MOBILE

RADIO

CHANNEL

2 Antenna Fundamentals

Basic Concepts

Isotropic Source: source radiating energy in all directions

Poyting Vector = Power Density: power / area

Radiated Power through Surface

$$W = \iint \vec{P} \cdot d\vec{s} = \iint_S P \cdot ds$$

∴ Isotropic Source

$$W = P \, 4\pi d^2$$

Free Space Transmission Formula

$$P = \frac{W}{4\pi d^2}$$

Radiation Intensity (Power / solid angle)

Isotropic $\quad U_0 = \frac{W}{4\pi} = d^2 P$

Radiation Pattern: Geographical distribution of power radiated by a source (azimuthal, elevation)

Isotropic Source: circle both planes

Real Antenna: directional properties

Omnidirectional: circle horizontal plane

Directivity

$$D = \frac{\text{maximum radiation intensity}}{\text{radiation intensity from isotropic source radiating same power}}$$

$$D = \frac{U_M}{U_0} = \frac{U_M}{W/4\pi} = \frac{4\pi}{W/U_M} = \frac{4\pi}{B}$$

$$B = \frac{W}{U_M} = \text{Beamwidths} = \frac{\text{Watts}}{\text{Watts/solidangle}} = \text{solidangle}$$

$$B = \frac{\iint P\,ds}{U_m} = \frac{\iint (U/d^2)\,ds}{U_m}$$

$$= \frac{\iint \frac{U}{d^2}\, d^2 \sin\theta\, d\theta\, d\phi}{U_m}$$

$$B = \frac{\iint U\, d\Omega}{U_m} \; ; \; d\Omega = \sin\theta\, d\theta\, d\phi$$

Example

Conical radiation pattern

$$W = \int_0^{2\pi}\int_0^{\theta} U_m \sin\theta'\, d\theta'\, d\phi = 2\pi\,(1-\cos\theta)\,U_m$$

$$D = \frac{U_m}{\frac{2\pi\,(1-\cos\theta)\,U_m}{4\pi}} = \frac{2}{1-\cos\theta}$$

$\theta = \pi \Rightarrow D = 1$; $\theta = \pi/2 \Rightarrow D = 2$

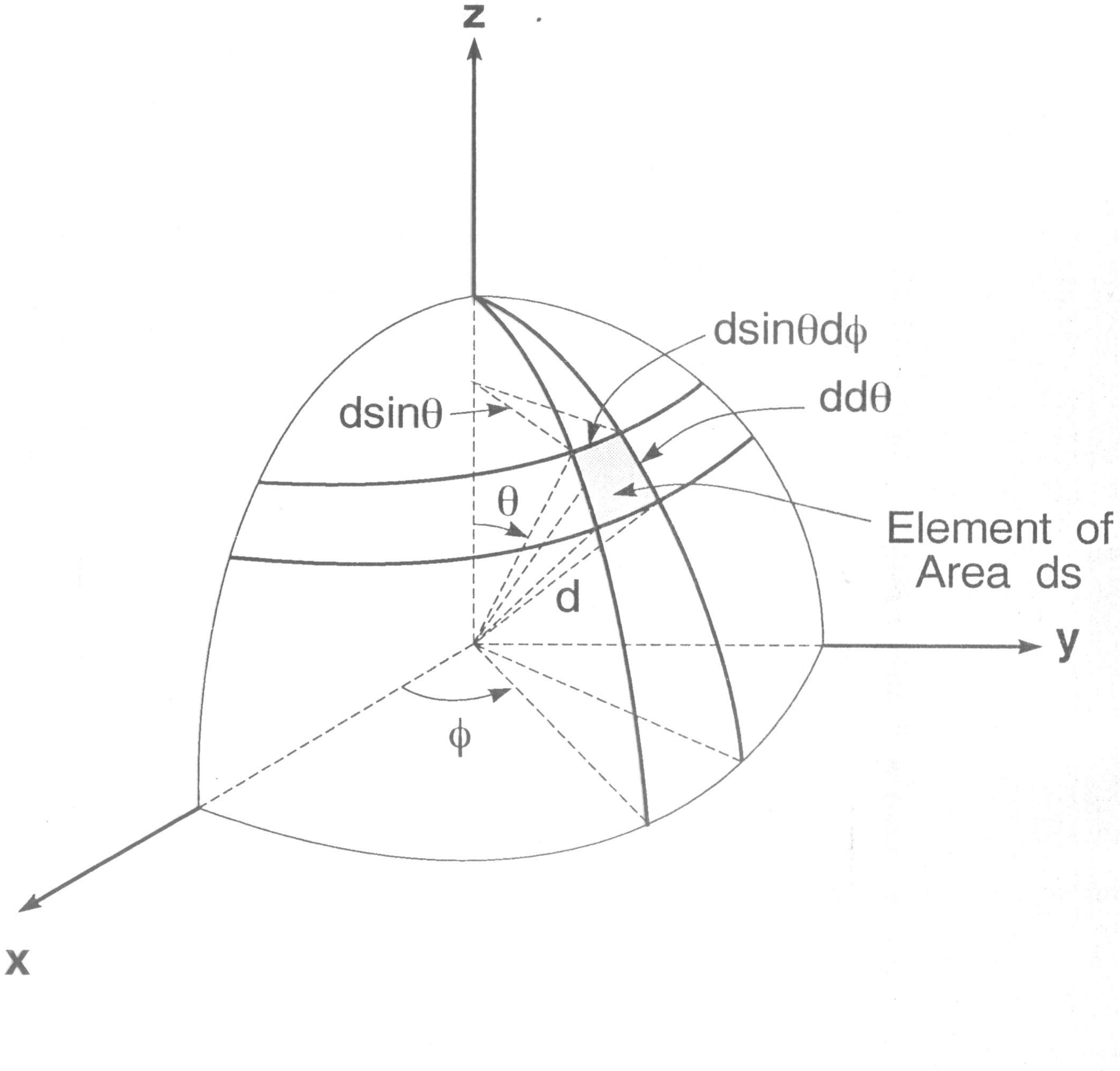


FIG3-1 200%

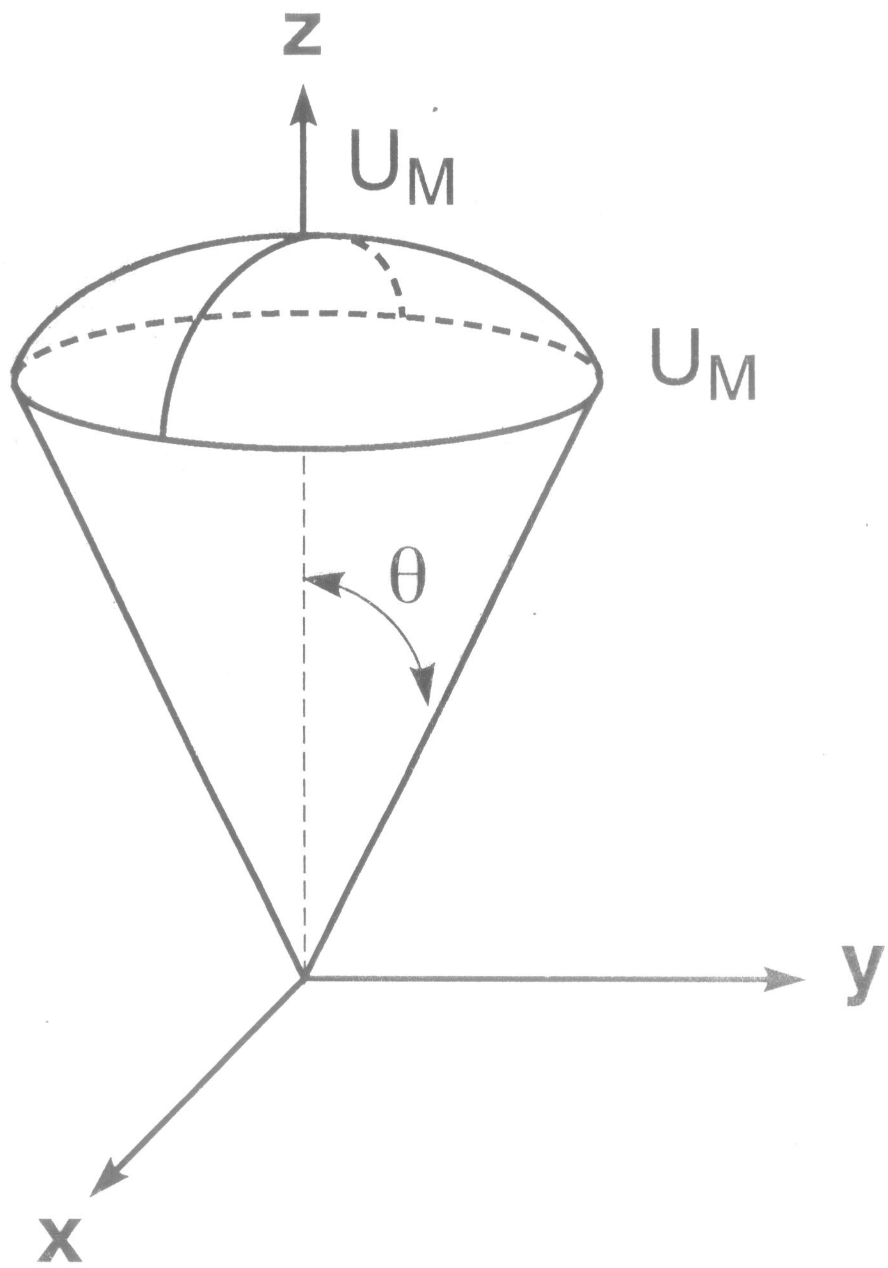


FIG3.2 300%

Approximation

$B \simeq \Delta\theta \, \Delta\varphi$

$\Delta\theta, \Delta\varphi$: half-power bandwidths (radians)

Example: $U = U_m \cos\theta$

$$W = \int_0^{2\pi} \int_{\theta=0}^{\pi/2} U_m \cos\theta \sin\theta \, d\theta \, d\varphi = \pi U_m$$

$$\boxed{D = 4} \qquad D = \frac{U_m}{W/4\pi} = \frac{U_m}{\frac{\pi U_m}{4\pi}} = 4$$

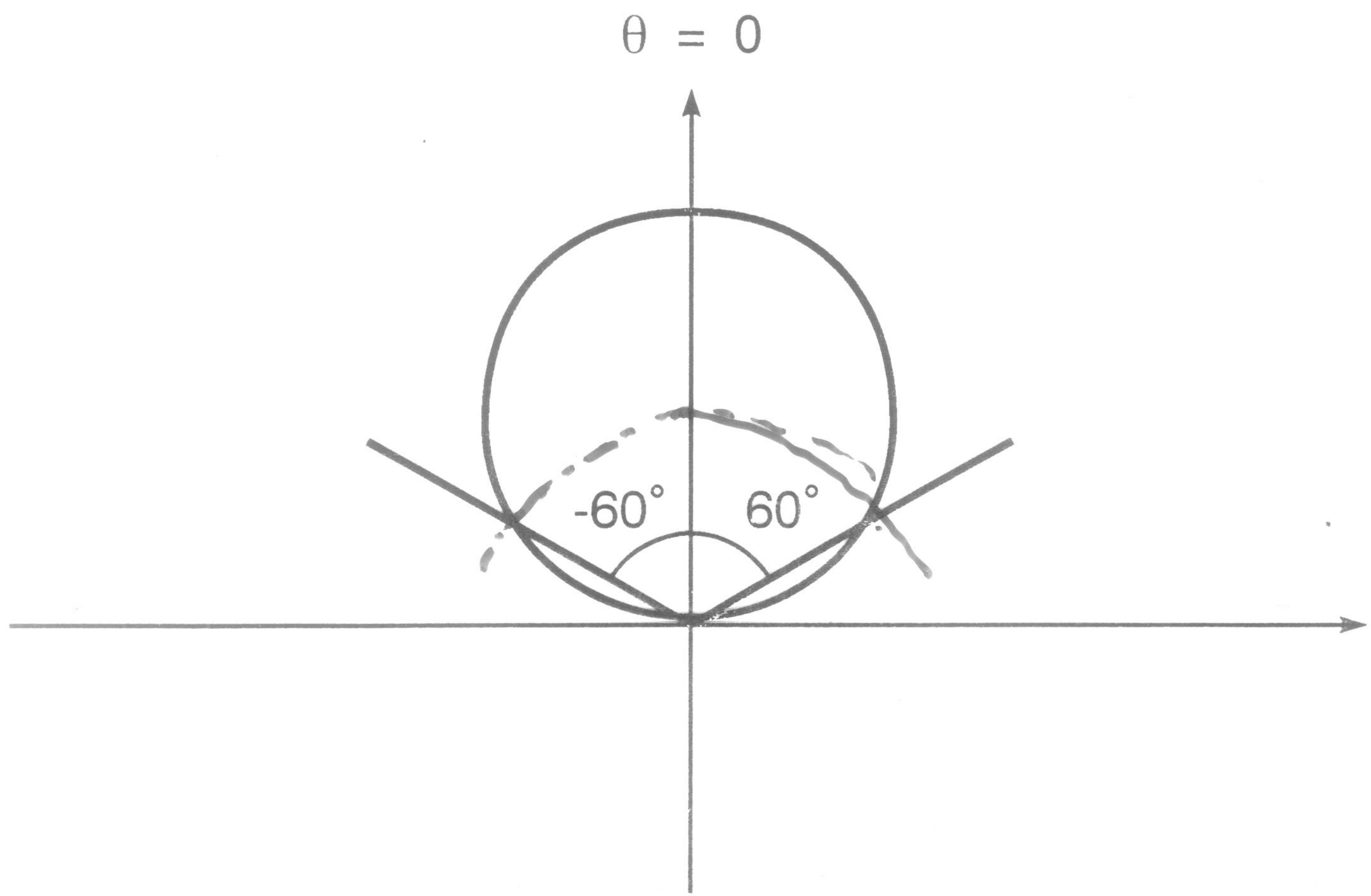
Half-power method

$$\Delta\theta = \Delta\varphi = \pi/3 - (-\pi/3) = 2\pi/3$$

$$D = \frac{4\pi}{B} = \frac{4\pi}{(2\pi/3)^2} = 2.87$$

Gain : antenna efficiency

$$G = \eta \frac{U_m}{U_0} = \eta D \quad 0 \leq \eta \leq 1$$



TIG3-3 200%

Aperture : antenna has terminating impedance + intrinsic impedance

terminating : load receiving power

intrinsic : losses (heat, reflection)

A : virtual area immersed in electro-magnetic field

$$A = \frac{\text{Power loss}}{\text{Power density}} \quad (m^2).$$

a) $\text{Effective Aperture} = \dfrac{\text{Power at term. imp.}}{\text{Power density}}$

b) $\text{Loss Aperture} = \dfrac{\text{Power dissipated as heat}}{\text{Power density}}$

c) scattering ~~Collecting~~ $\text{Apert.} = \dfrac{\text{Power reflected back}}{\text{Power density}}$

d) Collecting Apert = a+b+c

⑥

Power extracted from magnetic field increases with directivity

$D = K A$

$$\frac{D_i}{D_j} = \frac{A_i}{A_j}$$

$$\frac{\eta_j G_i}{\eta_i G_j} = \frac{D_i}{D_j} = \frac{A_i}{A_j}$$

Isotropic: $A = \lambda^2/4\pi$; $D = 1$

Short Dipole: $A = 3\lambda^2/8\pi$; $D = 3/2$

Linear $\frac{1}{2}\lambda$: $A = 30\lambda^2/73\pi$, $D = 1.64$

$$\therefore \boxed{A_i = \frac{\lambda^2}{4\pi} D_i}$$

Friis Free-Space Transmission Formula

Given W_t, G_t

At d

$$P = \frac{G_t W_t}{4\pi d^2}$$

But

$$W_r = A_r P$$

$$\therefore W_r = \frac{A_r G_t W_t}{4\pi d^2}$$

Note

$$A_r = \frac{\lambda^2}{4\pi} D_r$$

And

$$\frac{W_r}{W_t} = D_r G_t \left(\frac{\lambda}{4\pi d}\right)^2$$

Suppose $\eta = 1$

$$\boxed{\frac{W_r}{W_t} = G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2}$$

INTRODUCTION

PROPAGATION PATH LOSS

$$l \triangleq W_r/W_t$$

In dB

$$L = -10\log l = -10\log W_r + 10\log W_t$$

Free-Space Path Loss

$$\frac{W_r}{W_t} = G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2$$

$$L = -10\log G_t - 10\log G_r - 20\log\lambda + 20\log d + 21.98$$

Example:
f = 1 GHz
d = 10 km

$$L = 20\log f + 20\log d + 32.44 \text{ dB}$$

$\therefore L = 112.44$

$$(G_t = G_r = 1 \; ; \; f \text{ in MHz} \; ; \; d \text{ in km})$$

Plane Earth Path Loss

$$\frac{W_r}{W_t} = G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \left|1 + \rho e^{j\Delta\varphi} + (1-\rho)Ae^{j\Delta\varphi} + \dots\right|^2$$

GRAZING ANGLES: $\theta \simeq 0°, \rho \simeq -1$
$f > 100$ MHz and $\theta < 10°$ $\rho \simeq -1$

$$\rho = \frac{\sin\theta - K}{\sin\theta + K}$$

$$\frac{W_r}{W_t} \simeq G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \left|1 - e^{j\Delta\varphi}\right|^2$$

$\Delta\varphi = 2\pi f \Delta t = 2\pi \frac{c}{\lambda} \Delta t$

$$\Delta\varphi = 2\pi \frac{\Delta d}{\lambda}$$

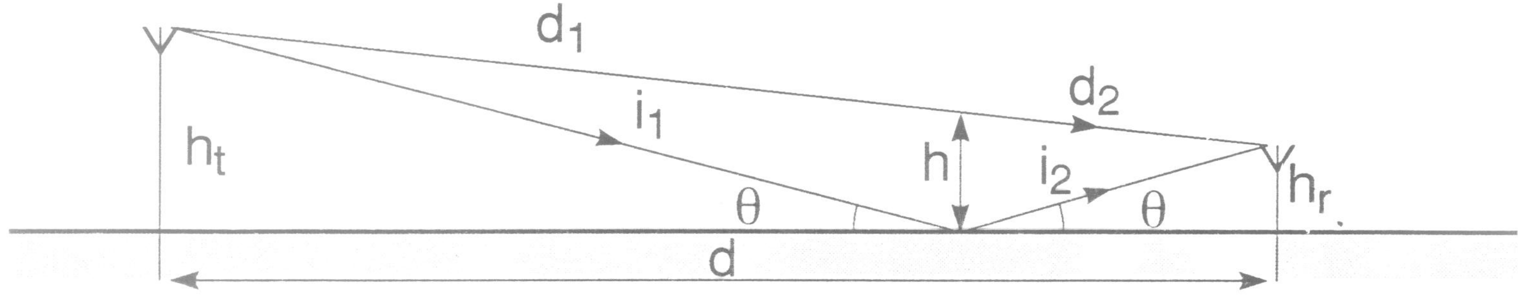


FIG3-4 200%

where

$$\Delta d = (i_1 + i_2) - (d_1 + d_2)$$

$$\Delta\varphi = \frac{2\pi d}{\lambda}\left\{\left[\left(\frac{h_t + h_r}{d}\right)^2 + 1\right]^{1/2} - \left[\left(\frac{h_t - h_r}{d}\right)^2 + 1\right]^{1/2}\right\}$$

Using the approximation $(1 + x)^{1/2} \simeq 1 + x/2$, for small x

For small $\Delta\varphi$, $\sin\frac{\Delta\varphi}{2} \simeq \frac{\Delta\varphi}{2}$. Then

Ex: $h_t = 30$ m
$h_r = 1.5$ m
$d = 10$ km
$\therefore L = 127$ dB

$$\sin^2\frac{\Delta\varphi}{2} \simeq \left(\frac{\Delta\varphi}{2}\right)^2$$

$$\boxed{\frac{W_r}{W_t} = G_tG_r\left(\frac{h_th_r}{d^2}\right)^2}$$

$$L = -10\log G_t - 10\log G_r - 20\log(h_th_r) + 40\log d$$

Rayleigh criterion for smoothness

$$S = \frac{4\pi\sigma\theta}{\lambda},\ S < 0.1, \text{ then smooth; } S > 10 \text{ then routh}$$

Knife Edge Diffraction Loss

$$E = E_0Fe^{j\Delta\varphi}$$

$$F = \frac{S(x) + 0.5}{\sqrt{2}\,\sin(\Delta\varphi + \pi/4)}$$

$$\Delta\varphi = \tan^{-1}\left[\frac{S(x) + 0.5}{C(x) + 0.5}\right] - \frac{\pi}{4}$$

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}u^2\right)du$$

$$S(x) = \int_0^x \sin\left(\frac{\pi}{2}u^2\right) du$$

$$x = -h\sqrt{\frac{2}{\lambda}\left(\frac{d_1 + d_2}{d_1 d_2}\right)}$$

$$L = 10\log|E/E_0|^2 = 20\log F$$

Multiple Knife Edge Diffraction Loss

a) *Bullington's Model* (two-obstuctions)

b) *Epstein - Peterson's Model* (consecutive obstructions)

c) *Deygout's Model* (Dominant obstruction)

