MULTIPATH PROPAGATION EFFECTS

INTRODUCTION

VELOCITIES OF WAVE PROPAGATION

α: ~~phase~~ attenuation constant
β: phase constant

$$e = E_M \exp(j\omega t) = E_0 \exp(-\alpha r) \exp\left[j(\omega t - \beta r)\right]$$

Phase Velocity

$$\omega t - \beta r = \text{constant}$$

μ: magnetic permeability
ε: electric permittivity

Therefore,

$$v_p = \frac{dr}{dt} = \frac{\omega}{\beta}$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

Thus

$$v_p = 1/\sqrt{\mu\varepsilon} \quad ; \quad \lambda = v_p T = v_p \frac{1}{f} = \frac{\omega}{\beta} \cdot \frac{1}{f} = \frac{2\pi f}{\beta f}$$

$$\therefore \boxed{\lambda = \frac{2\pi}{\beta}}$$

Group Velocity

$$v_g = d\omega/d\beta$$

$$v_p = \frac{\omega}{\beta} \; ; \; \omega = \beta v_p \; ; \; \frac{d\omega}{d\beta} = \beta \frac{\partial v_p}{\partial \beta} + v_p$$

$$v_g = \frac{v_p}{1 - \frac{\omega}{v_p} \frac{dv_p}{d\omega}}$$

$$\frac{d\omega}{d\beta} = v_p + \beta \frac{\partial v_p}{\partial \omega} \frac{d\omega}{d\beta} = v_p + \frac{\omega}{v_p} \frac{\partial v_p}{\partial \omega} v_g$$

$$\therefore v_g = v_p + \frac{\omega}{v_p} \frac{\partial v_p}{\partial \omega} v_g$$

DOPPLER FREQUENCY

$$v_p = \lambda f$$

$$v'_p = v_p - v\cos\theta \quad \therefore \quad \lambda f' = \lambda f - v \cos\theta$$

$$f' = f - \frac{v}{\lambda}\cos\theta$$

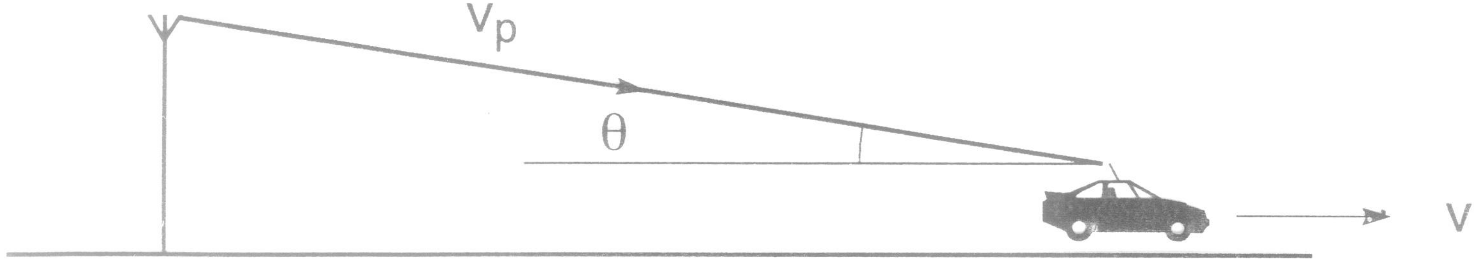


FIG4-1 200%

$$f' = f - f_D$$

where

$$f_D = \frac{v}{\lambda}\cos\theta$$

$$\omega_D = \omega_m \cos\theta$$

where $\omega_m \overset{\Delta}{=} \beta v$ is the maximum Doppler shift. $v = 72\text{km/h}$, $f = 900\text{MHz}$, $f_D = 60$ Hz

