ be the variance of the samples of the random process $X$, given by $\sigma^2_x = E[(\mu - x)^2]$, where $\mu$ is the mean value of the process. The type-1 mean square error (NMSE1) is defined as

$$NMSE_1 = \frac{NMSE}{\sigma^2_x} = \frac{E[(\hat{x} - x)^2]}{E[(\mu - x)^2]}.$$  

(23)

Notice that a predictor that presents an $NMSE_1$ value less than or equal to 1 offers a prediction performance superior or equal to a predictor that simply takes the past sample average as the estimate for future process value.

**Definition 11.** Let $\hat{x}_{pa}$ be the predicted value of the process $X$, given by the simplified predictor $\hat{x}(n + 1) = x(n)$. The type-2 normalized mean square error (NMSE2) is defined as

$$NMSE_2 = \frac{E[(\hat{x} - x)^2]}{E[(\hat{x}_{pa} - x)^2]}.$$  

(24)

For an $NMSE_2$ measure close to 1, it means that the corresponding predictor offers a prediction performance close to that given by the simplified predictor.

### 6.1. Description of the compared traffic predictors

In this section, we briefly describe the other considered traffic predictors.

- **Wiener predictor.**
  The Wiener predictor coefficients are determined based on autocorrelation of the filter input and the cross-correlation between the desired output and the input values [15].

- **Self-similar Wiener predictor.**
  The self-similar Wiener predictor was suggested by Hirchoren and Arantes [17] which is an optimal linear predictor for a fGn process in discrete time. The estimations of the correlation for the input sample sequence, of the cross-correlation between the input sample series and the desired input series are analytically obtained based on the autocorrelation function of the fGn process which is a function of the estimated Hurst parameter. In other words, once assuming the incremental process of the fBt (fractional Brownian traffic) as a valid traffic model, it is possible to obtain a linear time invariant predictor using the fGn correlation structure and the estimated Hurst parameter.

- **Adaptive LMS predictor.**
  The design of a Wiener filter is basically defined by the Wiener–Hopf equation (Eq. (18)). When the network environment is unknown or traffic statistic information is not clearly defined, an adaptive filter should be considered as an alternative to the Wiener filter design, capable of recursively learning the environment conditions and adjusting filter coefficients.
The least mean-square (LMS) algorithm is an adaptive implementation of the Wiener filter, providing a recursive solution for the ‘normal equations’ (Wiener–Hopf equation). The filter coefficients vary recursively in time according to the following equation [15]:

\[
\hat{h}(k, n + 1) = \hat{h}(k, n) + \mu \cdot e(n) x(n - k), \quad k = 1, 2, \ldots, M,
\]

where \( \hat{h}(k, n) \) corresponds to the filter coefficient estimates, \( \mu \in \mathbb{R}_+^\ast \) is an adaptation parameter, \( e(n) = d(n) - y(n) \) is the error signal, \( d(n) \) is the desired signal, \( y(n) \) is the filter’s output and \( x(n - k) \) denotes the filter input values.

In this work, all filter coefficients at time \( n = 0 \) are set to zero as the initial condition. Besides, the value of the adaptation parameter \( \mu \) is chosen in order to guarantee the convergence and best performance of the LMS algorithm.

### 6.2. Prediction results for real traffic traces

The real traffic traces used in the predictor evaluation are the same previously mentioned: TCP/IP traces (aggregation time scales of 512 ms) and Petrobrás traffic traces (aggregation time scales of 100 ms). All predictors considered in this work have 10 input elements and are applied to one-step ahead prediction tasks. Due to Eq. (14), we divide the intensity samples of the traffic traces by the process standard deviation in order to compute the required mBm autocorrelation function of our prediction algorithm.

Figure 6(a) exhibits the estimated Hölder function for the first 2048 traffic samples of the TCP/IP dec-pkt-1 trace under 512 ms time scale aggregation. Figure 6(b) compares the input traffic trace and the predicted traffic trace (generated by the proposed traffic predictor) on the time interval between the 650th and 700th samples. Notice that the Hölder function \( H(t) \) in Fig. 6(a) has the mean value equal to 0.80 and oscillates inside the interval \([0.62, 0.95]\). In addition, Fig. 6(b) visually demonstrates that the proposed predictor is capable of tracking the input traffic trace in spite of its variations. The same analysis was done for other traffic traces and traffic predictors. The results of the performance evaluation analysis of the considered prediction algorithms are summarized in Tables 2 and 3 for the dec-pkt-1 and Petrobrás 10-7-S-10 traffic trace, respectively.

In fact, the proposed prediction algorithm provides the best prediction performance (both the lowest NMSE1 and NMSE2) among the analyzed predictors, namely the proposed adaptive multifractal predictor, the Wiener Classic

![Fig. 6. (a) Hölder function obtained from the first 2048 dec-pkt-1 samples; (b) (square) dec-pkt-1 traffic trace, (circle) proposed predictor output signal.](image)
<table>
<thead>
<tr>
<th>Predictor</th>
<th>NMSE1</th>
<th>NMSE2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed predictor</td>
<td>0.5635</td>
<td>0.6431</td>
</tr>
<tr>
<td>Classic Wiener predictor</td>
<td>0.5711</td>
<td>0.6671</td>
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<tr>
<td>Self-similar Wiener predictor ($H_{Abry Veitch} = 0.704$)</td>
<td>0.6182</td>
<td>0.7220</td>
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<tr>
<td>Adaptive LMS predictor ($\mu = 5e-13$)</td>
<td>0.6154</td>
<td>0.7188</td>
</tr>
</tbody>
</table>

Table 3

<table>
<thead>
<tr>
<th>Predictor</th>
<th>NMSE1</th>
<th>NMSE2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed predictor</td>
<td>0.7704</td>
<td>0.6888</td>
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<tr>
<td>Classic Wiener predictor</td>
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<tr>
<td>Self-similar Wiener predictor ($H_{Abry Veitch} = 0.806$)</td>
<td>0.7830</td>
<td>0.7001</td>
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<tr>
<td>Adaptive LMS predictor ($\mu = 1e-12$)</td>
<td>0.9247</td>
<td>0.8268</td>
</tr>
</tbody>
</table>

7. Conclusion

Due to its relative simplicity, the fractional Brownian traffic model proposed in [25] was widely used for self-similar traffic modeling and synthesis [12]. Unfortunately, the fBm as a monofractal traffic model cannot faithfully describe many traffic traces generated by modern network applications. For instance, some studies have shown that sequences of MPEG-4 encoded video [19] and TCP/IP traffic [11,31] present statistical behavior that cannot be appropriately described by monofractal models; instead, they can be better represented via multifractal analysis.

In summary, this work presents two major contributions to the area of network traffic analysis. The first contribution is a traffic modeling proposal that involves traffic’s multifractal characteristics and enhances the capability of the classical fractional Brownian traffic model. The second innovation is a new traffic prediction approach based on the rich correlation structure of the extended fractional Brownian traffic model. Comparative analyses between the fractional Brownian traffic and its extended version have shown that the incorporation of multifractal characteristics offers better traffic modeling. As a consequence of this more accurate modeling, the prediction approach based on the extended fractional Brownian traffic model has presented the smallest prediction error in comparison to the other linear predictors used in the simulations.

One possible complement to this work is to make a comparative evaluation between the extended fractional Brownian traffic model and non-Gaussian traffic models. Regarding the proposed prediction approach, an interesting extension includes the application of the correlation structure of the extended fractional Brownian traffic in the design of other filter predictor types.

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References