Path Clearance Conditions

Loss Due to Reflection/Refraction

$$h\sqrt{\frac{2}{\lambda}\left(\frac{d_1 + d_2}{d_1 d_2}\right)} > \sqrt{2}$$

Therefore,

$$h > \sqrt{\lambda\left(\frac{d_1 d_2}{d_1 + d_2}\right)}$$

$$d < 4\, h_t h_r/\lambda$$

Avoiding Nulls at the Reception

$$d = \frac{2h_t h_r}{n\lambda}$$

$$d > \frac{2h_t h_r}{\lambda}$$

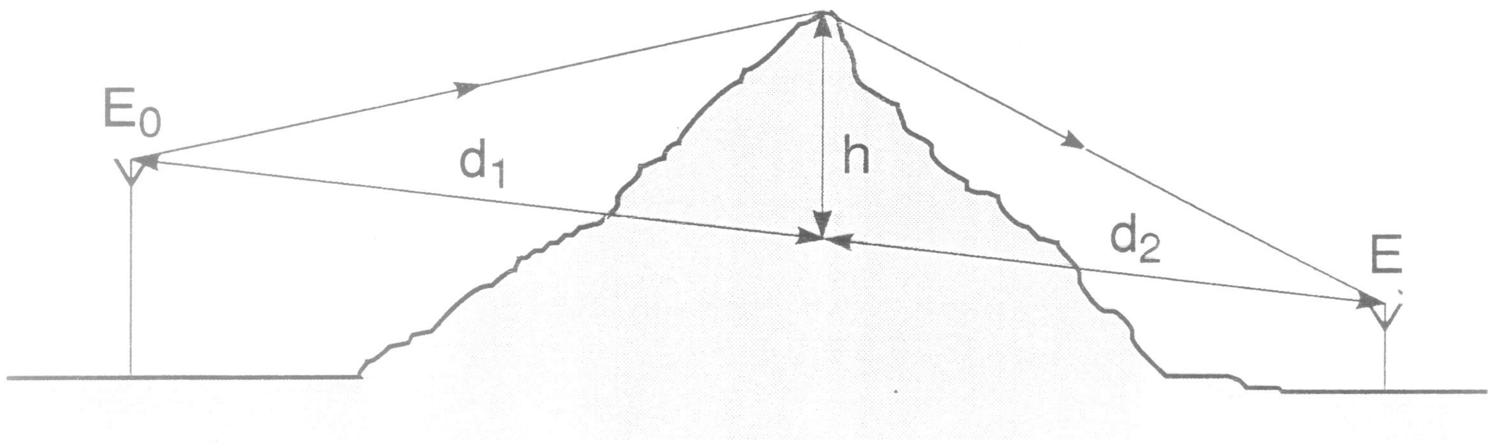


FIG3-5 200%

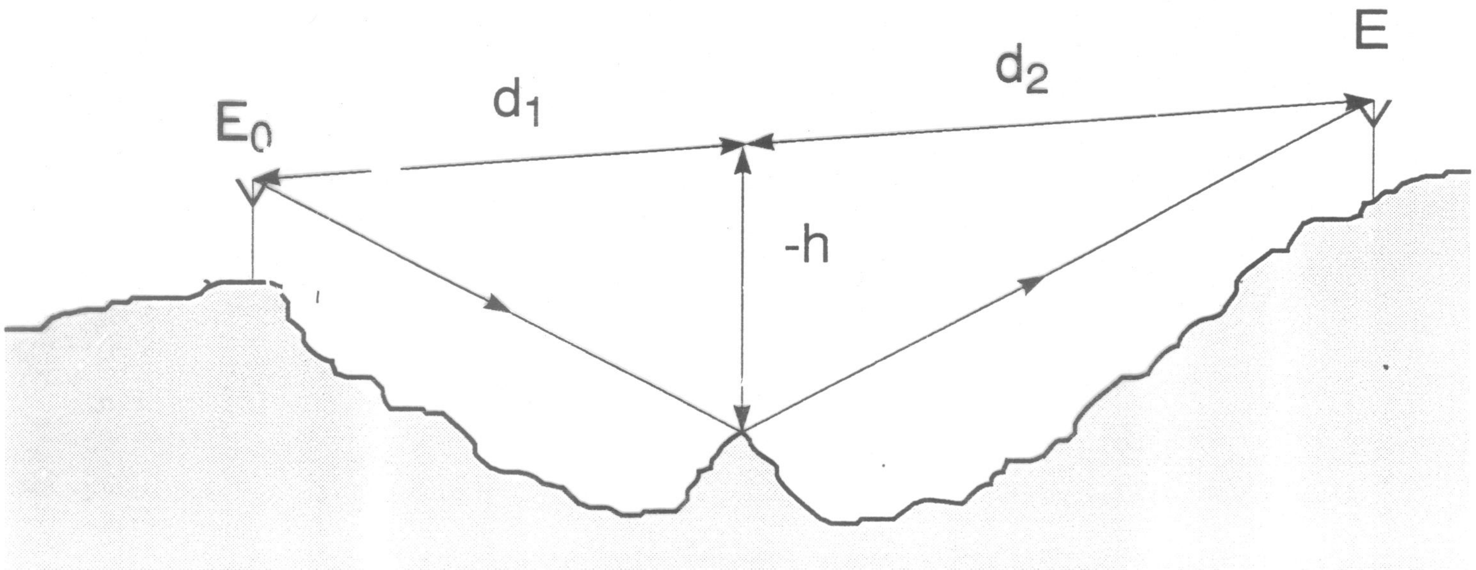
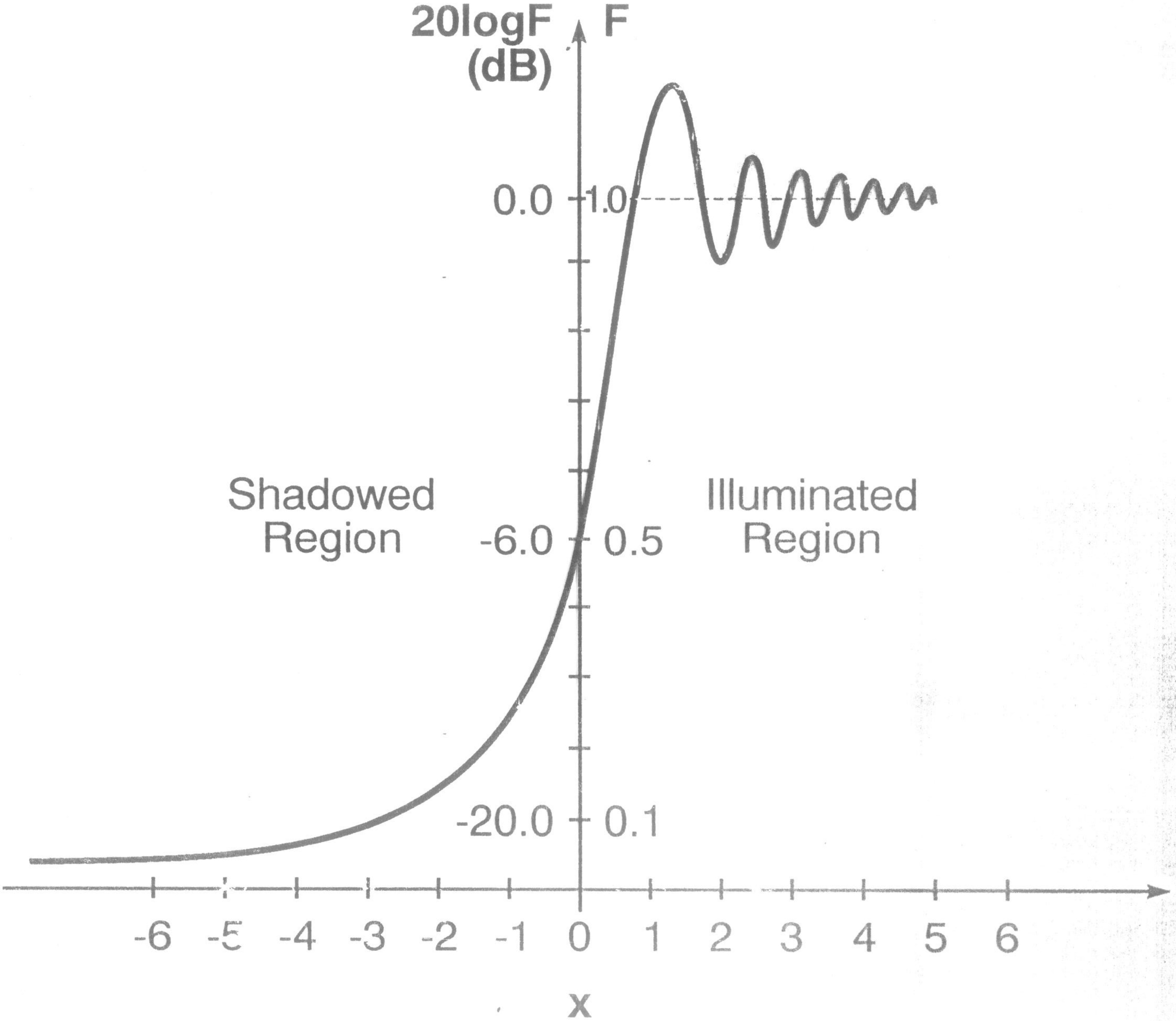


FIG3-6 200%



20logF (dB)
F
0.0
1.0
-6.0
0.5
-20.0
0.1
Shadowed Region
Illuminated Region
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6
x

FIG3 7 200%

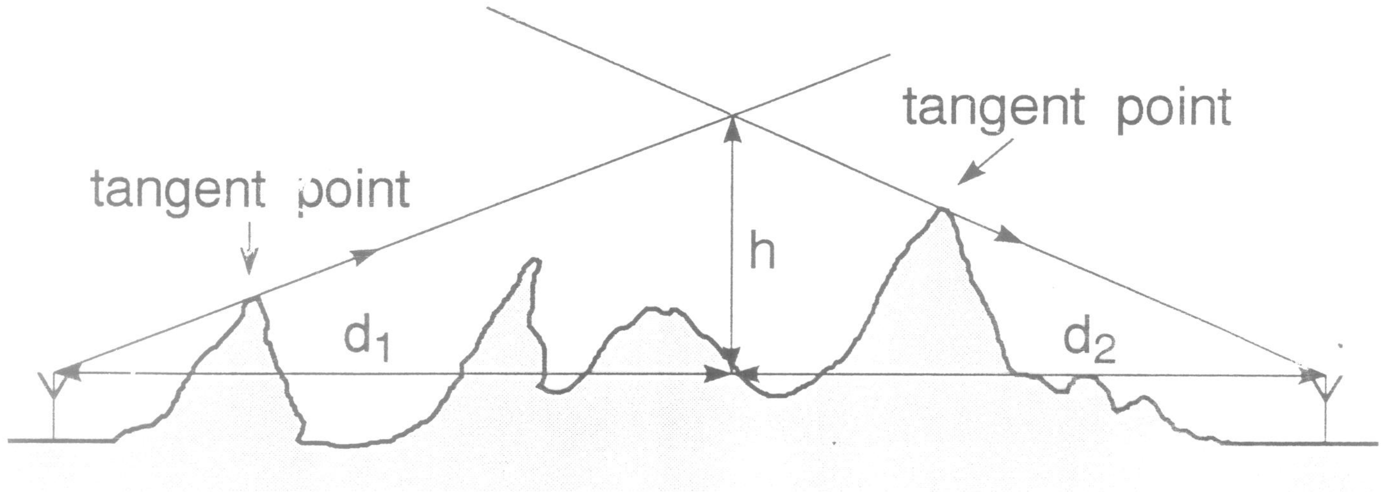


FIG3-8 200%

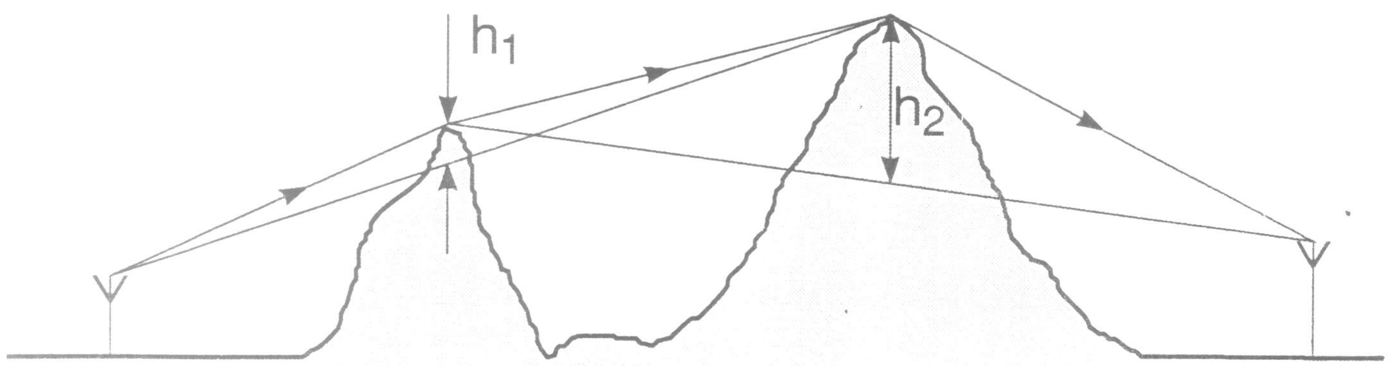


FIG3-9 200%

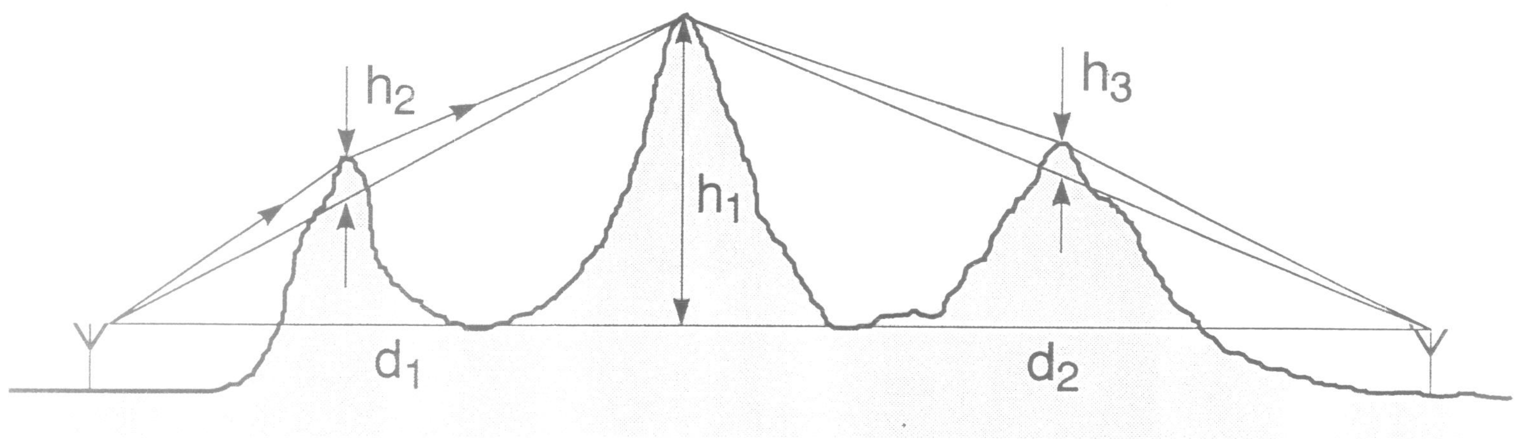
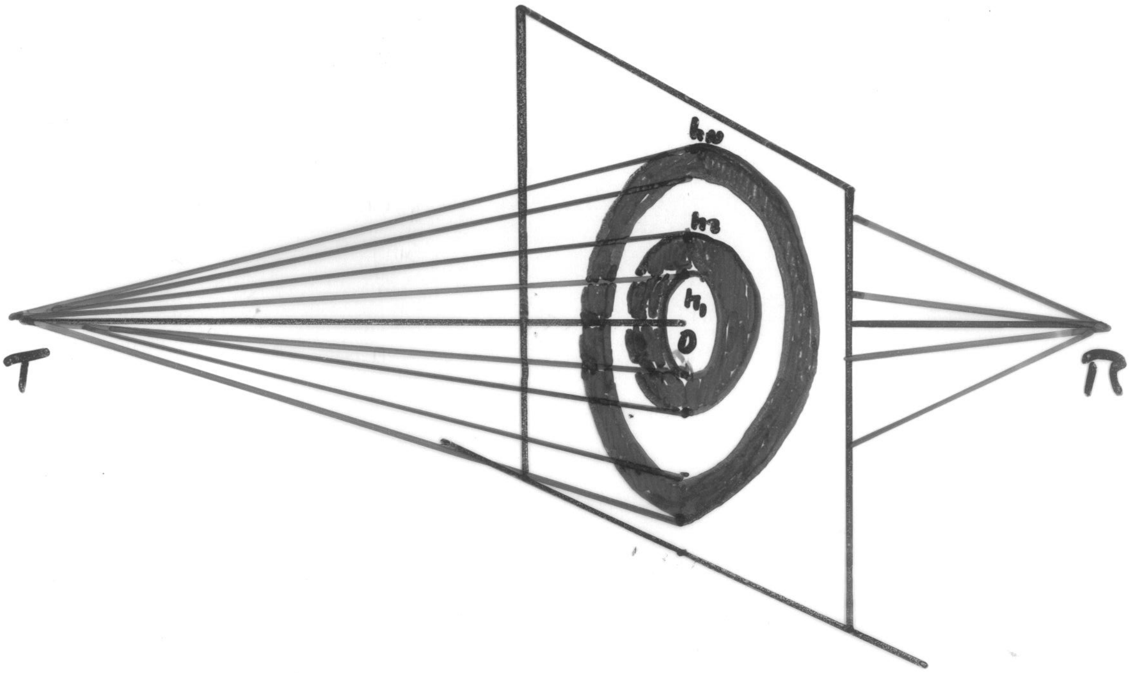


FIG3-10 200%



$$\frac{r_1 + r_2}{d_1 + d_2} = \frac{n\lambda}{2} \quad , n = 1, 2, \ldots$$

3.3.5.1 Loss Due to Diffraction

A measure of path clearance for propagation over hilly terrain may be given by the Fresnel zone radii. Consider a plane placed perpendicularly at an arbitrary point O between transmitter T and receiver R. Let d_1 = TO and d_2 = OR be the respective distances. Draw concentric circles of radii h_n, n = 1, 2, ..., such that the indirect path from T to R via each circle is $n\lambda/2$ greater than the direct path $d_1 + d_2$. Using an approach similar to that carried out in Section 3.3.2, it is straightforward to show that

$$h_n = \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}} \tag{3.46}$$

Equation 3.46 defines the radius of the nth Fresnel zone. The segments delimited by adjacent circles are the Fresnel zones. It is recognized that the first Fresnel zone (radius h_1) bounds the volume contributing significantly to wave propagation. Therefore, path clearance is obtained if

$$h > h_1 = \sqrt{\frac{\lambda d_1 d_2}{d_1 + d_2}} \tag{3.47}$$

From Equation 3.42, it can be noted that the condition $h > h_1$ corresponds to $x > \sqrt{2}$*, implying a negligible loss (refer to Figure 3.7) due to diffraction.

3.3.5.2 Minima and Maxima at Reception

Analysis of the plane earth propagation conditions leads to some interesting conclusions. Referring to Equation 3.32, it is readily seen that the field strength may pass through minima and maxima depending on the phase difference between direct and indirect waves. Minima will occur when the argument $\Delta\varphi/2$ of the sine function equals $n\pi$, where n is an integer. In the same way, maxima will occur for this argument equal to $(2n-1)\pi/2$. Using these conditions in Equation 3.31, the distances of minima, $d\min_n$, and of maxima, $d\max_n$, are, respectively

$$d\min_n = \frac{2h_t h_r}{n\lambda}$$

---

* Note that in Figure 3.11 the height h corresponds to -h in Figure 3.6.

$$d\max_n = \frac{4h_t h_r}{(2n-1)\lambda}$$

Note that, as n increases, both $d\min_n$ and $d\max_n$ decrease. Hence, for distances

$$d > d\min_1 = \frac{2h_t h_r}{\lambda} \qquad (3.48)$$

minima will no longer occur. In the same way, for

$$d > d\max_1 = \frac{4h_t h_r}{\lambda} \qquad (3.49)$$

maxima will cease to occur. In such a region, the loss due to reflection exceeds the free space loss when $d > 12h_t h_r / \lambda$, where Equation 3.33 (or, equivalently, Equation 3.34) can be used.