Relativity Theory

$$f' = f\,\frac{1 \pm v/c}{\sqrt{1 - (v/c)^2}}$$

$$f' = f\left[1 - \frac{v}{c} + \frac{1}{2}\left(\frac{v}{c}\right)^2 - \ldots\right]$$

$$f' \simeq f - fv/c$$

Given that $c = \lambda f$, then

$$f' \simeq f - v/\lambda$$

DELAY SPREAD, MEAN TIME DELAY

$$E[T^k] \overset{\Delta}{=} \int_0^\infty T^k p(T)\,dT$$

$$h(t) = \sum_{i=1}^{n} a_i\,\delta(t - T_i)$$

$$\bar{T} = E[T]$$

$$\sigma_T^2 = E[T^2] - E^2[T]$$

$$p(T) \simeq \frac{1}{\bar{T}}\exp\left(-\frac{T}{\bar{T}}\right)$$

COHERENCE BANDWIDTH

Definition; Narrowband Systems; Wideband Systems

Received Signal

$\bar{T} = 0.2\mu s$ open area
$0.5\mu s$ suburban
$3\mu s$ urban

$$e_i = E_0\exp(-\alpha_i r_i)\exp\left[j(\omega t + \omega_i t - \omega T_i)\right]$$

$$\omega T_i = 2\pi f T_i = \frac{2\pi}{\lambda} v_p T_i = \frac{2\pi}{\lambda} r_i;\ \beta r_i; \quad \omega_i = \beta v\cos\theta_i$$