Example: $h_t = 30\,m$

$h_r = 1{,}5\,m$

$f = 900\,MHz$

$\therefore\ d > \frac{12 \times 30 \times \cancel{1}\ 1{,}5}{1/3}$

$\boxed{d > 1620\,m}$

3. $$\frac{W_r}{W_t} = G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \times 4 \sin^2\left(\frac{\Delta\varphi}{2}\right)$$

$$\Delta\varphi = 4\pi \frac{h_t h_r}{\lambda d}$$

$$\boxed{\frac{W_r}{W_t} = G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \times 4 \sin^2\left(\frac{2\pi h_t h_r}{\lambda d}\right)}$$

a) Excesso de perda

$$\text{Excesso} = 4 \sin^2\left(\frac{2\pi h_t h_r}{\lambda d}\right)$$

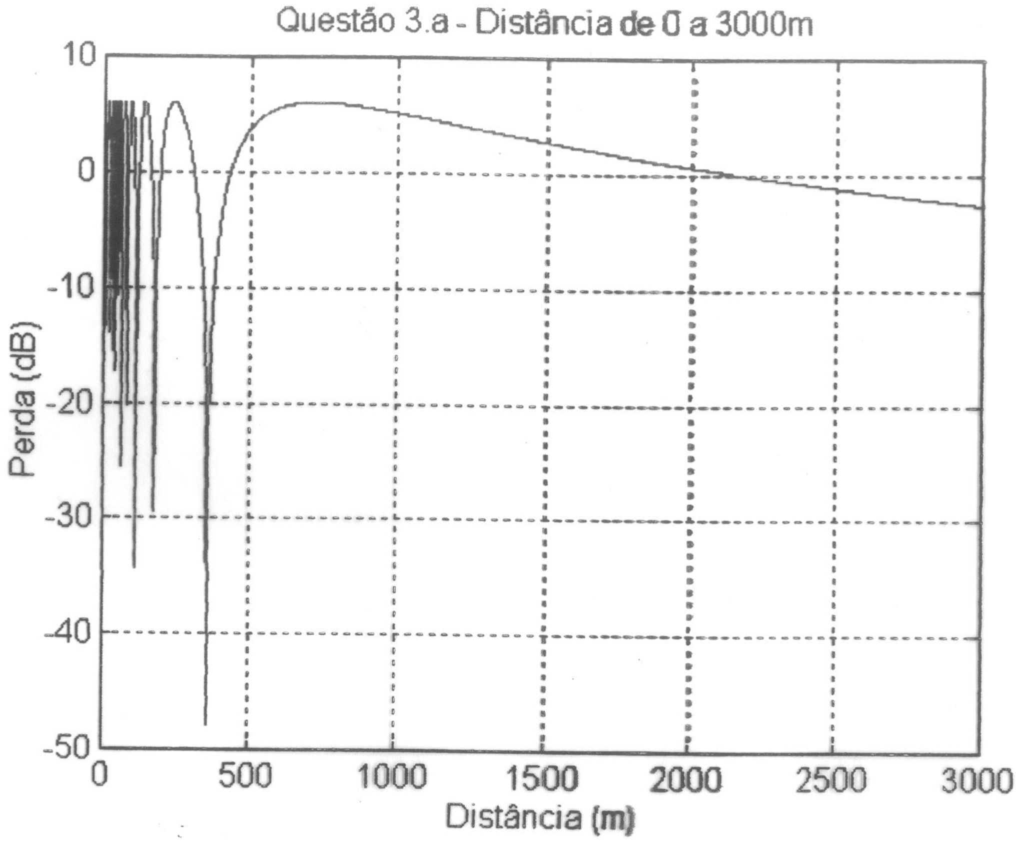
b) Perda total

$h_t = 40$ m

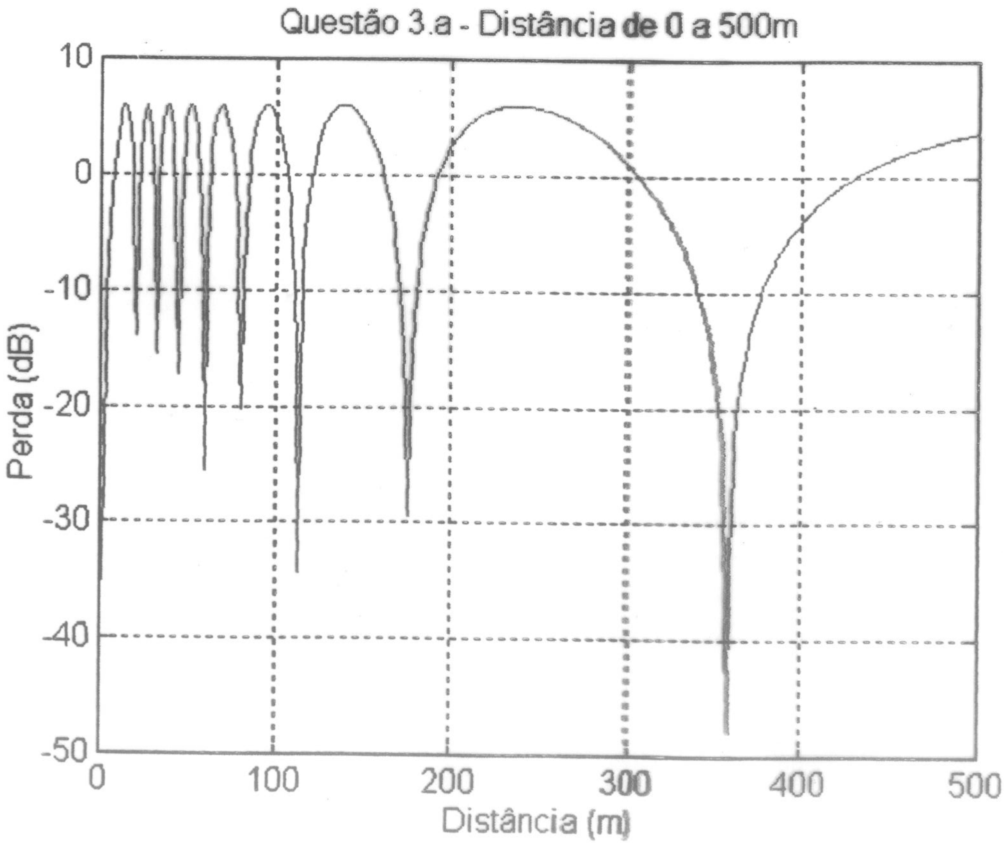
$h_r = 1{,}5$ m

$\lambda = 1/3$

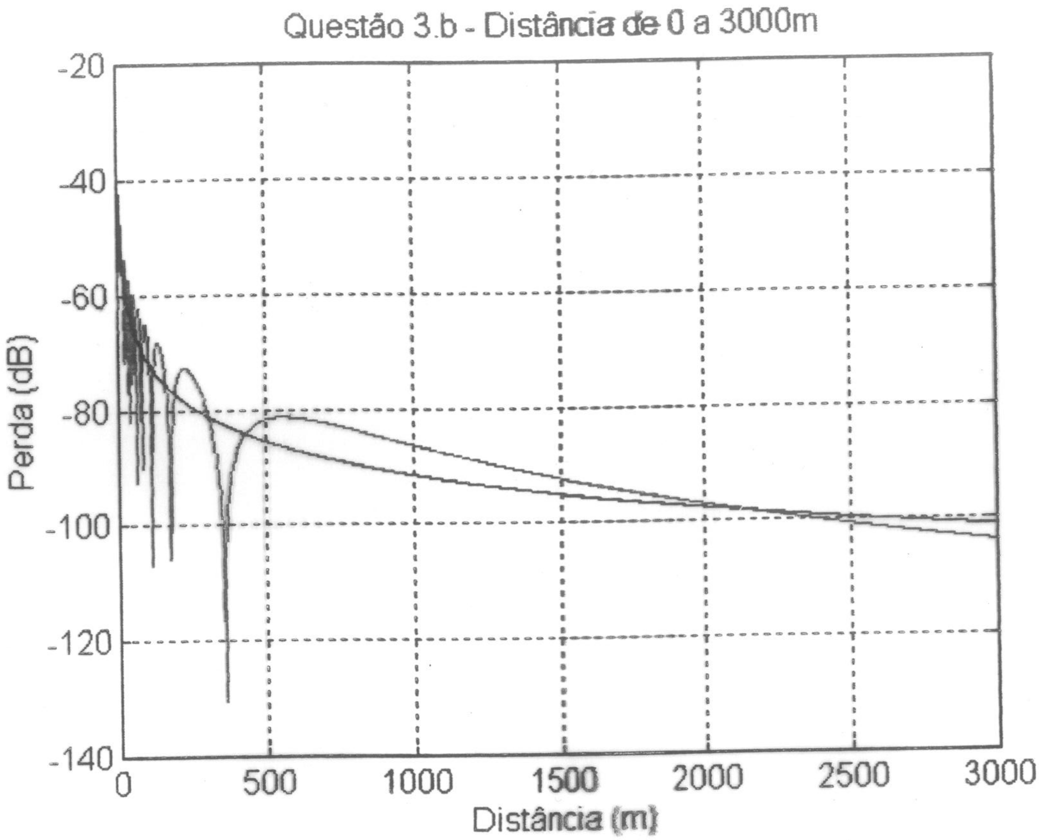
$$\boxed{\frac{W_r}{W_t} = G_t G_r \left(\frac{1}{12\pi d}\right)^2 \times 4 \sin^2\left(\frac{2\pi \times 180}{d}\right)}$$



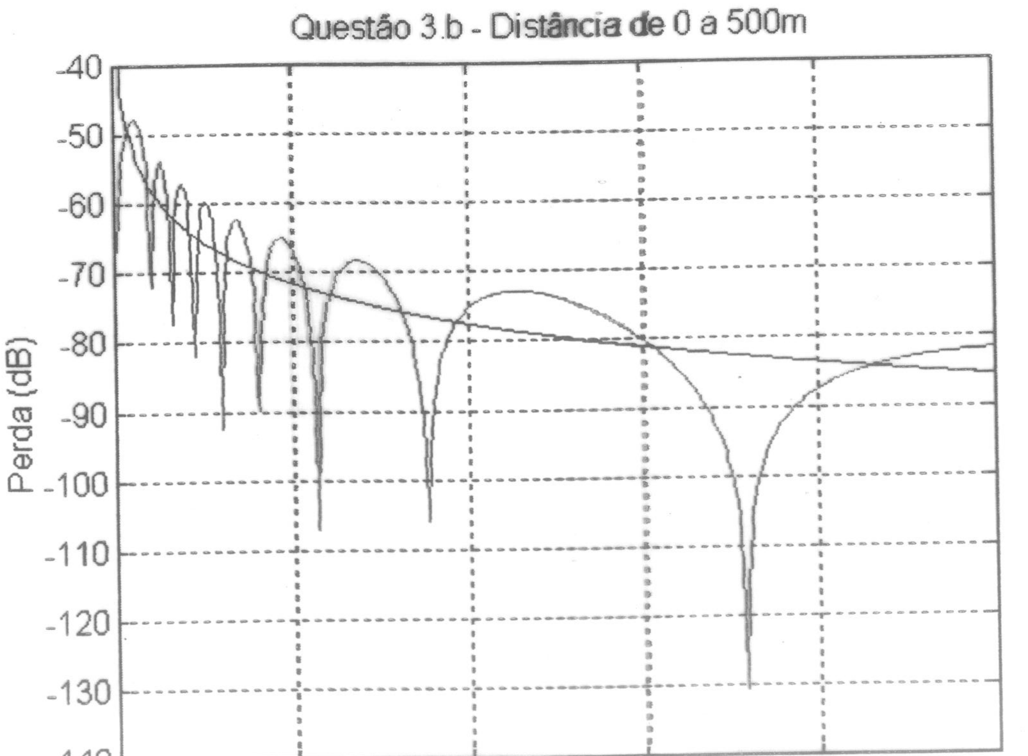
Questão 3.a - Distância de 0 a 3000m
Perda (dB)
Distância (m)
10
0
-10
-20
-30
-40
-50
0
500
1000
1500
2000
2500
3000



Questão 3.a - Distância de 0 a 500m
Perda (dB)
Distância (m)
10
0
-10
-20
-30
-40
-50
0
100
200
300
400
500


Questão 3.b - Distância de 0 a 3000m
Perda (dB)
Distância (m)
-20
-40
-60
-80
-100
-120
-140
0
500
1000
1500
2000
2500
3000

Questão 3.b


Questão 3.b - Distância de 0 a 500m
Perda (dB)
Distância (m)
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
-140
0
100
200
300
400
500

Sizes of Cells

Above Roof Tops

Large Cells

Small Cells, $1\text{km} \leq d \leq 3\text{km}$

Below Roof Tops

Microcells $0.5\text{km} \leq d \leq 1\text{km}$

Field Strength Prediction Models

Analytical Models + Field Data

Adjustment Parameters

No Theoretical Explanations

Base Station Antennas

Above Roof Topps

Diffractions + Scatterings at roof tops vicinity of mobiles

Below Roof Tops

Diffractions + Scatterings around buildings

$$h < \sqrt{2\lambda\left(\frac{d_1 d_2}{d_1 + d_2}\right)}$$

Prediction Models

1) ***Egli Method***

$$L = 139.1 - 20 \log h_t + 40 \log d$$

2) ***Blomquist - Ladell Method***

$$L = L_0 + \mathrm{Max}(L_p, L_k)$$

or

$$L = L_0 + \sqrt{L_p^2 + L_k^2}$$

3) ***Longley - Rice Method***

20 MHz ≤ Frequency ≤ 40 GHz

0.5 m ≤ Antenna height ≤ 3 km

1 km ≤ Distance ≤ 2000 km

$$\Delta h' = \Delta h\left[1 - \exp(-0.02\ d)\right]$$

4) ***Okumura Method***

Gain $G(h_t)$ of 6 dB per octave for base station antenna height

$$G(h_t) = 20\log(h_t/200), \qquad h_t > 10\ \text{m}.$$

Gain $G(h_r)$ of 3 dB or 6 dB per octave for mobile station antenna height depending on the range, i.e.,

$$G(h_r) = 10\log(h_r/3), \qquad h_r < 3\text{m}$$

$$G(h) = 20\log(h_r/3), \qquad 3\text{m} \le h_r \le 10\ \text{m}$$

$$L = L_0 + A(f,d) - G_{area} - G(h_t) - G(h_r)$$

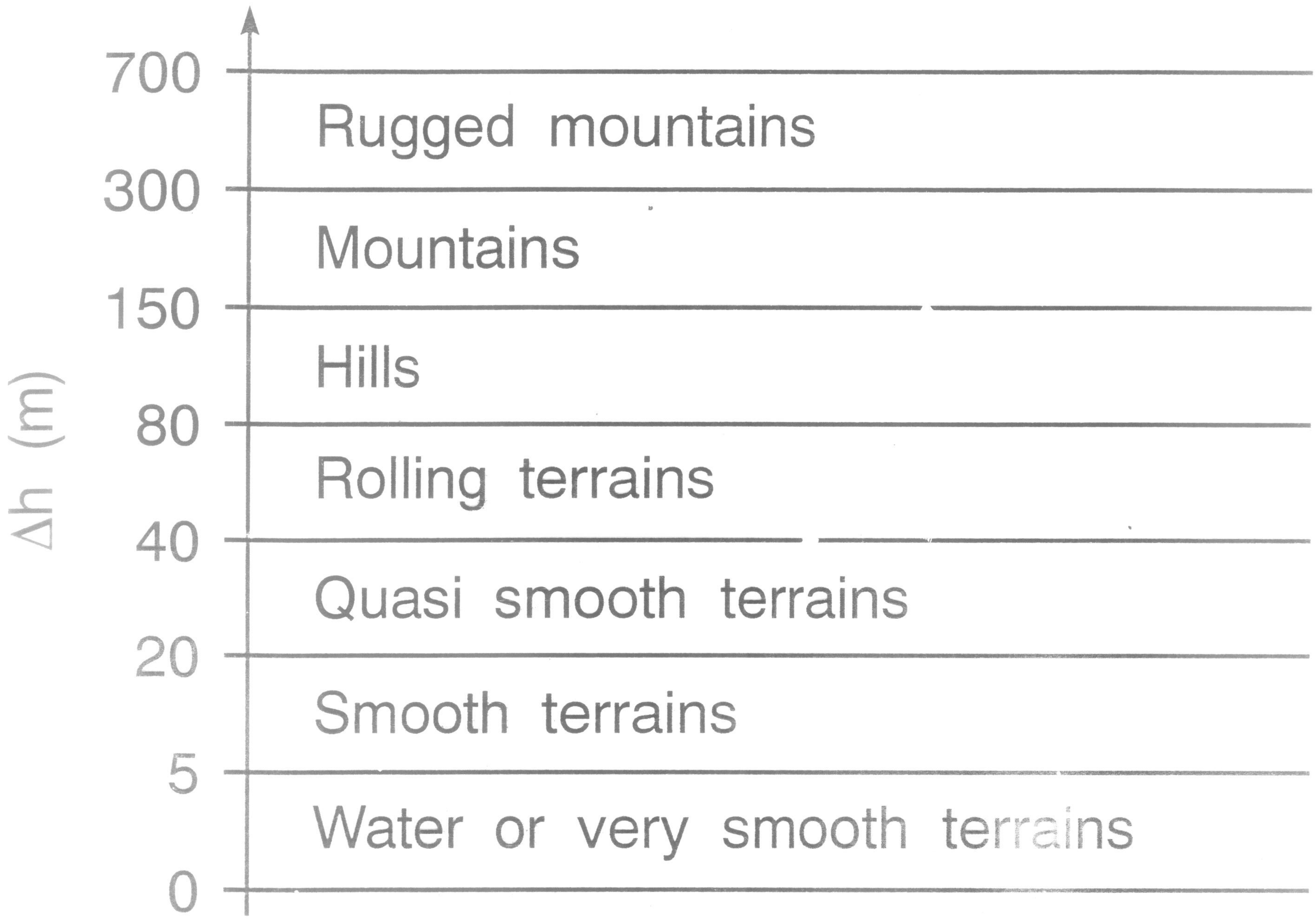

Δh (m)
700
300
150
80
40
20
5
0
Rugged mountains
Mountains
Hills
Rolling terrains
Quasi smooth terrains
Smooth terrains
Water or very smooth terrains

FIG3-13 300%

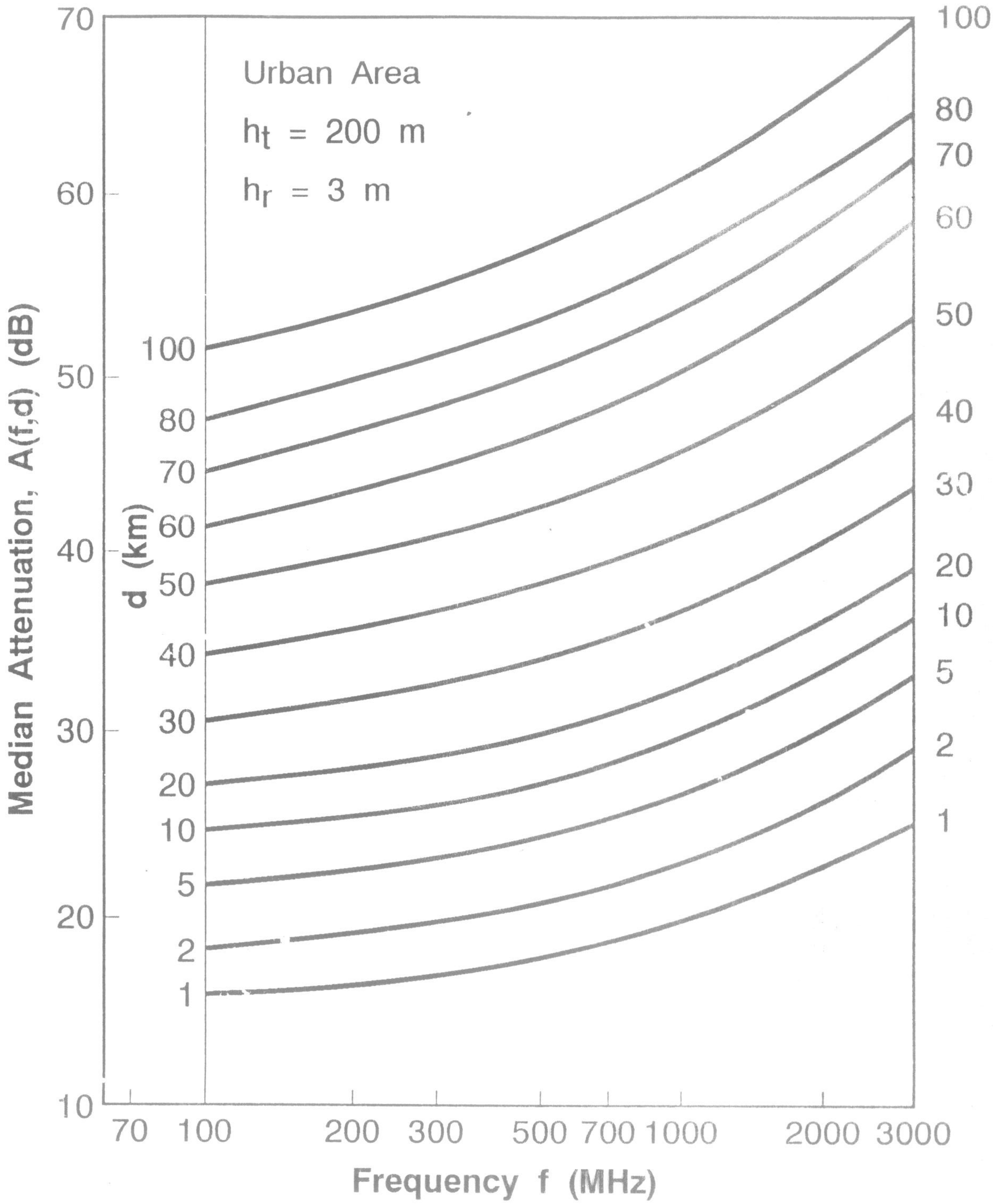

Urban Area
h_t = 200 m
h_r = 3 m
Median Attenuation, A(f,d) (dB)
d (km)
Frequency f (MHz)
70
60
50
40
30
20
10
100
80
70
60
50
40
30
20
10
5
2
1
100
80
70
60
50
40
30
20
10
5
2
1
70
100
200
300
500
700
1000
2000
3000

FIG3.14 150%

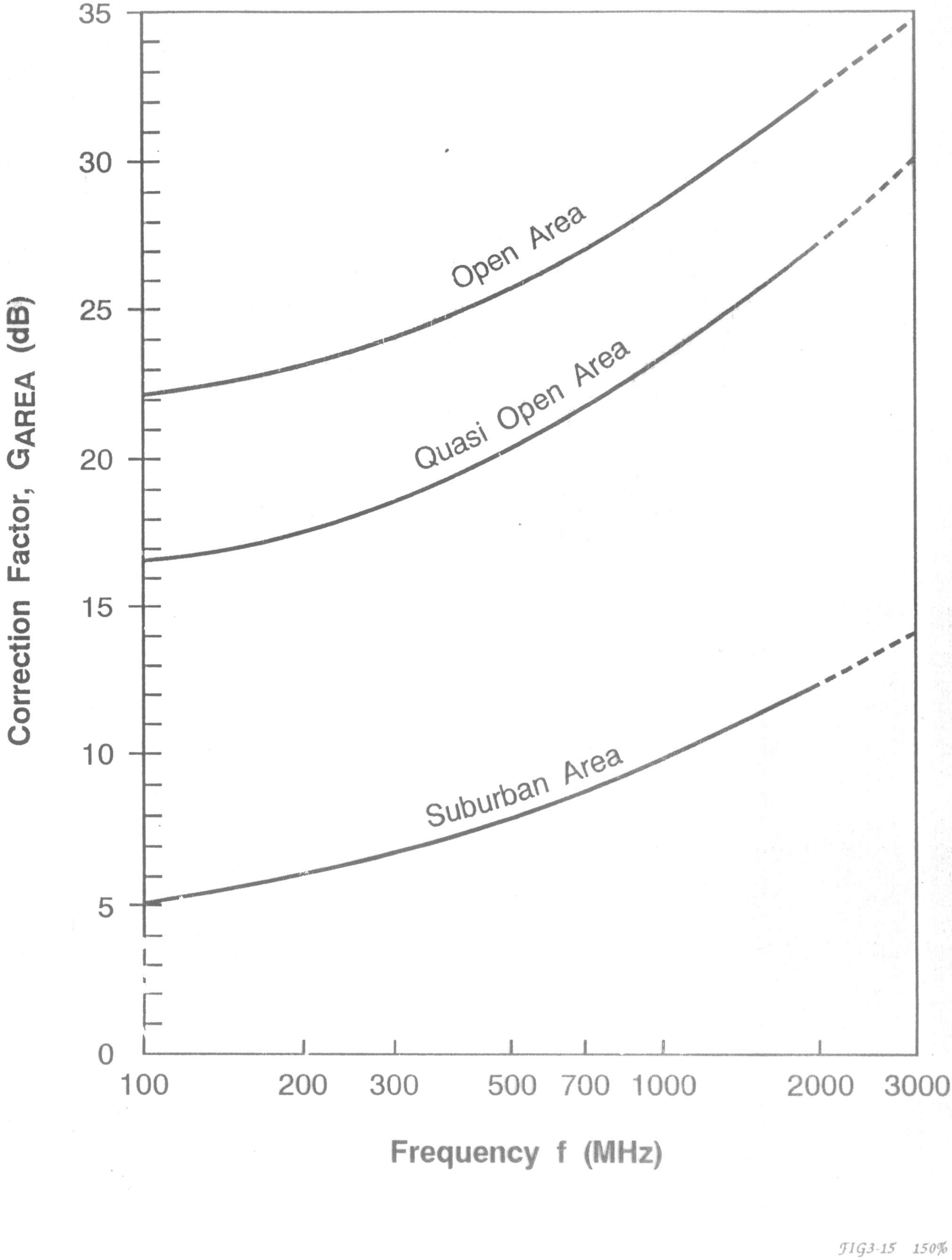

Open Area
Quasi Open Area
Suburban Area
Correction Factor, GAREA (dB)
Frequency f (MHz)
0
5
10
15
20
25
30
35
100
200
300
500
700
1000
2000
3000

FIG3-15 150%

HATA MODEL

$$150\,MHz \leq f \leq 1000\,MHz$$

$$30\,m \leq h_t \leq 200\,m$$

$$1\,m \leq h_r \leq 10\,m$$

$$1\,km \leq d \leq 20\,km$$

URBAN Area

$$L = 69.55 + 26.16 \log f - 13.82 \log h_t - A(h_r) + (44.9 - 6.55 \log h_t) \log d$$

$$A(h_r) = \,?$$

Small or Medium-Sized City

$1m \le h_r \le 10m$

$$A(h_r) = (1.1 \log f - 0.7) h_r - (1.56 \log f - 0.8)$$

Large City $f \le 200$ MHz

$$A(h_r) = 8.29 \log^2(1.54 h_r) - 1.1$$

Large City $f \geq 400$ MHz

$$A(h_r) = 3.2 \log^2(11.75 h_r) - 4.97$$

Suburban Area

$$L = L_{urban} - 2 \log^2(f/28) - 5.4$$

Rural (quasi-open) area

$$L = L_{urban} - 4.78 \log^2(f) + 18.33 \log f - 35.94$$

Rural (open) area

$$L = L_{urban} - 4.78 \log^2(f) + 18.33 \log f - 40.94$$

COST-231-HATA MODEL

European Coop. in the Field of Scientific and Technical Research - Euro-Cost-, COST 231 subgroup Prop. Models

Okumura Curves higher frequencies

Large and Small Cells

$1500 \text{ MHz} \leq f \leq 2000 \text{ MHz}$

URBAN Area

$$L = 46.3 + 33.9 \log f - 13.82 \log h_t - A(h_r) + (44.9 - 6.55 \log h_r) \log d$$

$$A(h_r) = ?$$

Medium-Sized and Suburban Centers
with Moderate Tree Density

$$A(h_r) = (1.1 \log f - 0.7) h_r - (1.58 \log f - 0.8)$$

Metropolitan Centers

$$A(h_r) = (1.1 \log f - 0.7) h_r - (1.58 \log f + 2.2)$$

Lee Model

Area-to-Area Mode

Basic Loss (initial conditions)
+ Loss due to distance
+ Correction Factors

Point-to-Point Mode

Area-to-Area + Diffraction
or Reflection

$$L = L_b + 10\alpha \log\left(\frac{d}{d_b}\right) - C - m\left(o L_K + \bar{o} L_R\right)$$

m = mode $\rightarrow$ 1: POINT TO POINT

$\rightarrow$ 0: Area to Area

o : binary $\rightarrow$ 1: obstructive

$\rightarrow$ 0: NON-obstructive

$$C = 10 \log\left[\left(\frac{h_t}{h_{tb}}\right)^2 \left(\frac{h_r}{h_{rb}}\right)^x \left(\frac{G_t}{G_{tb}}\right)\left(\frac{G_r}{G_{rb}}\right)\right]$$

$$x = \begin{cases} 2 & h_r > 10\,m \\ 1 & h_r < 10\,m \end{cases}$$

Initial Conditions

$P_t = 10\ W$ (40 dBm)

$h_{tb} = 100\ ft$ (30.48 m)

$h_{rb} = 10\ ft$ (3 m)

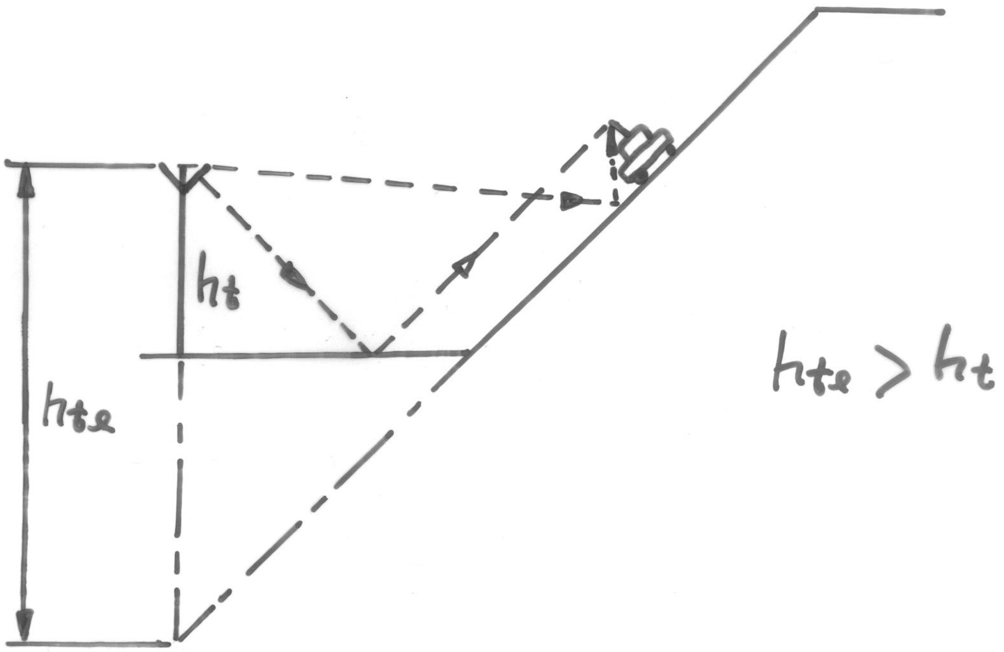
$G_{tb} = 4$ (6 dB) wrt $\lambda/2$ dipole

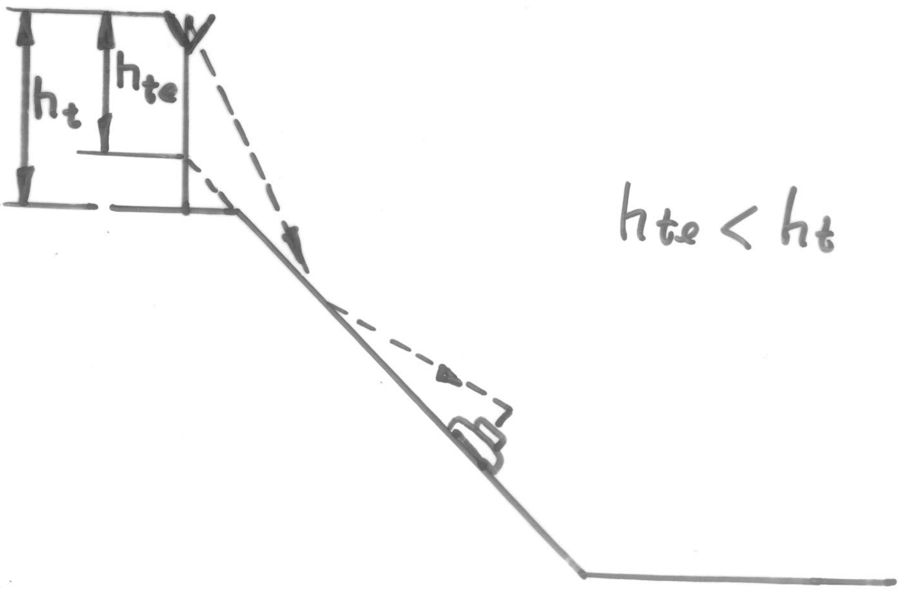
$G_{rb} = 1$ (0 dB) wrt $\lambda/2$ dipole

$d_b = 1$ mile (1.6 km)

Environment	L_b (dB)	α
Free Space	85	2.0
Open Area	89	4.35
Suburban Area	101.7	3.84
Urban		
Newark	104	4.31
Philadelphia	110	3.68
Tokyo	124	3.05
New York	117	4.8

$$L_R = 20 \log\left(\frac{h_{te}}{h_t}\right)$$





COST 231 - Walfish-Ikegami Model

Street Canyon (Stockholm)

Obstruction by Buildings (Walfish + Ikegami)

$800\,\mathrm{MHz} \le f \le 2000\,\mathrm{MHz}$

$4\,\mathrm{m} \le h_t \le 50\,\mathrm{m}$

$1\,\mathrm{m} \le h_r \le 3\,\mathrm{m}$

$0.02\,\mathrm{km} \le d \le 5\,\mathrm{km}$

Line-of-Sight (Street Canyon) Case

$$L = 42.6 + 26 \log d + 20 \log f$$

(km, MHz, dB)

Obstructed Case

$$L = L_F + L_D + L_S$$

$L_F = L_{Free\ space}$

$L_F = 20 \log f + 20 \log d + 32.44 \text{ dB}$

$L_D = L_{\text{roof top-to-street diffraction + scatter}}$

$$L_D = -16.9 - 10 \log r_w + 10 \log f + 20 \log (h_B - h_r) + L_\varphi$$

r_w = road width

h_B = mean building height

f = frequency

h_r: receiver antenna height

$L_\varphi = L_{\text{road orientation wrt direct path}}$

$$L_\varphi = -10 + 0.354\,\varphi \qquad 0^\circ \le \varphi < 35^\circ$$
$$= 2.5 + 0.075(\varphi - 35) \qquad 35^\circ \le \varphi < 55^\circ$$
$$= 4.0 - 0.114(\varphi - 55) \qquad 55^\circ \le \varphi < 90^\circ$$

L_s = L multiscreen diffraction loss

$$L_s = -18\log(1 + h_t - h_B) - 9\log d_B + k_a + k_d \log d + k_f \log f$$

d_B = mean distance between buildings

$k_a = 54 \qquad h_t - h_B > 0$

$k_a = 54 - 0.8(h_t - h_B) \qquad d \geq 0.5,\ h_t - h_B \leq 0$

$k_a = 54 - 0.8(h_t - h_B)\dfrac{d}{0.5} \qquad d < 0.5,\ h_t - h_B \leq 0$

$k_d = 18 \qquad h_t - h_B > 0$

$k_d = 18 - 15\dfrac{h_t - h_B}{h_B} \qquad h_t - h_B \leq 0$

$k_f = -4 + 0.7\left(\dfrac{f}{925} - 1\right)$ medium-sized city and sub-urban centers with moderate tree densities

$k_f = -4 + 1.5\left(\dfrac{f}{925} - 1\right)$ metropolitan centers

Default values

$d_B = 20\,m \ldots 50\,m$

$r_w = d_B/2$

$h_B = 3 \times$ no. floors + roof heights

$$\text{roof heights} = \begin{cases} 3\,m & \text{pitched roofs} \\ 0\,m & \text{flat roofs} \end{cases}$$

$\varphi = 90°$

5) *Hata's Formula*

$$L = 69.55 + 26.16\log f - 13.82\log h_t - A(h_r) + (44.9 - 6.55\log h_t)\log d \text{ dB}$$

where $150 \text{ MHz} \le f \le 1500 \text{ MHz}$

$30 \text{ m} \le h_t \le 300 \text{ m}$

$1 \text{ km} \le d \le 20 \text{ km}$

For small or medium size city

$$A(h_r) = (1.1\log f - 0.7)h_r - (1.56\log f - 0.8) \text{ dB}$$

where $1\text{m} \le h_r \le 10 \text{ m}$

For a large city

$$A(h_r) = 8.29\log^2(1.54h_r) - 1.1 \text{ dB} \qquad (f \le 200 \text{ MHz})$$

and

$$A(h_r) = 3.2\log^2(11.75h_r) - 4.97 \text{ dB} \qquad (f \le 400 \text{ MHz})$$

6) *Ibrahim - Parsons Method*

A Simplified Path Loss Model

$$\frac{W_r}{W_t} \approx h_r^x \quad , \quad \begin{array}{lll} x = 1 & \text{for} & h_r < 3\text{m} \\ x = 2 & \text{for} & 3\text{m} \le h_r \le 10 \text{ m} \end{array}$$

$$\frac{W_r}{W_t} \approx \frac{1}{d^\alpha} \quad , \quad 2 \le \alpha \le 4$$

$$\frac{W_r}{W_t} \approx \frac{1}{f^y} \quad , \quad 2 \le y \le 3$$

$$\frac{W_r}{W_t} = K \frac{G_t G_r h_t^2 h_r^x}{d^\alpha f^y}$$

$$W_{ri} = K \frac{W_{ti} G_{ti} G_{ri} h_{ti}^2 h_{ri}^x}{d_i^\alpha f_i^y}$$

$$\frac{W_{rj}}{W_{ri}} = \left(\frac{d_j}{d_i}\right)^{-\alpha} \beta_j$$

where

$$\beta_j = \frac{W_{tj}G_{tj}G_{rj}}{W_{ti}G_{ti}G_{ri}}\left(\frac{h_{tj}}{h_{ti}}\right)^2\left(\frac{h_{rj}}{h_{ri}}\right)^x\left(\frac{f_j}{f_i}\right)^{-y}$$

$$\frac{W_{rj}}{W_{ri}} = \left(\frac{d_j}{d_i}\right)^{-\alpha}$$

$$10\log\left(\frac{W_{rj}}{W_{ri}}\right) = -\alpha 10\log\left(\frac{d_j}{d_i}\right)$$

$$\frac{W_{r1}}{W_{r0}} = \left(\frac{d_1}{d_0}\right)^{-\alpha_1}$$

$$\frac{W_{r2}}{W_{r1}} = \left(\frac{d_2}{d_1}\right)^{-\alpha_2}$$

$$\vdots$$

$$\frac{W_{rn}}{W_{rn-1}} = \left(\frac{d_n}{d_{n-1}}\right)^{-\alpha_n}$$

Then

$$\frac{W_{rn}}{W_{r0}} = \prod_{i=1}^{n}\left(\frac{d_i}{d_{i-1}}\right)^{-\alpha_i}$$

$$\frac{W_{rn}}{W_{r0}} = \left(\frac{d_n}{d_0}\right)^{-\alpha_1} C_1$$

where

$$C_1 = d_1^{-\alpha_1} d_n^{\alpha_1} \prod_{i=2}^{n}\left(\frac{d_i}{d_{i-1}}\right)^{-\alpha_i}$$

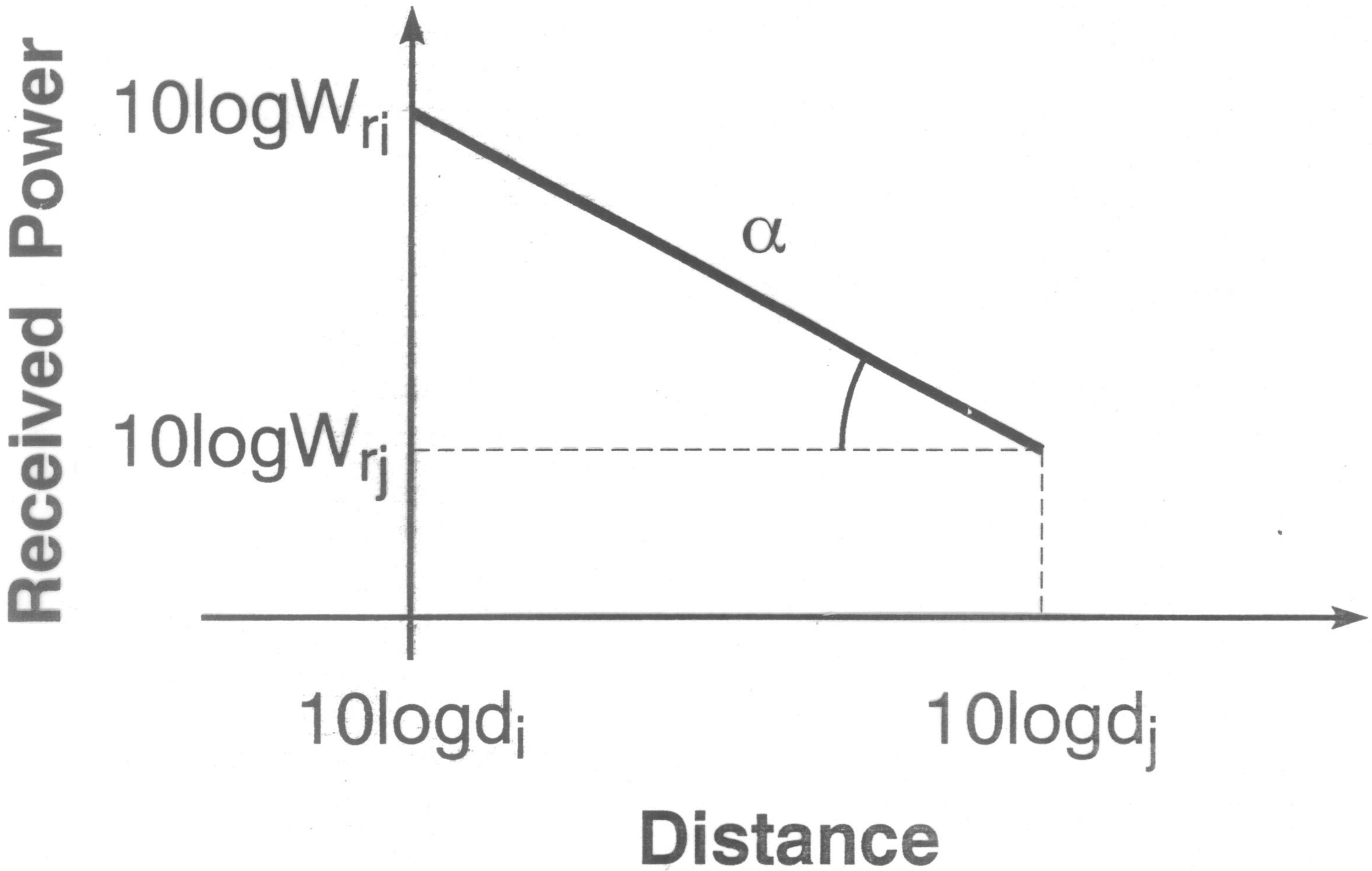

Received Power
10logW_ri
10logW_rj
α
10logd_i
10logd_j
Distance

FIG3-16 300%

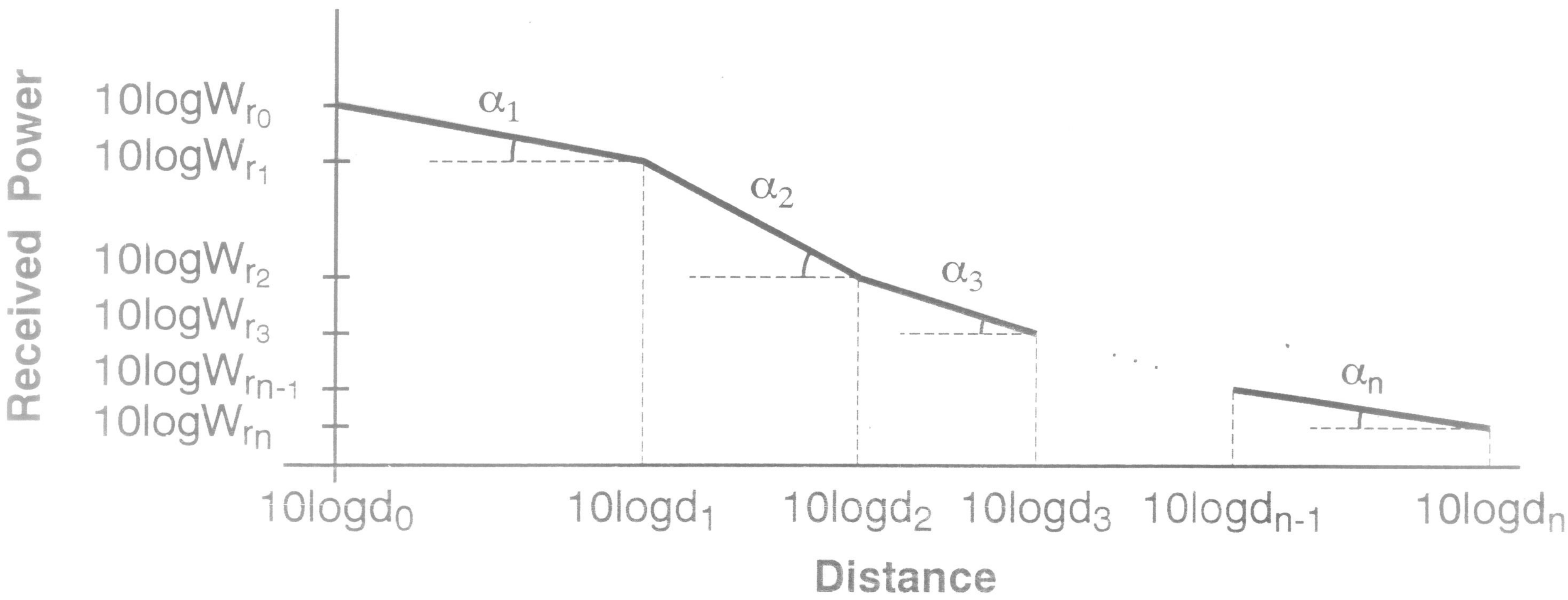

Received Power
10logWr0
10logWr1
10logWr2
10logWr3
10logWrn-1
10logWrn
α1
α2
α3
αn
10logd0
10logd1
10logd2
10logd3
10logdn-1
10logdn
Distance