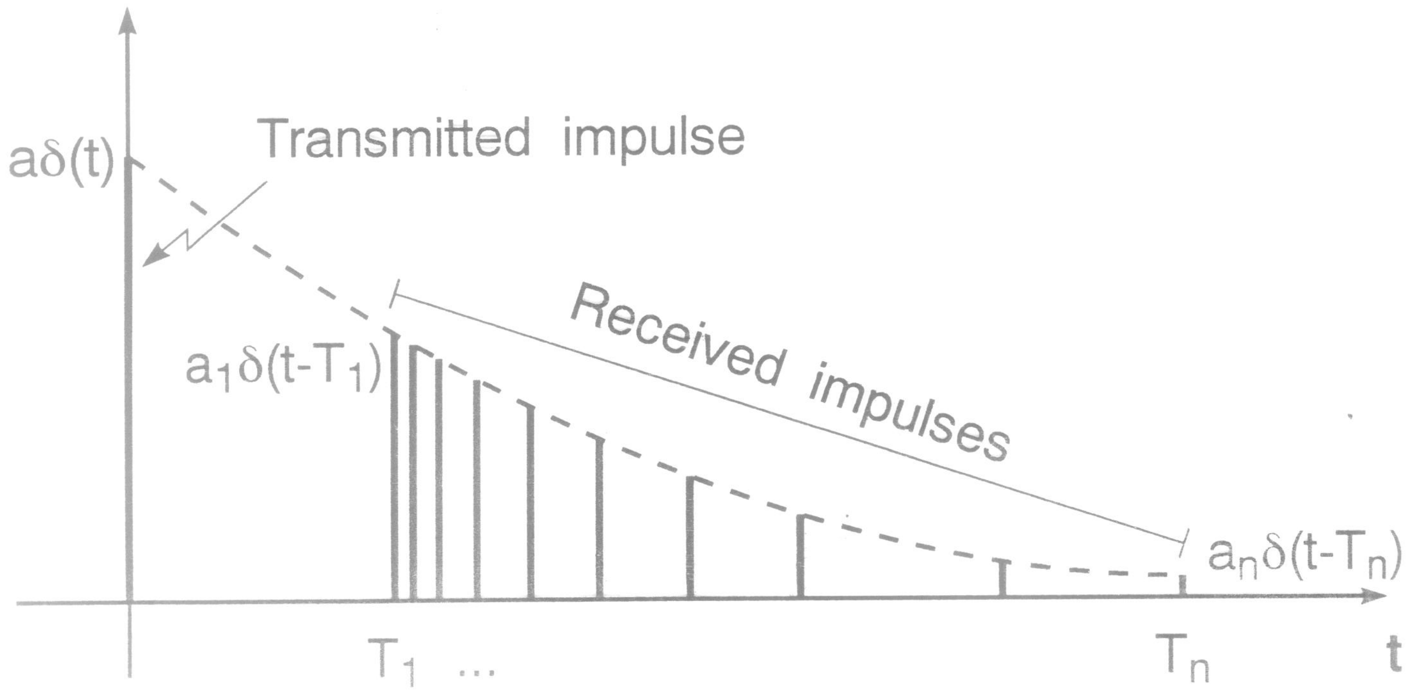

aδ(t)
Transmitted impulse
Received impulses
a1δ(t-T1)
anδ(t-Tn)
T1 ...
Tn
t

FIG4-2 200%

$$e = E_0 \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij} \exp\left[j(\omega t + \omega_{ij} t + \omega T_i)\right]$$

where $a_{ij}^2 =$ [redacted] $p(\theta_{ij})d\theta p(T_i)dT$

and $\omega_{ij} = \beta v \cos\theta_{ij}$

$$\lim_{j\to\infty} \lim_{i\to\infty} a_{ij}^2 := \frac{1}{2\pi\overline{T}} \exp\left(-\frac{T}{\overline{T}}\right) d\theta \; dT$$

$$s(t) \triangleq \mathrm{Re}(e) = X \cos\omega t - Y \sin\omega t$$

where

$$X = E_0 \sum_{j=1}^{m} \sum_{i=1}^{n} a_i \cos(\omega_i t - \omega T_i) \triangleq r \cos\varphi$$

$$Y = E_0 \sum_{j=1}^{m} \sum_{i=1}^{n} a_i \sin(\omega_i t - \omega T_i) \triangleq r \sin\varphi$$

Joint Probability Density Function ; $s_1(t)$ in t and ω_1
$s_2(t)$ in $t+\tau$ and ω_2

$$X_1 = E_0 \sum \sum_{i=1}^{n} a_i \cos(\omega_i t - \omega_1 T_i) \triangleq r_1 \cos\varphi_1$$

$$Y_1 = E_0 \sum \sum_{i=1}^{n} a_i \sin(\omega_i t - \omega_1 T_i) \triangleq r_1 \sin\varphi_1$$

$$X_2 = E_0 \sum \sum_{i=1}^{n} a_i \cos(\omega_i t + \omega_i \tau - \omega_2 T_i) \triangleq r_2 \cos\varphi_2$$

$$Y_2 = E_0 \sum \sum_{i=1}^{n} a_i \sin(\omega_i t + \omega_i \tau - \omega_2 T_i) \triangleq r_2 \cos\varphi_2$$

$$p(R_1, R_2, \ldots, R_n) = \frac{\exp\left[-\frac{1}{2|\Lambda|} \sum_{j=1}^{n} \sum_{k=1}^{n} |\Lambda|_{jk} (R_j - M_j)(R_k - M_k)\right]}{(2\pi)^{n/2} \, |\Lambda|^{1/2}}$$

where R_j, $j = 1, \ldots, n$ are the random variables

$M_j = E\left[R_j\right]$ are their mean values

and $|\Lambda|_{jk}$ is the cofactor of the element λ_{jk} of the determinant $|\Lambda|$. The matrix

Λ is called *Covariance Matrix* and its elements λ_{jk} are the covariances, defined as follows.