FIG3-17 200%

Considerations on Other Effects

1) *Atmospheric Conditions* Moisture attenuates
• 10 GHz: rain ; • 24 GHz water vapour
• 60 GHz oxygen

2) *Foliage* : • diffracts
• reflects • absorbs

$$L = 1637\sigma + \frac{\exp(-90/f)\log(1 + f/10)}{2.99} \text{ dB/m}$$

Depends on:
• Height
• Shape
• density
• Season
• humidity
•

and

$$L = 1637\sigma + \frac{\exp(-210/f)\log(1 + f/200)}{2.34} \text{ dB/m}$$

Conductivity of various types of foliages

Foliage	σ (x 10^{-5}) Dry	σ (x 10^{-5}) Wet
Bare tree, branches	0.5 - 1	2 - 10
Deciduous, full leaf	1	5 - 20
Evergreen forest	2 - 5	5 - 20
Thin jungle scrub	1 - 10	3 - 20
Dense rain forest	10 - 50	50

Loss low for low VHF
High for UHF

$\boxed{\text{10 dB ok!}}$

3) *Street Orientation* wave guides radially
parallel
perpendicular 10-20 dB ↑

4) *Tunnel*

STATISTICAL DISTRIBUTIONS OF THE MOBILE RADIO SIGNAL

Log-Normal Distribution

Dispersive medium electric permittivity
magnetic permeability
ϵ, μ

$$E = E_0 e^{\pm \gamma . r}$$

$$\gamma = \sqrt{(\sigma + j\omega\varepsilon) j\omega\mu} = \alpha + j\beta$$

γ: prop. cte.
α: attenuation cte neper/m
β: phase cte. rad/m

where

$$\alpha^2 = \omega^2 \frac{\mu\varepsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon} \right)^2} - 1 \right)$$

$$\beta^2 = \omega^2 \frac{\mu\varepsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon} \right)^2} + 1 \right)$$

$$E = E_0 \exp(-\alpha r)$$

Log-Normal Distribution

$$E_i = E_{i-1} \exp(-\alpha_i \Delta r_i)$$

$$E_n = E_0 \exp\left(- \sum_{i=1}^{n} \alpha_i \Delta r_i \right)$$

$$x \triangleq - \sum_{i=1}^{n} \alpha_i \Delta r_i$$

Then

$$E_n = E_0 \exp(x)$$

$$p(x) = \frac{1}{\sqrt{2\pi}\, \sigma_x} \exp\left[-\frac{1}{2} \left(\frac{x - m_x}{\sigma_x} \right)^2 \right]$$

$$y = \frac{E_n}{E_0} = \exp(x)$$

$$p(Y)|dY| = p(x)|dx|$$

$$p(Y) = \frac{1}{\log e}\, p(x) = \frac{1}{\sqrt{2\pi}\, \sigma_x \log e} \exp\left[-\frac{1}{2} \left(\frac{x - m_x}{\sigma_x} \right)^2 \right]$$

Therefore,

$$p(Y) = \frac{1}{\sqrt{2\pi}\,\sigma_Y} \exp\left[-\frac{1}{2}\left(\frac{Y - M_Y}{\sigma_Y}\right)^2\right]$$

$$p(R) = \frac{1}{\sqrt{2\pi}\,\sigma_R} \exp\left[-\frac{1}{2}\left(\frac{R - M_R}{\sigma_R}\right)^2\right]$$

$$p(y) = \frac{1}{\sqrt{2\pi}\,y\sigma_X} \exp\left[-\frac{1}{2}\left(\frac{\ln y - m_X}{\sigma_X}\right)^2\right]$$

In the same way, if $R = \log r$ then

$$p(r) = \frac{1}{\sqrt{2\pi}\,r\sigma_X} \exp\left[-\frac{\ln^2(r/m_r)}{2\sigma_X^2}\right]$$

The cumulative distribution is

$$P(Y_0) = \text{prob}(Y \leq Y_0) = \int_{-\infty}^{Y_0} p(Y)\, dY$$

Rayleigh Distribution

$$s = a\exp(j\omega_0 t)$$

$$s_r = \sum_{i=1}^{n} a_i \exp\left[j(\omega_0 t + \theta_i)\right] = \left[\sum a_i \exp(j\theta_i)\right] \exp(j\omega_0 t)$$

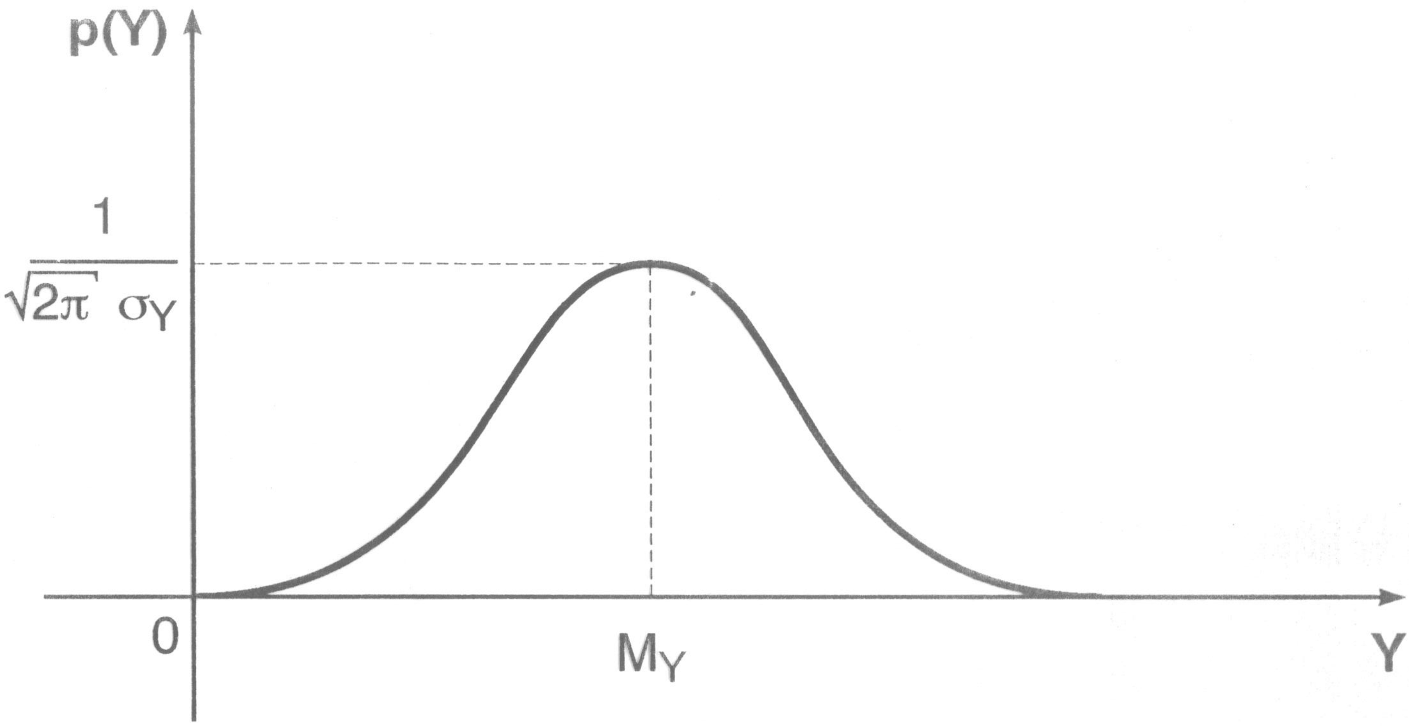
Equivalently,

$$s_r \triangleq r\exp\left[j(\omega_0 t + \theta)\right]$$

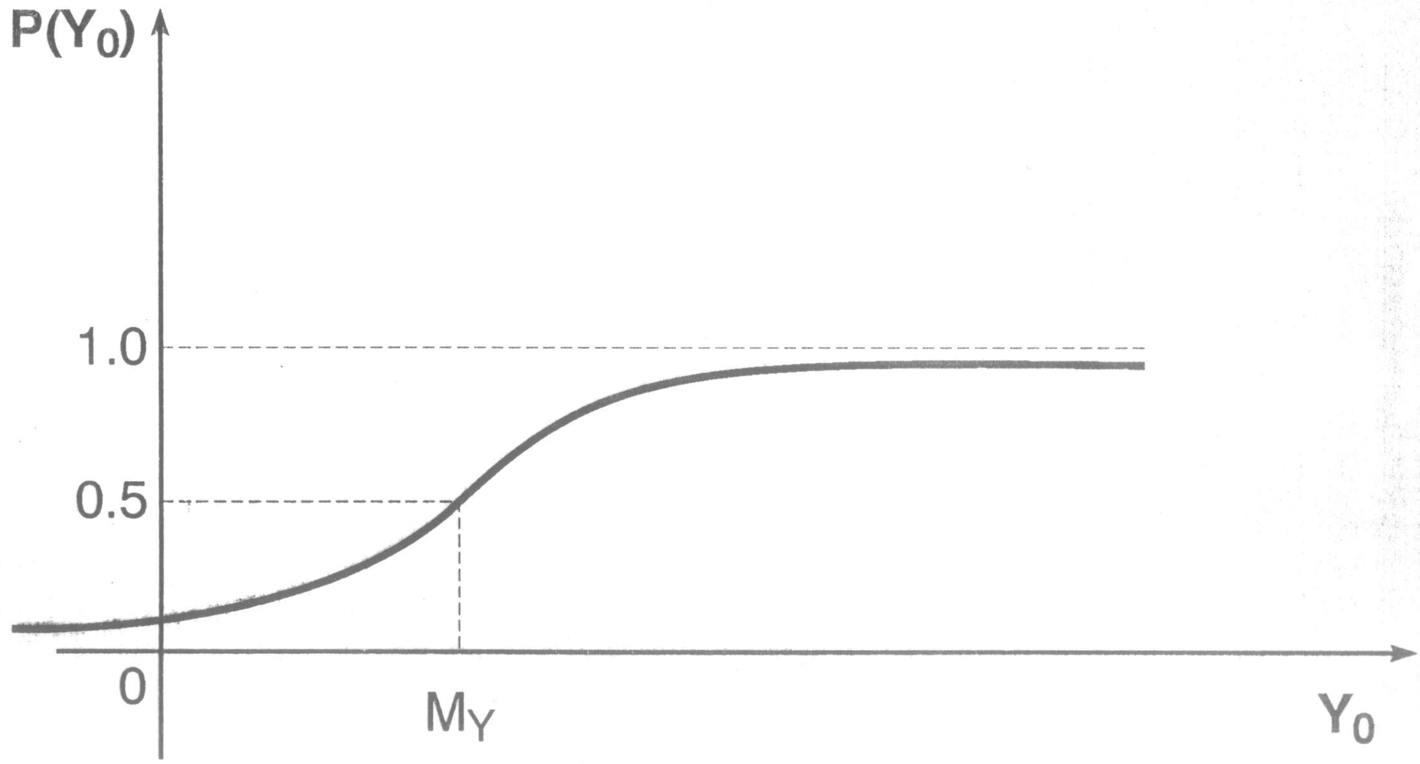
where

$$r\exp(j\theta) \triangleq \sum_{i=1}^{n} a_i \exp(j\theta_i)$$

But



(a)



(b)

FIG3-19 200%

$$r\exp(j\theta) = \sum_{i=1}^{n} a_i\cos\theta_i + j\sum_{i=1}^{n} a_i\sin\theta_i \triangleq x + jy$$

Then

$$x \triangleq \sum_{i=1}^{n} a_i\cos\theta_i \qquad \text{and} \qquad y \triangleq \sum_{i=1}^{n} a_i\sin\theta_i$$

where

$$r^2 = x^2 + y^2$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$p(z) = \frac{1}{\sqrt{2\pi}\,\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

$$p(x,y) = p(x)p(y) = \frac{1}{2\pi\sigma_r^2}\exp\left(-\frac{x^2+y^2}{2\sigma_r^2}\right)$$

$$p(r,\theta) = |J|\,p(x,y)$$

where

$$J \triangleq \begin{vmatrix} \partial x/\partial r & \partial x/\partial\theta \\ \partial y/\partial r & \partial y/\partial\theta \end{vmatrix}$$

$$p(r,\theta) = \frac{r}{2\pi\sigma_r}\exp\left(-\frac{r^2}{2\sigma_r^2}\right)$$

$$p(r) = \int_0^{2\pi} p(r,\theta)\,d\theta$$

$$= \begin{cases} \dfrac{r}{\sigma_r^2}\exp\left(-\dfrac{r^2}{2\sigma_r^2}\right) & , r \geq 0 \\ 0 & , \text{otherwise} \end{cases}$$

$$P(r_0) = \text{prob}(r \leq r_0) = \int_0^{r_0} p(r)\,dr = 1 - \exp\left(-\frac{r_0^2}{2\sigma_r^2}\right)$$

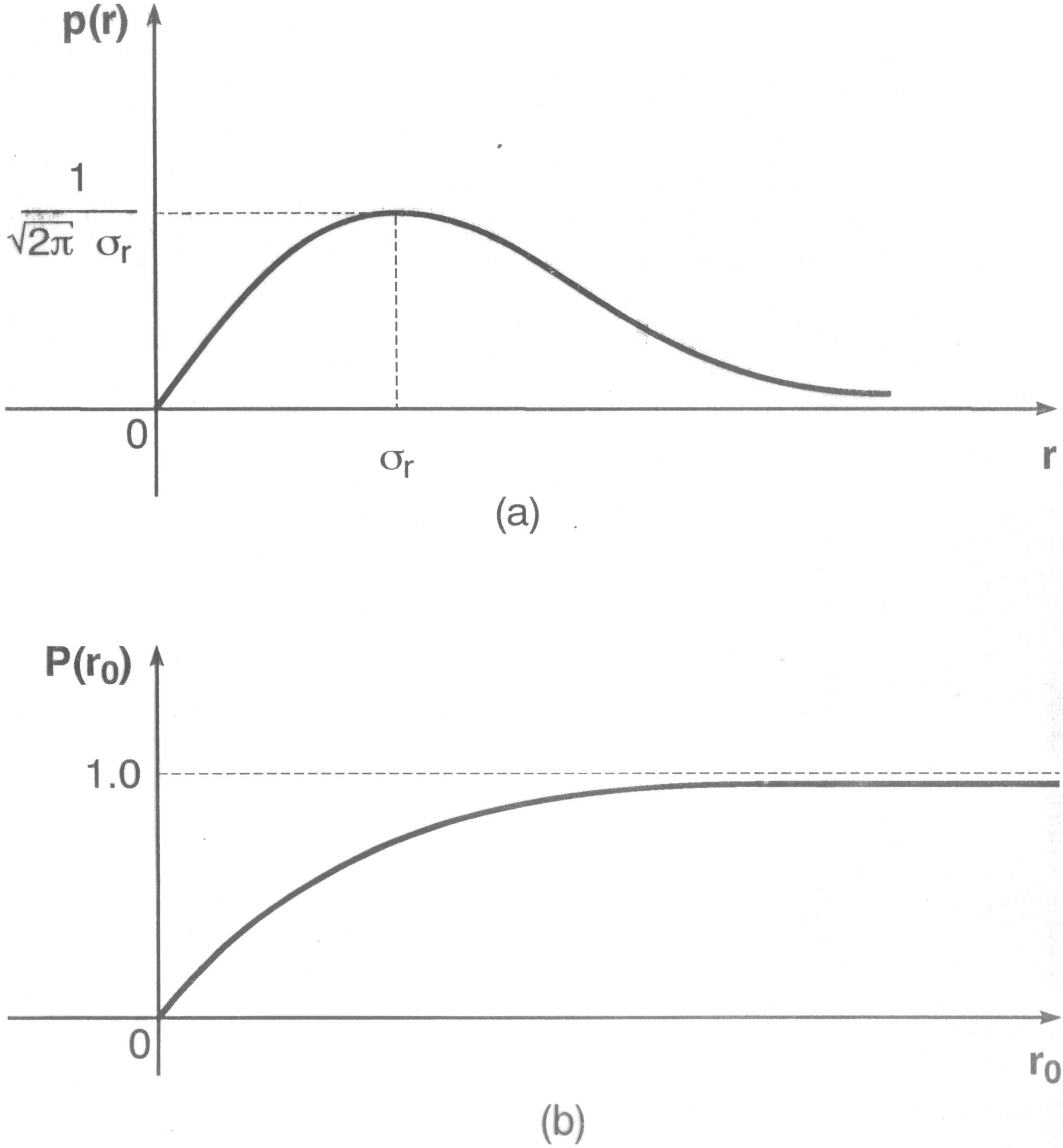


FIG3-20 200%

Ricean Distribution

$$s_r = \overbrace{r\exp(j\omega_0 t + \theta)}^{\text{scattered waves}} + \overbrace{a\exp(j\omega_0 t)}^{\text{direct waves}}$$

or, equivalently

$$s_r = \Big[(x + a) + jy\Big]\exp(j\omega_0 t)$$

Note that, in this case,

$$r^2 = (x + a)^2 + y^2$$

$$x + a = r\cos\theta$$

$$y = r\sin\theta$$

$$p(r) = \frac{r}{\sigma_r^2}\exp\left(-\frac{r^2 + a^2}{2\sigma_r^2}\right) I_0\left(\frac{ar}{\sigma_r^2}\right)$$

where

$$I_0\left(\frac{ar}{\sigma_r^2}\right) = \frac{1}{2\pi}\int_0^{2\pi}\exp\left(\frac{ar\cos\theta}{\sigma_r^2}\right)d\theta$$

Suzuki Distribution

$$p(r|R) = \frac{r}{\sigma_r^2}\exp\left(-\frac{r^2}{2\sigma_r^2}\right)$$

Therefore

$$20\log\left(E\Big[r|R\Big]\right) = 20\log\left(\sqrt{\pi/2}\,\sigma_r\right) = R$$

or, equivalently

$$\sigma_r = \sqrt{2/\pi}\;10^{R/20}$$

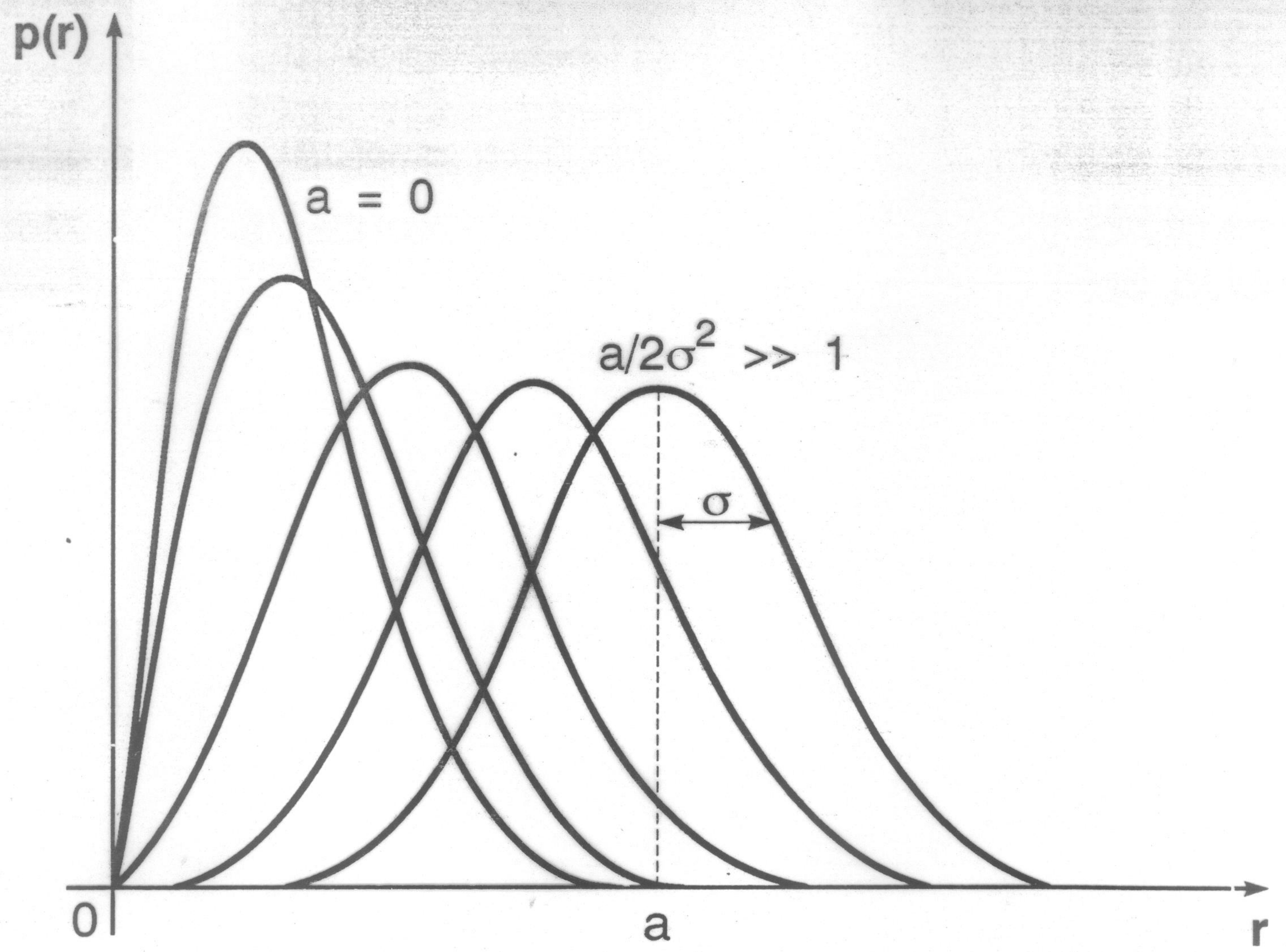


FIG3-21 200%

Nakagami Distribution

$$p(r) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^{m} r^{2m-1} \exp\left(-\frac{m}{\Omega} r^2\right)$$

$\Omega = E[r^2]$ mean square value

$$m = \frac{\Omega^2}{Var(r^2)} \geq 1$$

$$\Gamma(m) = \int_0^{\infty} x^{m-1} \exp(-x)\, dx$$

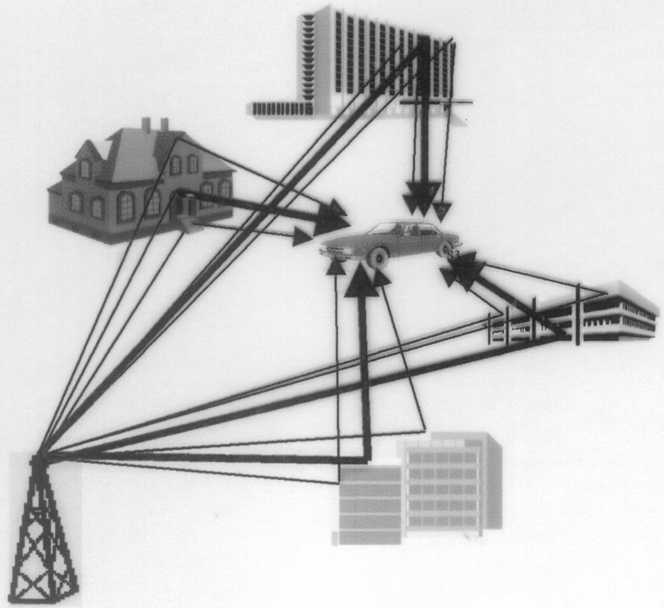
Rice & Nakagami

$$k = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}} \qquad m \geq 1$$

UNICAMP

Physical Model

- Clusters of multipath waves propagating in an non-homogeneous and non-linear environment.
- Within any one cluster, phases are random and have similar delay times with delay-time spreads of different clusters being relatively large.
- The clusters of multipath waves are assumed to have the scattered waves with identical powers
- Envelope is a non-linear function of the modulus of the sum of the multipath components



UNICAMP

Introduction

- **The κ-μ Distribution**

$$p(\rho)=\frac{2\mu(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}}\exp(\mu\kappa)}\rho^{\mu}\exp\left(-\mu(1+\kappa)\rho^{2}\right)I_{\mu-1}\left(2\mu\sqrt{\kappa(1+\kappa)}\rho\right)$$

$\kappa \geq 0$

$\mu \geq 0$

$$\mu=\frac{1}{Var\left(\rho^{2}\right)}\times\frac{1+2\kappa}{(1+\kappa)^{2}}$$

κ =0 $\Rightarrow$ Nakagami

μ =1 $\Rightarrow$ Rice

UNICAMP

Introduction

The η-μ Distribution

$$p(\rho) = \frac{4\sqrt{\pi}\,\mu^{\mu+\frac{1}{2}} h^{\mu}}{\Gamma(\mu) H^{\mu-\frac{1}{2}}} \rho^{2\mu} \exp\left(-2\mu h \rho^2\right) I_{\mu-\frac{1}{2}}\left(2\mu H \rho^2\right)$$

$$h = \frac{2+\eta^{-1}+\eta}{4} \qquad H = \frac{\eta^{-1}-\eta}{4} \qquad \mu = \frac{E^2(r^2)}{Var(r^2)} \times \frac{1+\eta^2}{(1+\eta)^2}$$

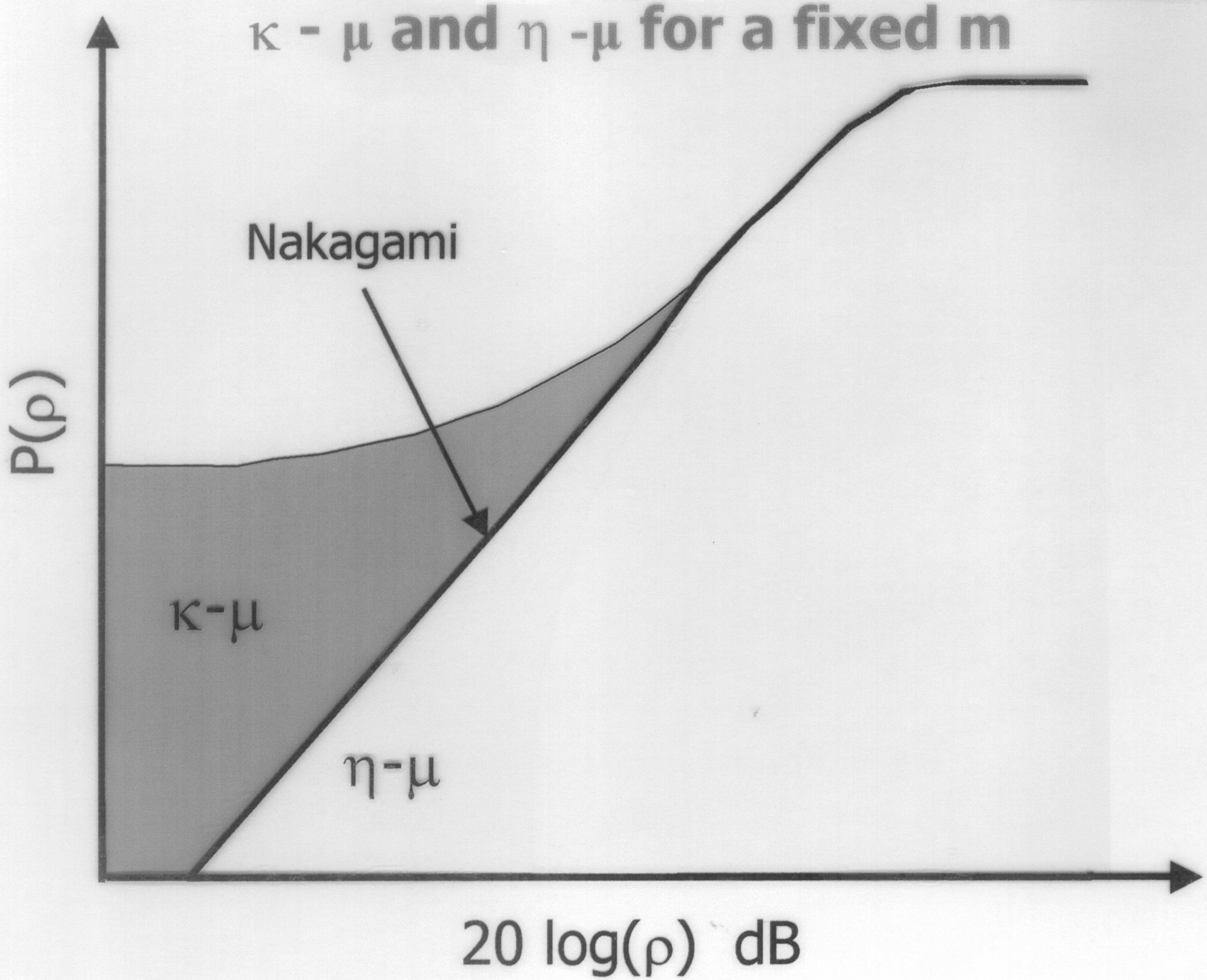
$$0 \le \eta \le 1 \qquad \mu \ge 0$$

η =0 ⇒ Nakagami **η =1 ⇒ Nakagami**

μ =1/2 ⇒ Hoyt

Introduction

κ - μ and η -μ for a fixed m



UNICAMP

Outline of the Derivation

$$r = \left[\sum_{i=1}^{n}\left(x_i^2 + y_i^2\right)\right]^{\frac{1}{\alpha}}$$

$$E(x_i) = E(y_i) = 0 \qquad E\left(x_i^2\right) = \sigma^2 \qquad E\left(y_i^2\right) = \sigma^2$$

$$E\left(r^{\alpha}\right) = 2\sigma^2 n \qquad E\left(r^{2\alpha}\right) = \left(2\sigma^2\right)^2 (n+1)n \qquad \mu \stackrel{\Delta}{=} \frac{E^2\left(r^{\alpha}\right)}{E\left(r^{2\alpha}\right) - E^2\left(r^{\alpha}\right)} = n$$

$$p(\rho) = \frac{\alpha\mu^{\mu}\rho^{\alpha\mu-1}}{\Gamma(\mu)}\exp\left(-\mu\rho^{\alpha}\right)$$

UNICAMP

The α-μ Distribution

$$p(\rho) = \frac{\alpha\mu^{\mu}\rho^{\alpha\mu-1}}{\Gamma(\mu)}\exp\left(-\mu\rho^{\alpha}\right) \qquad P(r) = \frac{\Gamma\left(\mu, \mu r^{\alpha}/\hat{r}^{\alpha}\right)}{\Gamma(\mu)}$$

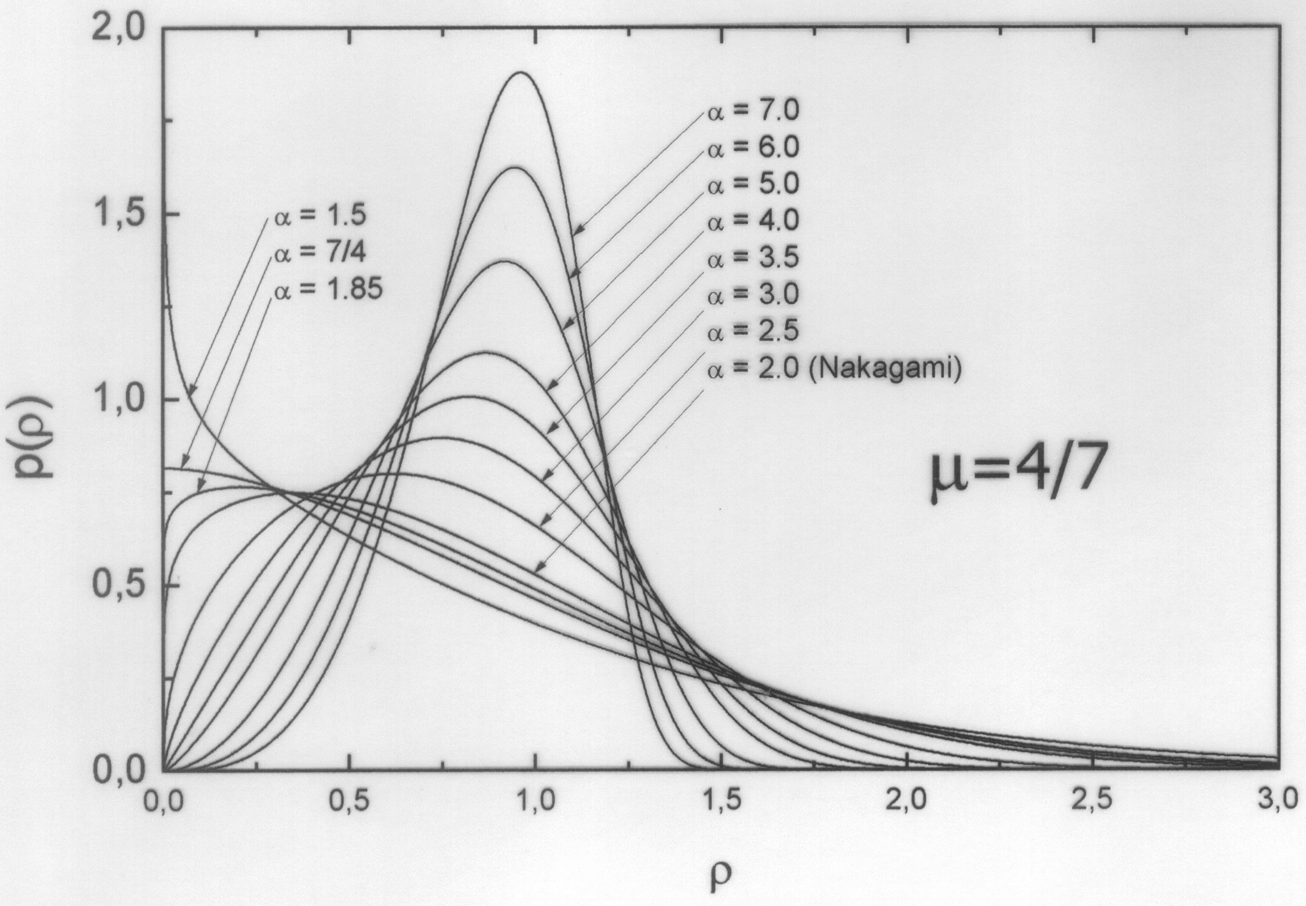
$$\rho = r/\hat{r} \qquad \hat{r} = \sqrt[\alpha]{E\left(r^{\alpha}\right)} \qquad \alpha > 0 \qquad \mu \geq 0$$

$$\mu = \frac{E^2\left(r^{\alpha}\right)}{E\left(r^{2\alpha}\right) - E^2\left(r^{\alpha}\right)} \qquad \mu = \frac{1}{Var\left(\rho^{\alpha}\right)} \qquad E\left(\rho^{k}\right) = \frac{\Gamma\left(\mu + k/\alpha\right)}{\mu^{k/\alpha}\Gamma(\mu)}$$