$$\lambda_{jk} \triangleq \mathrm{Cov}\,(R_j,R_k) = E\left[(R_j-M_j)(R_k-M_k)\right]$$

$$= E\left[R_jR_k\right] - M_kE\left[R_j\right] - M_jE\left[R_k\right] + M_jM_k =$$

$$= E\left[R_jR_k\right] - E\left[R_j\right]\,E\left[R_k\right]$$

$$\Lambda = \begin{bmatrix} \mathrm{Cov}(X_1,X_1) & \mathrm{Cov}(X_1,Y_1) & \mathrm{Cov}(X_1,X_2) & \mathrm{Cov}(X_1,Y_2) \\ \mathrm{Cov}(Y_1,X_1) & \mathrm{Cov}(Y_1,Y_1) & \mathrm{Cov}(Y_1,X_2) & \mathrm{Cov}(Y_1,Y_2) \\ \mathrm{Cov}(X_2,X_1) & \mathrm{Cov}(X_2,Y_1) & \mathrm{Cov}(X_2,X_2) & \mathrm{Cov}(X_2,Y_2) \\ \mathrm{Cov}(Y_2,X_1) & \mathrm{Cov}(Y_2,Y_1) & \mathrm{Cov}(Y_2,X_2) & \mathrm{Cov}(Y_2,Y_2) \end{bmatrix}$$

$$E\left[X_1\right] = < X_1 > = E_0 \sum_{i=1}^{n} < a_i\cos(\omega_it - \omega_1T_i)> = 0$$

where

$$< x(t) > = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, dt$$

$E\left[X_1\right] = E\left[Y_1\right] = E\left[X_2\right] = E\left[Y_2\right] = 0$. Therefore, in this case $\mathrm{Cov}\left[R_j,R_k\right] = E\left[R_jR_k\right]$. Then

$$\mathrm{Cov}\left[X_1,X_1\right] = E\left[X_1^2\right] = < X_1^2 > = E_0^2 \sum_{i,j} < a_ia_j\cos(\omega_it - \omega_1T_i)\,\cos(\omega_jt - \omega_1T_i)>$$

It is clear that the above average will vanish unlcss $i = j$. In this case $< a_i^2\cos^2(\omega_it - \omega_1T_i) > = a_i^2/2$. Therefore,

$$\mathrm{Cov}\left[X_1,X_1\right] = \sigma^2 \sum_i a_i^2$$

where $\sigma^2 = E_0^2/2$

$$\mathrm{Cov}\left[X_1,X_1\right] = \sigma^2 \int_0^{2\pi}\int_0^{\infty} \frac{1}{2\pi\overline{T}} \exp\left(-\frac{T}{\overline{T}}\right)d\theta\, dT = \sigma^2$$

Similarly,

$$\mathrm{Cov}\left[X_i, X_i\right] = \mathrm{Cov}\left[Y_i, Y_i\right] = \sigma^2, \quad i = 1,\ 2$$

$$\mathrm{Cov}\left[X_1, Y_1\right] = \mathrm{Cov}\left[Y_1, X_1\right] = \mathrm{Cov}\left[X_2, Y_2\right] = \mathrm{Cov}\left[Y_2, X_2\right] = 0$$

$$\mathrm{Cov}\left[X_1, X_2\right] = \mathrm{Cov}\left[X_2, X_1\right] = \mathrm{Cov}\left[Y_1, Y_2\right] = \mathrm{Cov}\left[Y_2, Y_1\right] \triangleq \mu_1, \quad \text{to be determined}$$

$$\mathrm{Cov}\left[X_1, Y_2\right] = \mathrm{Cov}\left[Y_2, X_1\right] = -\ \mathrm{Cov}\left[Y_1, X_2\right] = -\ \mathrm{Cov}\left[X_2, Y_1\right] \triangleq \mu_2, \text{to be determined}$$

Using the same procedure

$$\mu_1 = E\left[X_1 X_2\right] = < X_1 X_2 > = E_0^2 \sum_{i,j} < a_i a_j \cos(\omega_i t - \omega_1 T_i) \cos(\omega_j t + \omega_j \tau - \omega_2 T_j) >$$

$$\mu_1 = E\left[X_1\ X_2\right] = \sigma^2 \sum_i a_i^2 \cos(\omega_i \tau - \Delta\omega T_i)$$

where $\Delta\omega = \omega_2 - \omega_1$ and $\sigma^2 = E_0^2/2$

In the limit, with $i \to \infty$

$$\mu_1 = \sigma^2 \int_0^{2\pi} \int_0^{\infty} \frac{1}{2\pi \overline{T}} \exp\left(-\frac{T}{\overline{T}}\right) \cos(\beta v \tau \cos\theta - \Delta\omega T)\ d\theta\ dT$$

Then

$$\mu_1 = \frac{\sigma^2\ J_0(\omega_m \tau)}{1 + (\Delta\omega \overline{T})^2}$$

$$J_n(z) = \frac{1}{\pi} \int_0^{\pi} \cos(z \sin x - nx) dx, \quad n \text{ integer}$$

$$\mu_2 = \frac{-\sigma^2 \Delta\omega \overline{T} J_0(\omega_m \tau)}{1 + (\Delta\omega \overline{T})^2} \quad ; \quad \rho^2 \triangleq \frac{\mu_1^2 + \mu_2^2}{\sigma^4}$$

$$\Lambda = \begin{bmatrix} \sigma^2 & 0 & \mu_1 & \mu_2 \\ 0 & \sigma^2 & -\mu_2 & \mu_1 \\ \mu_1 & -\mu_2 & \sigma^2 & 0 \\ \mu_2 & \mu_1 & 0 & \sigma^2 \end{bmatrix}$$

$$p(X_1,Y_1,X_2,Y_2) = \frac{1}{4\pi^2\sigma^4(1-\rho^2)} \exp\left\{- \frac{1}{2\sigma^8(1-\rho^2)^2} \left[\sigma^2(X_1^2 + Y_1^2 + X_2^2 + Y_2^2) - 2\mu_1(X_1X_2 + Y_1Y_2) - 2\mu_2(X_1Y_2 - X_2Y_1)\right]\right\}$$

$$p(r_1,r_2,\varphi_1,\varphi_2) = |J|\, p(X_1,Y_1,X_2,Y_2)$$

$$J = \begin{bmatrix} \frac{\partial X_1}{\partial r_1} & \frac{\partial X_1}{\partial \varphi_1} & \frac{\partial X_1}{\partial r_2} & \frac{\partial X_1}{\partial \varphi_2} \\ \frac{\partial Y_1}{\partial r_1} & \frac{\partial Y_1}{\partial \varphi_1} & \frac{\partial Y_1}{\partial r_2} & \frac{\partial Y_1}{\partial \varphi_2} \\ \frac{\partial X_2}{\partial r_1} & \frac{\partial X_2}{\partial \varphi_1} & \frac{\partial X_2}{\partial r_2} & \frac{\partial X_2}{\partial \varphi_2} \\ \frac{\partial Y_2}{\partial r_1} & \frac{\partial Y_2}{\partial \varphi_1} & \frac{\partial Y_2}{\partial r_2} & \frac{\partial Y_2}{\partial \varphi_2} \end{bmatrix}$$

$$|J| = r_1 r_2$$

Therefore,

$$p(r_1,r_2,\varphi_1,\varphi_2) = \frac{r_1 r_2}{4\pi^2\sigma^4(1-\rho^2)} \exp\left\{- \frac{1}{2\sigma^8(1-\rho^2)^2} \left[\sigma^2(r_1^2 + r_2^2) - 2r_1r_2\mu_1\cos(\varphi_2-\varphi_1) - 2r_1r_2\mu_2\sin(\varphi_2-\varphi_1)\right]\right\}$$

$$p(r_1,r_2) = \int_0^{2\pi}\int_0^{2\pi} p(r_1,r_2,\varphi_1,\varphi_2)\, d\varphi_1\, d\varphi_2$$