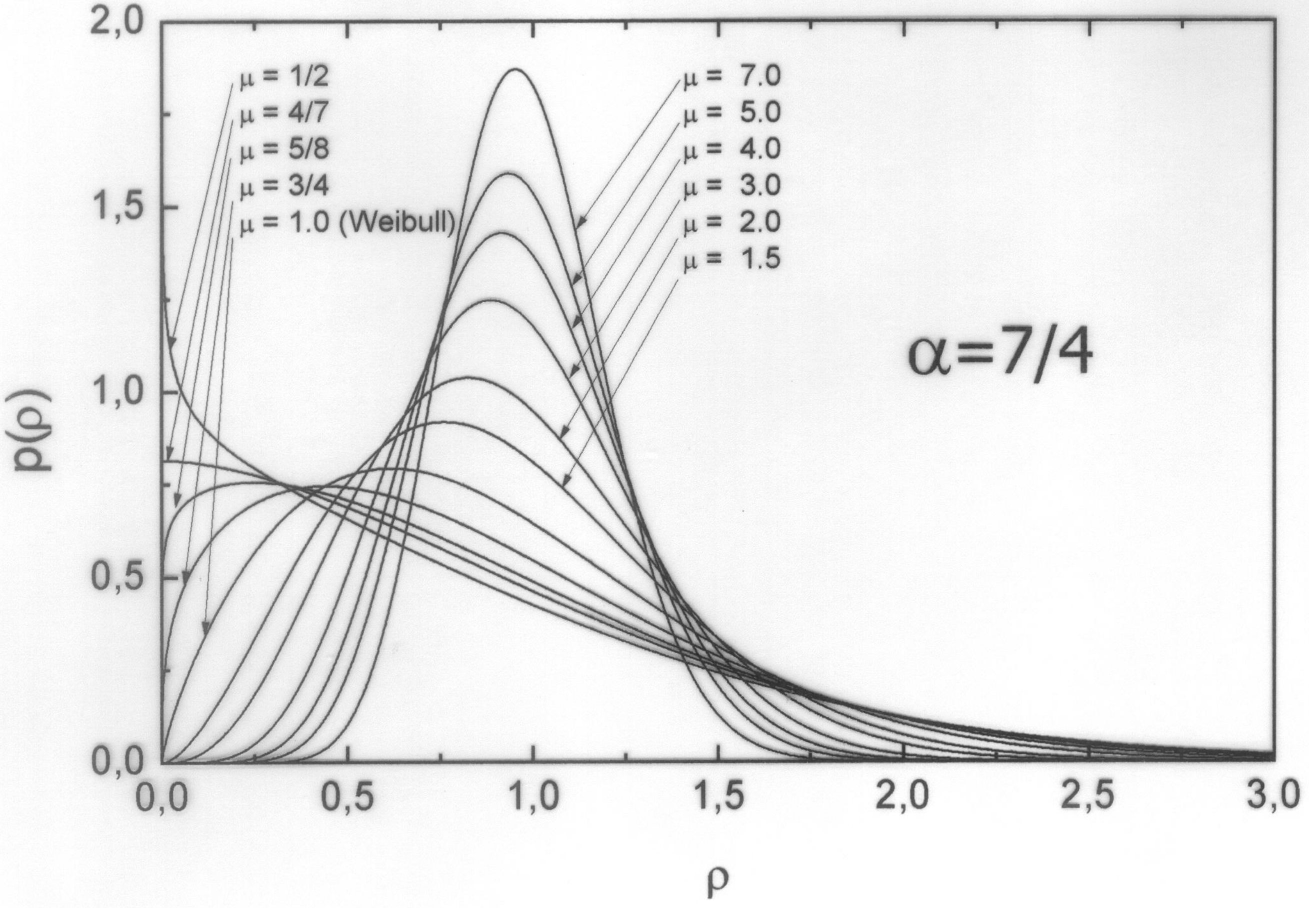
α=2 ⇒ Nakagami **μ =1 ⇒ Weibull**

UNICAMP

Sample Shapes of The α-μ Distribution

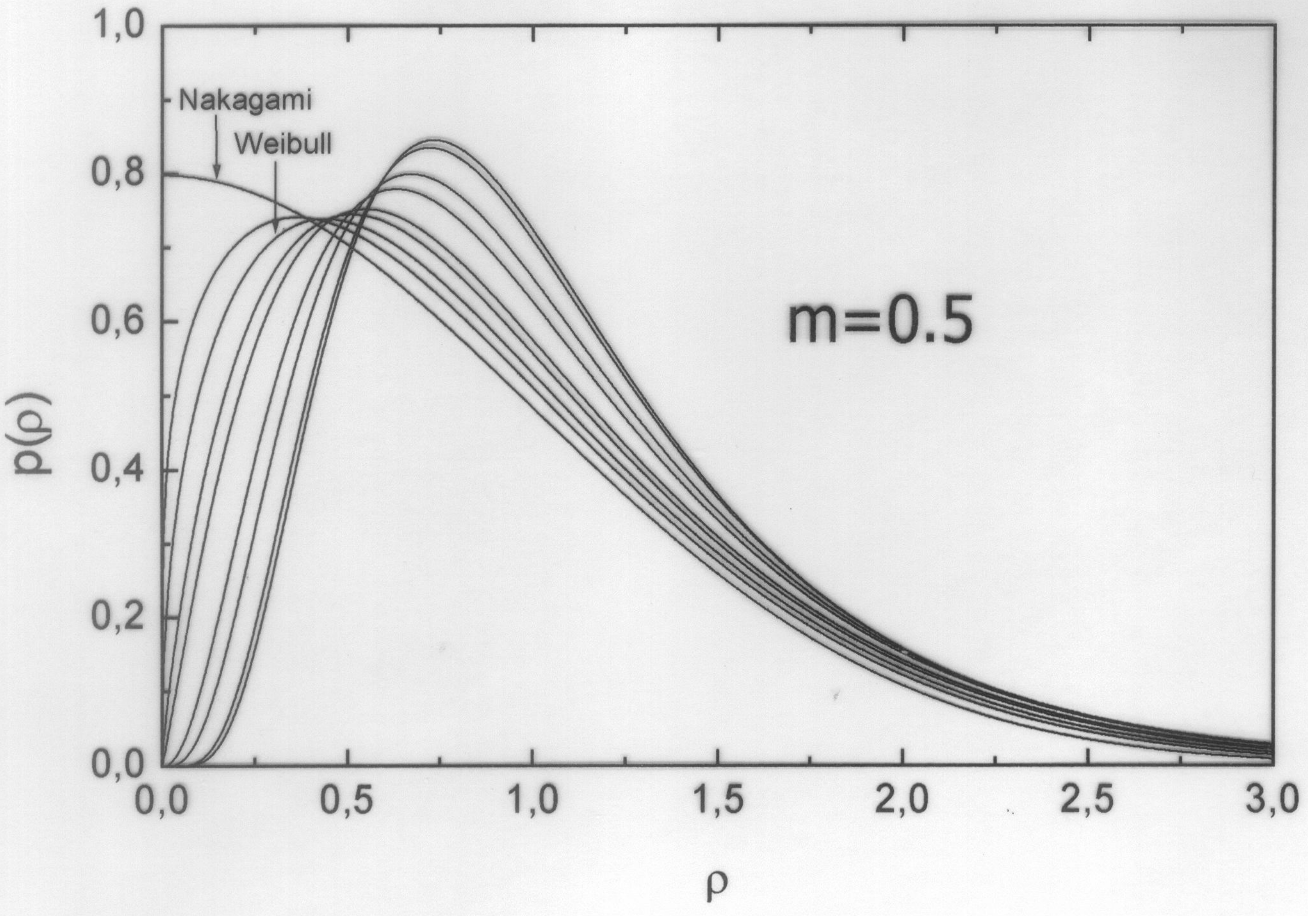


Sample Shapes of The α-μ Distribution

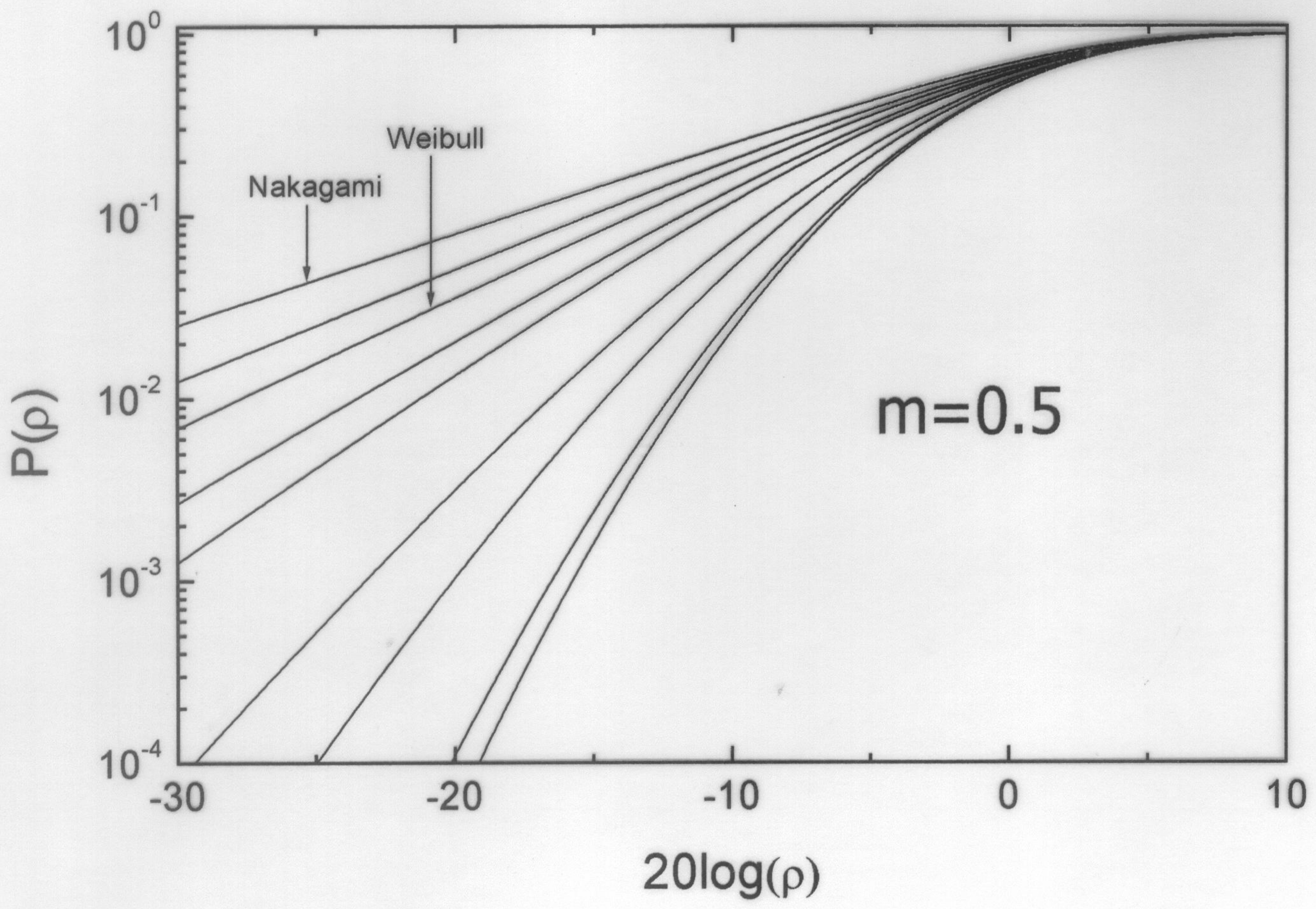


UNICAMP

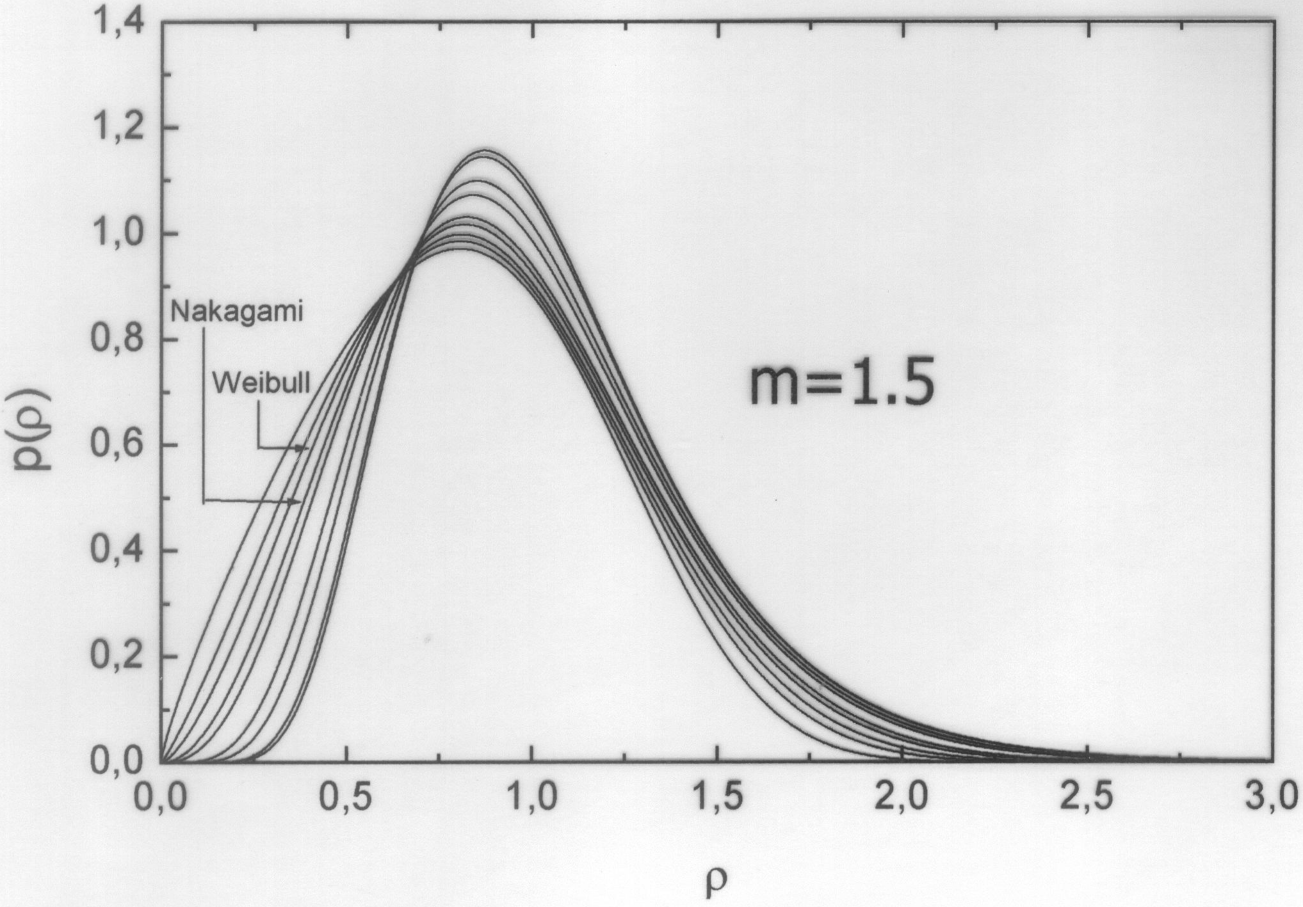
Sample Shapes of The α-μ Distribution



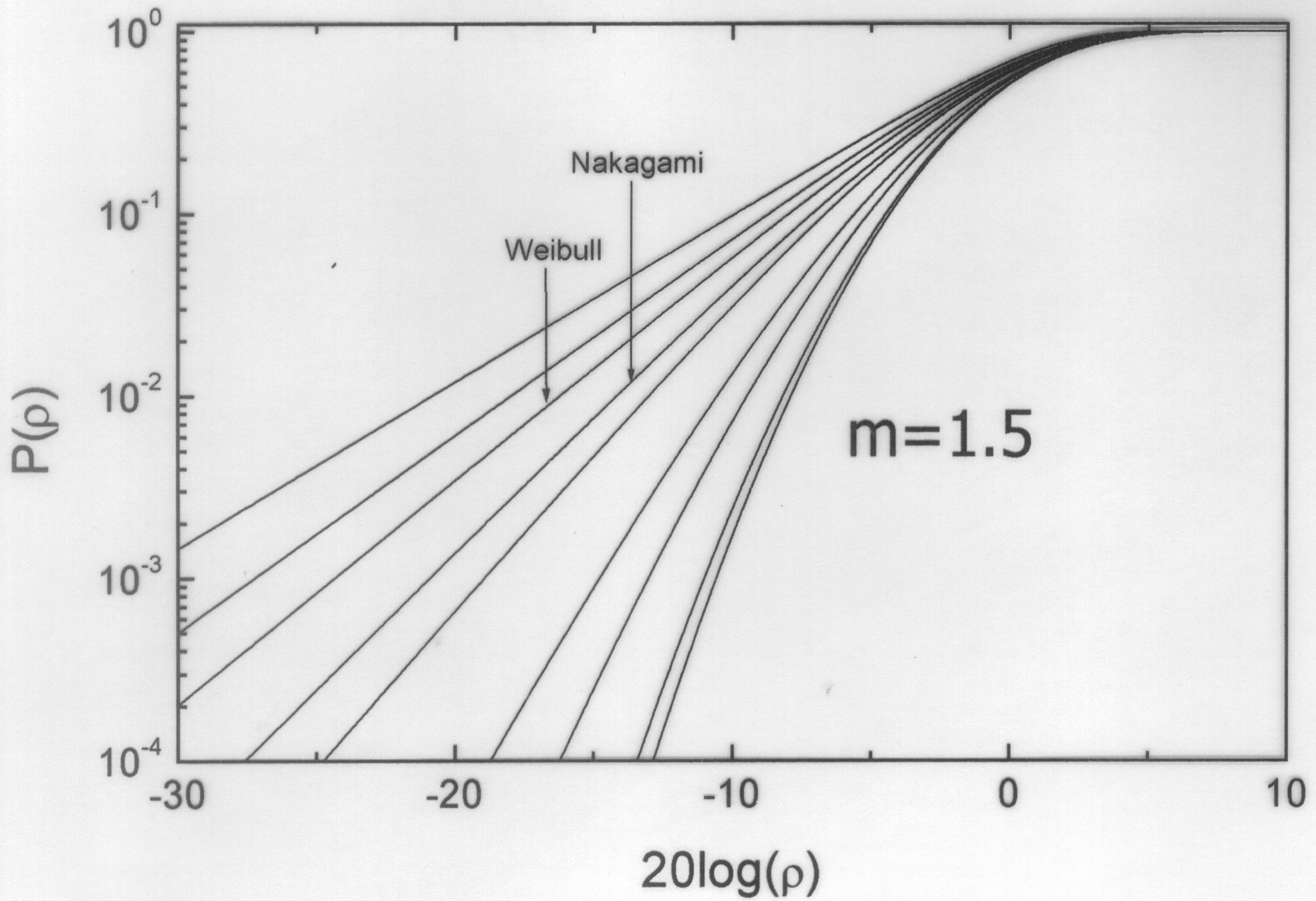
Sample Shapes of The α-μ Distribution



Sample Shapes of The α-μ Distribution



Sample Shapes of The α-μ Distribution



$$p(r|R) = \frac{\pi r}{2\ 10^{R/10}} \exp\left(-\frac{\pi r^2}{4\ 10^{R/10}}\right)$$

$$p(r) = \int_{-\infty}^{\infty} p(r|R)p(R)dR$$

$$p(r) = \sqrt{\frac{\pi}{8\sigma_R^2}} \int_{-\infty}^{\infty} \frac{r}{10^{R/10}} \exp\left(-\frac{\pi r^2}{4\ 10^{R/10}}\right) \exp\left[-\frac{1}{2}\left(\frac{R - M_R}{\sigma_R}\right)^2\right] dR$$

SIGNAL COVERAGE AREA (CELL AREA)

$$\frac{w_l}{w_L} = \left(\frac{l}{L}\right)^{-\alpha} c$$

$$m_w \triangleq k(l/L)^{-\alpha}$$

$$M_w \triangleq 10\log m_w = K - 10\alpha\log(l/L)$$

where $K = 10\log k$.

Proportion of Locations at Distance L

$$\beta \triangleq \text{prob}(w \geq w_0) = \int_{w_0}^{\infty} p(w)dw$$

or

$$\beta \triangleq \text{prob}(W \geq W_0) = \int_{W_0}^{\infty} p(W)dW$$

where $W = 10\log w$ and $W_0 = 10\log w_0$.

Proportion of Locations Within the Area Defined by L

$$\mu = \frac{1}{A} \int_A \text{prob}(w \geq w_0) dA$$

where $A = \pi L^2$ and $dA = l dl d\theta$. Therefore

$$\mu = \frac{1}{\pi L^2} \int_0^L \int_0^{2\pi} \text{prob}(w \geq w_0) l dl d\theta$$

$$= \frac{1}{L^2} \int_0^L \text{prob}(w \geq w_0) l dl$$

If W and W_0 are expressed in dB

$$\mu = \frac{1}{L^2} \int_0^L \text{prob}(W \geq W_0) l dl$$

Log-Normal Fading Case

$$p(W) = \frac{1}{\sqrt{2\pi}\, \sigma_w} \exp\left[-\frac{1}{2} \left(\frac{W - M_w}{\sigma_w} \right)^2 \right]$$

$$\beta = \text{prob}(W \geq W_0) = \frac{1}{2} \left[1 - \text{erf}\left(\frac{W_0 - M_w}{\sqrt{2}\, \sigma_w} \right) \right]$$

$$\mu = \frac{2}{L^2} \int_0^L \int_{u_0}^{\infty} \frac{1}{\sqrt{\pi}} \exp(-u^2) du l dl$$

where

$$u = \frac{W - K + 10\alpha \log(l/L)}{\sqrt{2}\, \sigma_W}$$

and

$$u_0 = \frac{W_0 - K + 10\alpha \log(1/L)}{\sqrt{2}\,\sigma_W}$$

$$\operatorname{erf}(u_0) \triangleq 2 \int_0^{u_0} \frac{1}{\sqrt{\pi}} \exp(-v^2)dv$$

$$\mu = \frac{2}{L^2} \int_0^L \left[\frac{1}{2} - \frac{1}{2}\operatorname{erf}(u_0)\right] l\,dl = \frac{1}{2} - \frac{1}{L^2} \int_0^L \operatorname{erf}(u_0) l\,dl$$

$$\mu = \frac{1}{2} - \frac{\sqrt{2}\,\sigma_W}{10\alpha \log e} \exp\left(-2\,\frac{W_0 - K}{10\alpha \log e}\right) \int_{-\infty}^{\frac{W_0 - K}{\sqrt{2}\,\sigma_W}} \exp\left(\frac{2\sqrt{2}\,u_0}{10\alpha \log e}\right) \operatorname{erf}(u_0)du_0$$

$$\mu = \frac{1}{2}\left\{1 + \operatorname{erf}\left(\frac{K - W_0}{\sqrt{2}\,\sigma_W}\right) + \exp\left[\frac{2(K - W_0)10\alpha \log e + 2\sigma_W^2}{100\alpha^2 \log^2 e}\right]\right.$$
$$\left.\left[1 - \operatorname{erf}\left(\frac{(K - W_0)10\alpha \log e + 2\sigma_W^2}{(10\alpha \log e)\sqrt{2}\,\sigma_W}\right)\right]\right\}$$

__Rayleigh Fading Case__

$$p(w) = \frac{1}{m_w} \exp\left(-\frac{w}{m_w}\right)$$

$$\beta = \operatorname{prob}(w \geq w_0) = \exp\left(-\frac{w_0}{m_w}\right)$$

$$\mu = \frac{2}{\alpha}\left(\frac{k}{w_0}\right)^{2/\alpha} \Gamma_c\left(\frac{2}{\alpha}, \frac{w_0}{k}\right)$$

where

$$\Gamma_c(x,y) = \int_0^y t^{x-1}\exp(-t)dt$$

<u>Some Examples</u>

1) Let the mean signal strength at $l = L$ be $M_w = K = -100$ dBm* in an environment where $\alpha = 3.5$. We want to estimate the probability that the received signal exceed a threshold $W_0 = -105$ dBm a) within the circular area delimited by L and b) at the perimeter of the corresponding circumference.

Log-Normal Fading Case

Assume that $\sigma_w = 5$ dB. Hence $(W_0 - K)/\sigma_w = (W_0 - M_w)/\sigma_w = -1$.

a) From Figure 3.22 $\mu = 96\%$

b) From (3.102) $\beta = 84.13\%$

Rayleigh Fading Case

Since $W_0 - K = -5$ dBm

a) From Figure 3.23 $\mu = 90\%$

b) From (3.110) $\beta = 73\%$

2) Let the mean signal strength at $l = 10$ Km be $M_w = -100$ dBm in an environment where $\alpha = 3.5$. We want to estimate the cell radius L such that the mobile stations receive a signal power above $W_0 = -110$ dBm 90% of the time a) within the circular area delimited by L and b) at the perimeter of the corresponding circumference.

Log-Normal Fading Case

Assume that $\sigma_w = 5$ dB.

a) From Figure 4.22 with $\mu = 90\%$ we find $(W_0 - K)/\sigma_w = -0.47$, yielding $K = -107.65$ dBm. Since $M_w - K = -10\alpha\log(l/L)$, then $L = 16.5$ Km.

b) From (3.102) with $\beta = 90\%$ we find $(W_0 - K)/\sigma_w = -1.28$, yielding $K = -103.6$ dBm. Then $L = 12.7$ Km.

Rayleigh Fading Case

a) From Figure 4.23 with $\mu = 90\%$ we find $W_0 - K = -5.2$ dBm, yielding $K = -104.8$ dBm. Then $L = 13.7$ Km.

b) From (3.110) with $\beta = 90\%$ we find $W_0 - K \simeq -100$ dBm. Then $L = 10$ Km.

BOUNDARIES BETWEEN CELLS

Joint Rayleigh Fading

$$p(r_i) = \frac{r_i}{m_i^2} \exp\left[\frac{-r_i^2}{2m_i^2}\right] \qquad i = 1,\ 2$$

$$p(r_1,r_2) = p(r_1)p(r_2)$$

$$b = \frac{m_2}{m_1}$$

$$B = 20\log b$$

$$A = 20\log a$$

$$C = -10\log(c^2/m_1 m_2)$$

$$P(a,b,c) = \frac{\exp\left[-\frac{c^2}{2m_2^2}\left(1 + \frac{b^2}{a^2}\right)\right]}{1 + \frac{b^2}{a^2}} + \frac{\exp\left[-\frac{c^2}{2m_1^2}\left(1 + \frac{1}{a^2 b^2}\right)\right]}{1 + \frac{1}{a^2 b^2}} - \exp\left[-\frac{c^2}{2}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\right]$$

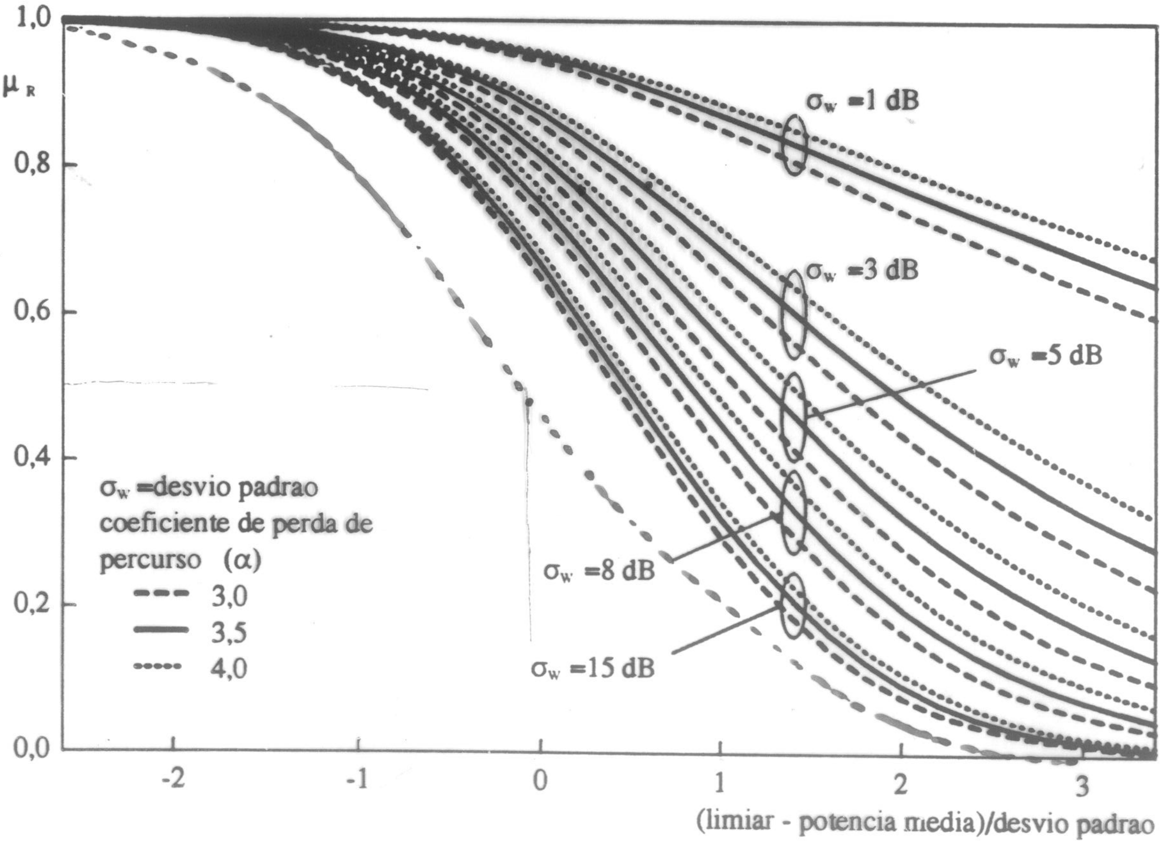
$$P(a,b,0) = \frac{a^2 - \frac{1}{a^2}}{\left(a^2 + \frac{1}{a^2}\right) + \left(b^2 + \frac{1}{b^2}\right)}$$