Then

$$p(r_1,r_2) = \frac{r_1 r_2}{\sigma^4(1-\rho^2)} \exp\left[-\frac{r_1^2 + r_2^2}{2\sigma^2(1-\rho^2)}\right] I_0\left(\frac{r_1 r_2 \rho}{\sigma^2(1-\rho^2)}\right)$$

Se $\rho = 0$; $p(r_1, r_2) = p(r_1)\, p(r_2)$

$$p(\varphi_1,\varphi_2) = \int_0^\infty \int_0^\infty p(r_1,r_2,\varphi_1,\varphi_2)\, dr_1\, dr_2$$

$$p(\varphi_1,\varphi_2) = \left(\frac{1-\rho^2}{4\pi^2}\right) \frac{\left(1 - U^2\right)^{1/2} + U\cos^{-1}(-U)}{\left(1 - U^2\right)^{3/2}}$$

where

$$U = \rho\cos\left[\varphi_2 - \varphi_1 + \tan^{-1}(\Delta\omega\overline{T})\right]$$

$$p(r_1,r_2) = p(r_1)\, p(r_2) = \frac{r_1 r_2}{\sigma^4} \exp\left[-\frac{r_1^2 + r_2^2}{2\sigma^2}\right]$$

In the same way, $U = 0$ and

$$p(\varphi_1,\varphi_2) = p(\varphi_1,\varphi_2) = \frac{1}{4\pi^2}$$

Envelope Correlation

$$\rho_r = \frac{Cov(r_1,r_2)}{\sqrt{Var(r_1)}\sqrt{Var(r_2)}} = \frac{E\left[r_1 r_2\right] - E\left[r_1\right] E\left[r_2\right]}{\sqrt{E\left[r_1^2\right] - E^2\left[r_1\right]}\ \sqrt{E\left[r_2^2\right] - E^2\left[r_2\right]}}$$

$$E\left[r_1\right] = E\left[r_2\right] = \sqrt{\pi/2}\ \sigma$$

$$Var(r_1) = Var(r_2) = (2 - \pi/2)\sigma^2$$

$$E\left[r_1\ r_2\right] = \int_0^\infty \int_0^\infty r_1 r_2\ p(r_1, r_2)\ dr_1\ dr_2$$

$$E\left[r_1\ r_2\right] = \frac{\pi}{2}\ \sigma^2\ F(-1/2,\ -1/2;\ 1;\ \rho^2)$$

Hipergeometric Function

$$F(a,\ b;\ c;\ z) = \sum_{n=0}^{\infty} \frac{(a)_n\ (b)_n}{(c)_n}\ \frac{z^n}{n!}$$

$$= 1 + \frac{ab}{c}\ z + \frac{a(a+1)\ b(b+1)}{c(c+1)}\ \frac{z^2}{2!} + \cdots\cdot$$

$$E\left[r_1\ r_2\right] = \frac{\pi}{2}\ \sigma^2\left[1 + \left(\frac{1}{2}\right)^2 \rho^2 + \left(\frac{1}{2}\right)^6 \rho^4 + \left(\frac{1}{2}\right)^9 \rho^6 + \cdots\right]$$

$$E\left[r_1\ r_2\right] \simeq \frac{\pi}{2}\ \sigma^2\left[1 + \left(\frac{\rho}{2}\right)^2\right]$$