3.1 AMBIENTE LOG-NORMAL

Resultado obtido por Reudink

$$\mu_n = \frac{1}{2}\left\{1 + \operatorname{erf}(a) + \exp\left(\frac{2\,a\,b+1}{b^2}\right)\left[1 - \operatorname{erf}\left(\frac{a\,b+1}{b}\right)\right]\right\}$$

$a = (K - W_T)/\sqrt{2}\,\sigma_W.$
$b = 10\alpha\log(e)/\sqrt{2}\,\sigma_W.$



Joint Log-Normal Fading

$$p(R_i) = \varphi\left(\frac{R_i - M_i}{\sigma_i}\right)$$

$$\varphi(u) = \frac{1}{(2\pi)^{1/2}} \exp\left(-\frac{u^2}{2}\right)$$

$$\phi(u) = \int_{-\infty}^{u} \varphi(x)dx$$

$$R = R_2 - R_1$$

$$B = M_2 - M_1$$

$$M = M_2 - M_1 = B$$

and

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho_R\sigma_1\sigma_2$$

$$p(A) = \varphi\left(\frac{A - B}{\sigma}\right) + \varphi\left(\frac{A + B}{\sigma}\right)$$

$$P(A) = \phi\left(\frac{B + A}{\sigma}\right) - \phi\left(\frac{B - A}{\sigma}\right)$$

The Geographical Distribution of the Mean Power Ratio

$$b^2 = \left(\frac{m_2}{m_1}\right)^2 = \left(\frac{x_1}{x_2}\right)^\alpha$$

$$h = \frac{x_1}{x_2} = b^{2/\alpha}$$

$$y^2 + \left(x - \frac{h^2 + 1}{h^2 - 1}\right)^2 = \left(\frac{2h}{h^2 - 1}\right)^2$$

$$\gamma = \frac{\text{area OBED}}{\text{area OBA}}$$

$$\gamma = 1 + \frac{2\sqrt{3}\, Y_E}{h^2 - 1} - 4\sqrt{3}\left(\frac{h}{h^2 - 1}\right)^2 \sin^{-1}\left(\frac{Y_E(h^2 - 1)}{2h}\right)$$

where

$$2\sqrt{3}\, Y_E = -3\left(\frac{1}{h^2 - 1}\right) + \sqrt{\left(\frac{2h^2 + 1}{h^2 + 1}\right)^2 - 4}$$

The Geographical Distribution of Instantaneous Power Ratio

$$\gamma = \int_0^1 d(x)P(a,x)dx$$

Proportion of the Cell Area With Three Alternative Paths

The Geographical Distribution of Mean Power Ratio

$$\delta = \frac{\text{area OBEG}}{\text{area OBA}}$$

$$\delta = 1 + \frac{2\sqrt{3}\, Y'_G}{h^2 - 1} - \sqrt{3}\, CD^2\left[\sin^{-1}\left(\frac{Y'_G}{CD}\right) - \sin^{-1}\left(\frac{Y_E}{CD}\right)\right]$$

$$- \frac{4\tan(\phi)\left[\sqrt{3}\, Y_E(h^2 - 1) - 3\right]}{(h^2 - 1)^2\,(\sqrt{3} - \tan(\phi))}$$

$$Y_E = \frac{-\sqrt{3} + \sqrt{4h^2 - 1}}{2(h^2 - 1)}$$

$$Y'_G = \frac{\sqrt{3}\left(-1 + \sqrt{4h^2 - 3}\right)}{2(h^2 - 1)}$$

$$CD = \frac{2h}{h^2 - 1}$$

$$\tan(\phi) = \frac{Y_E}{\sqrt{CD^2 - Y_E^2}}$$

$$h \geq \sqrt{3}\ .$$

In a similar way

$$\delta = \frac{\text{area BEG}}{\text{area OBA}}$$

$$\delta = 1 + \frac{12\tan(\theta)}{(\sqrt{3} - \tan(\theta))\,(h^2 - 1)^2} - \sqrt{3}\,CD^2\left[\sin^{-1}\left(\frac{Y'_G}{CD}\right) - \sin^{-1}\left(\frac{Y_E}{CD}\right)\right]$$

$$- \frac{4\tan(\phi)\left[\sqrt{3}\,Y_E(h^2 - 1) - 3\right]}{(h^2 - 1)^2\,(\sqrt{3} - \tan(\phi))} - (3 - 2\sqrt{3}\,Y'_G)\left[1 - \frac{4\tan(\theta)}{(h^2 - 1)\,(\sqrt{3} - \tan(\theta))}\right]$$

where Y_E, CD and $\tan(\phi)$ are given by (3.129b), (3.129d) and (3.129e) respectively, and

$$Y'_G = \frac{h\left(\sqrt{3}\,h - \sqrt{4 - h^2}\right)}{2(h^2 - 1)}$$

$$\tan(\theta) = \frac{Y'_G}{\sqrt{CD^2 - Y'^2_G}}$$

$$1 \leq h \leq \sqrt{3}$$

Joint Log-Normal Fading

$$p(R_1,R_2,R_3) = \frac{1}{\left(\sqrt{2\pi}\,\sigma\right)^3} \exp\left\{ \frac{-1}{2\sigma^2} \left[\left(R_1 - M_1\right)^2 + \left(R_2 - M_2\right)^2 + \left(R_3 - M_3\right)^2\right]\right\}$$

$$P(R_1,R_2,R_3) = \iiint_S p(R_1,R_2,R_3)dR_1dR_2dR_3$$

where S is the solid determined by

$$|R_1 - R_2| \le A$$

$$|R_2 - R_3| \le A$$

$$|R_3 - R_1| \le A$$

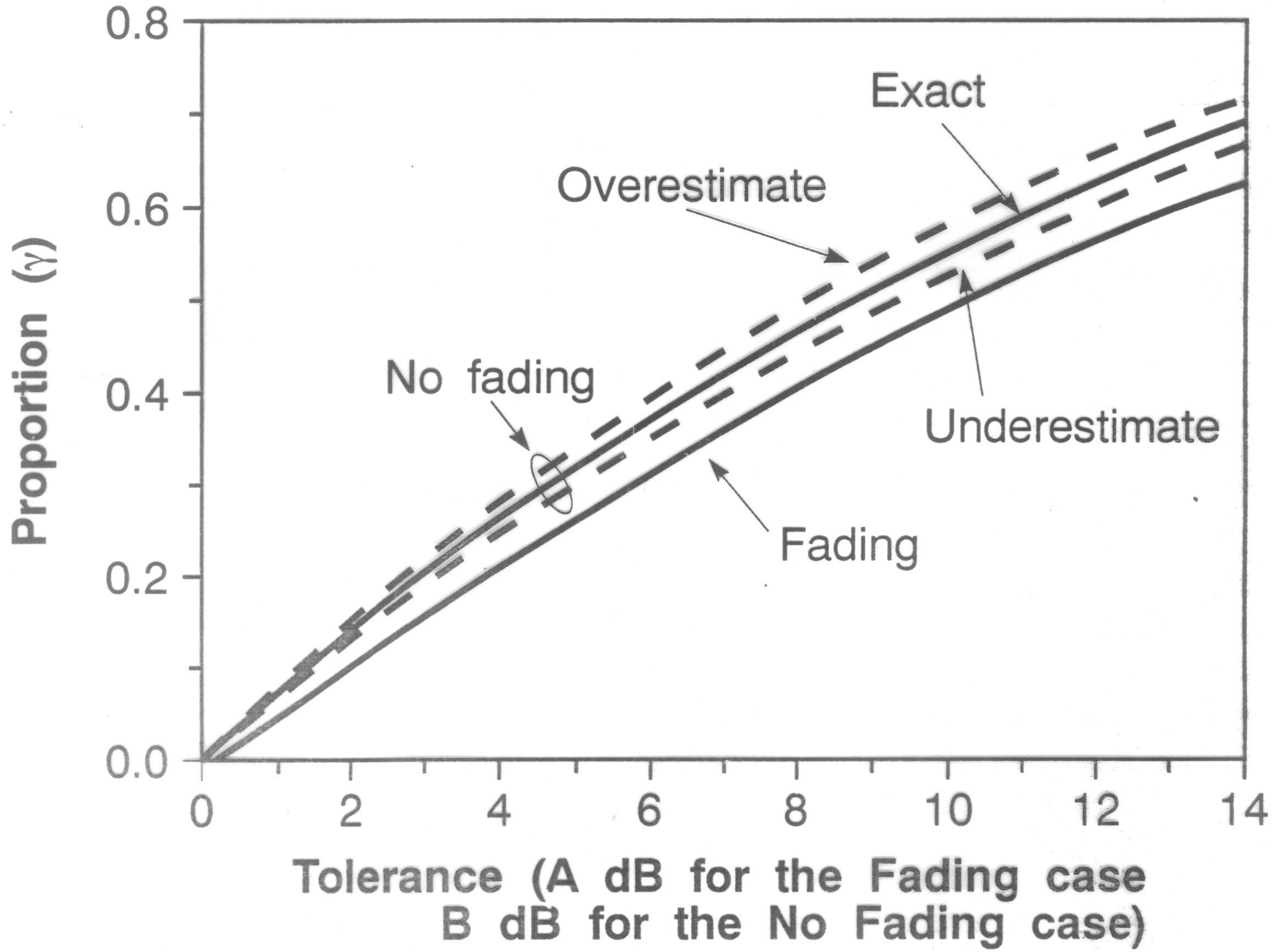
The Geographical Distribution of the Instantaneous Power Ratio

$$\delta = \frac{1}{T} \iint_T P(R_1,R_2,R_3,x,y)dxdy$$

$$\delta \simeq \frac{\Delta x \Delta y}{T} \sum_{i=0}^{\frac{1}{\Delta x}} \sum_{j=0}^{\frac{1}{\Delta x} - i} W\left(-i\Delta x, j\frac{\Delta x}{\sqrt{3}}\right) P\left(-i\Delta x, j\frac{\Delta x}{\sqrt{3}}\right)$$

δ as Function of γ

$$\delta = 1.25\,\gamma^2, \quad 0 \le \gamma \le 0.8.$$



Exact
Overestimate
No fading
Underestimate
Fading
0.0
0.2
0.4
0.6
0.8
0
2
4
6
8
10
12
14
Proportion (γ)
Tolerance (A dB for the Fading case
B dB for the No Fading case)

FIG3-28 200%

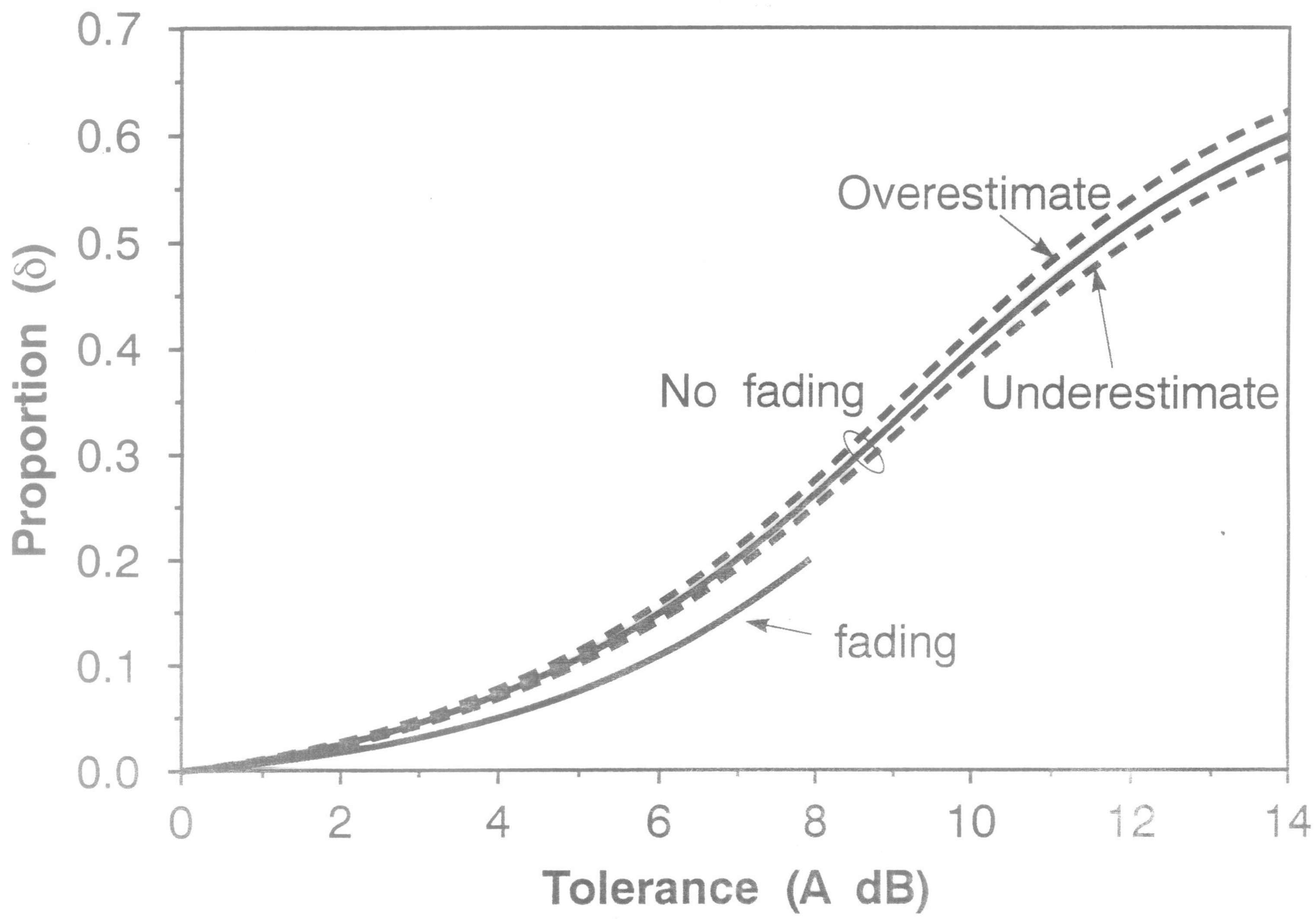

Proportion (δ)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
2
4
6
8
10
12
14
Tolerance (A dB)
Overestimate
No fading
Underestimate
fading

FIG3-33 200%

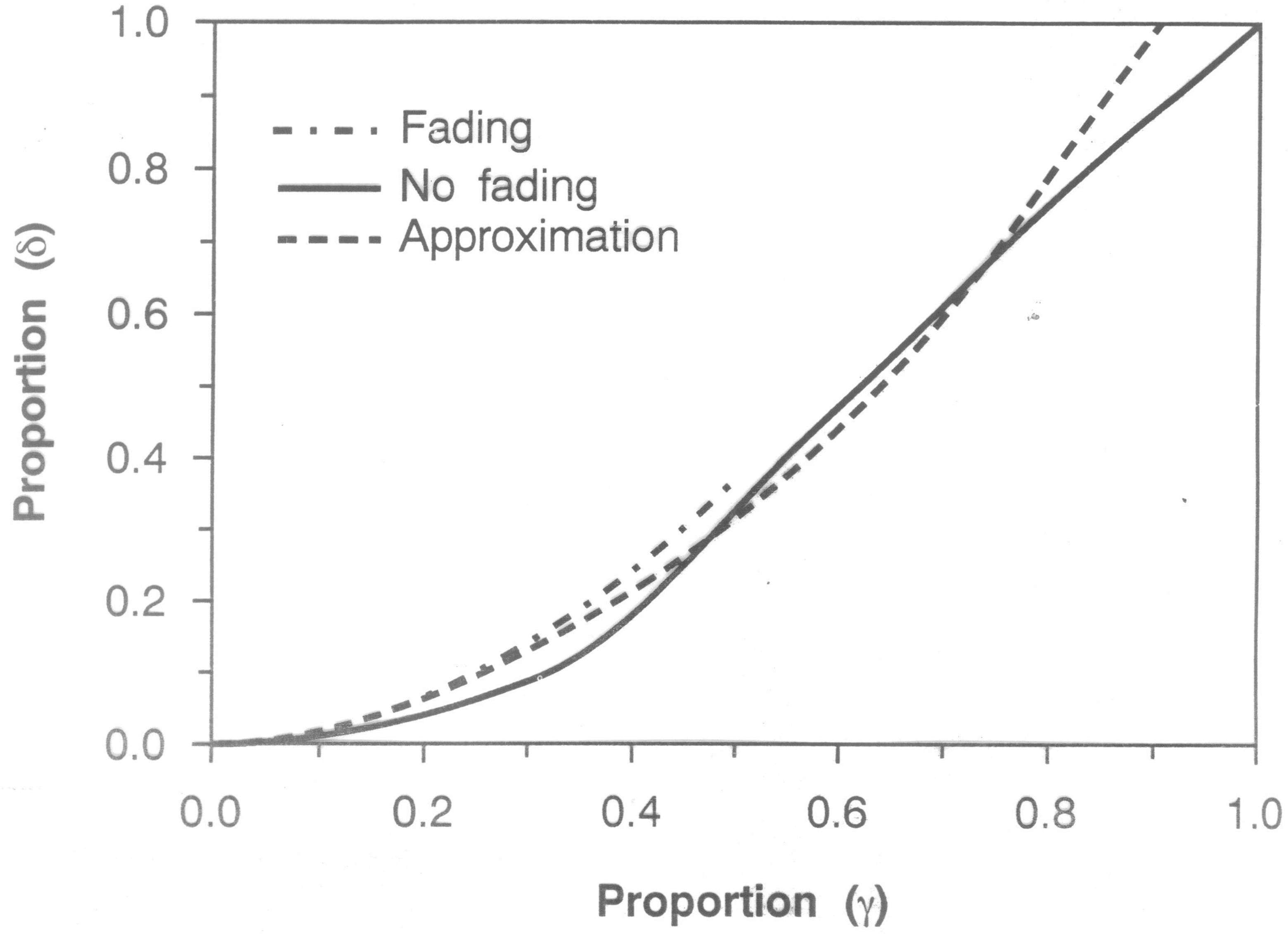

1.0
0.8
0.6
0.4
0.2
0.0
Proportion (δ)
Fading
No fading
Approximation
0.0
0.2
0.4
0.6
0.8
1.0
Proportion (γ)

FIG3-34 200%

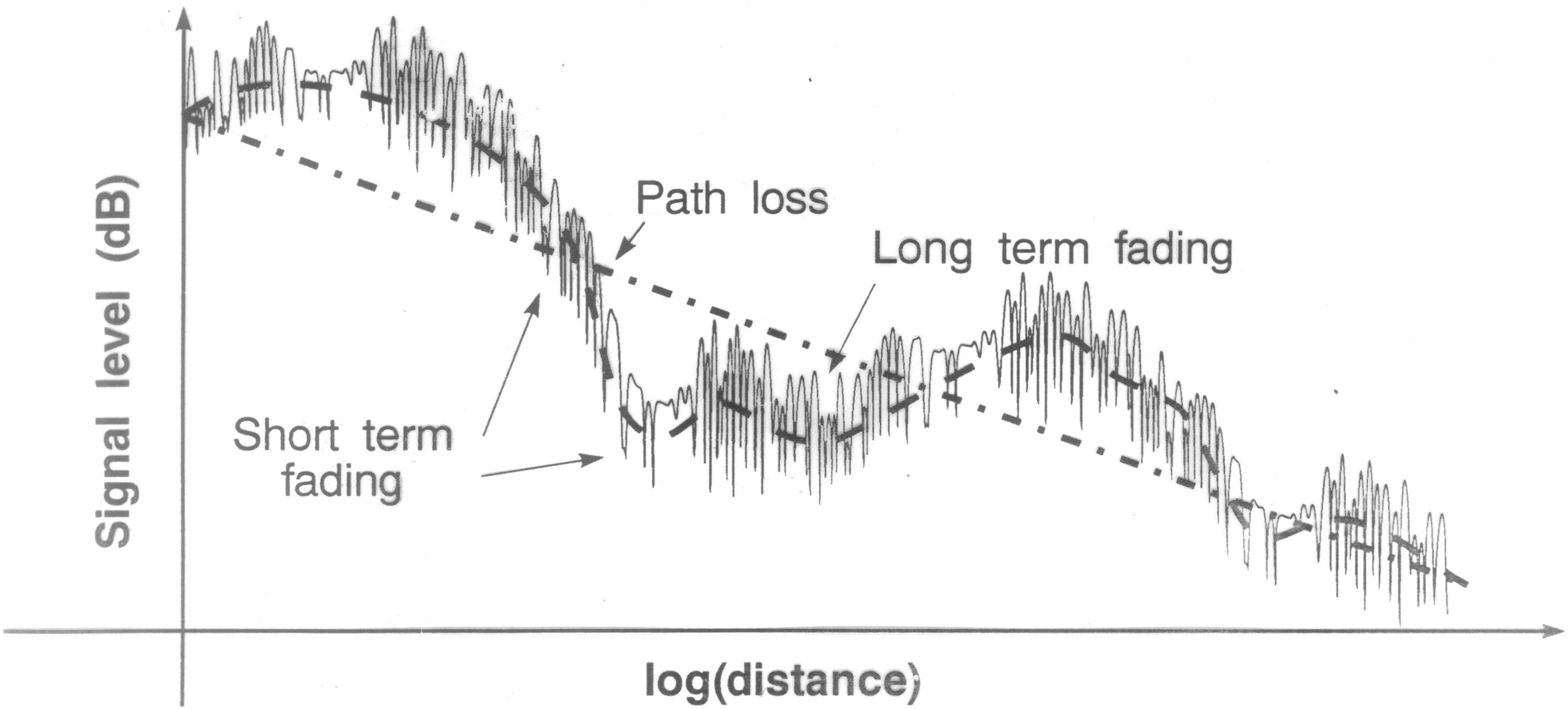

Signal level (dB)
Path loss
Long term fading
Short term fading
log(distance)

FIG3-35 200%