$$\rho_r = \frac{\pi}{4(4 - \pi)}\ \rho^2 \simeq \rho^2$$

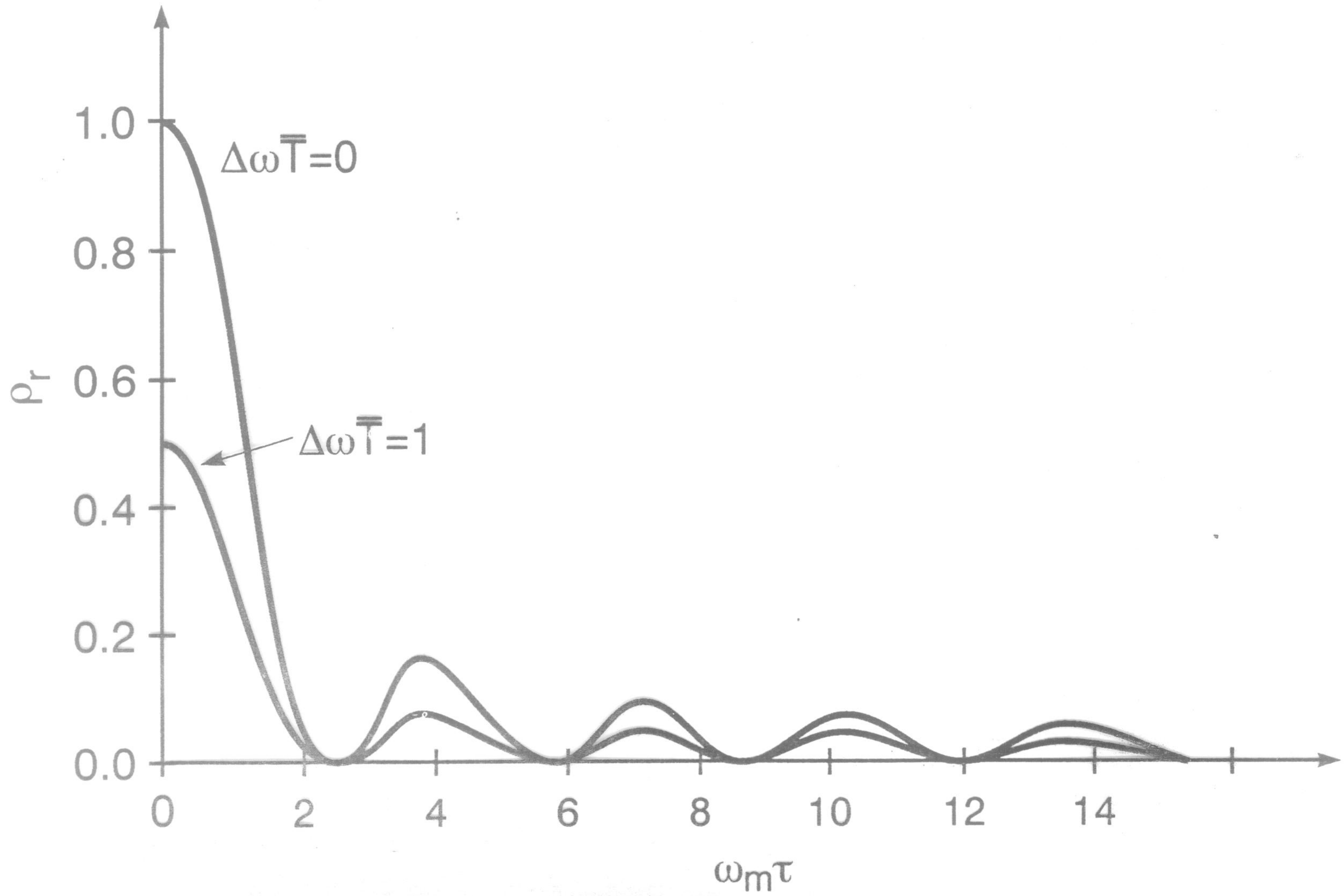
$$\rho_r = \frac{J_0^2(\omega_m \tau)}{1 + (\Delta\omega\overline{T})^2}$$

<u>Phase</u> <u>Correlation</u>

$$\rho_\varphi = \frac{E\left[\varphi_1 \varphi_2\right] - E\left[\varphi_1\right]\ E\left[\varphi_2\right]}{\sqrt{E\left[\varphi_1^2\right] - E^2\left[\varphi_1\right]}\ \sqrt{E\left[\varphi_2^2\right] - E^2\left[\varphi_2\right]}}$$

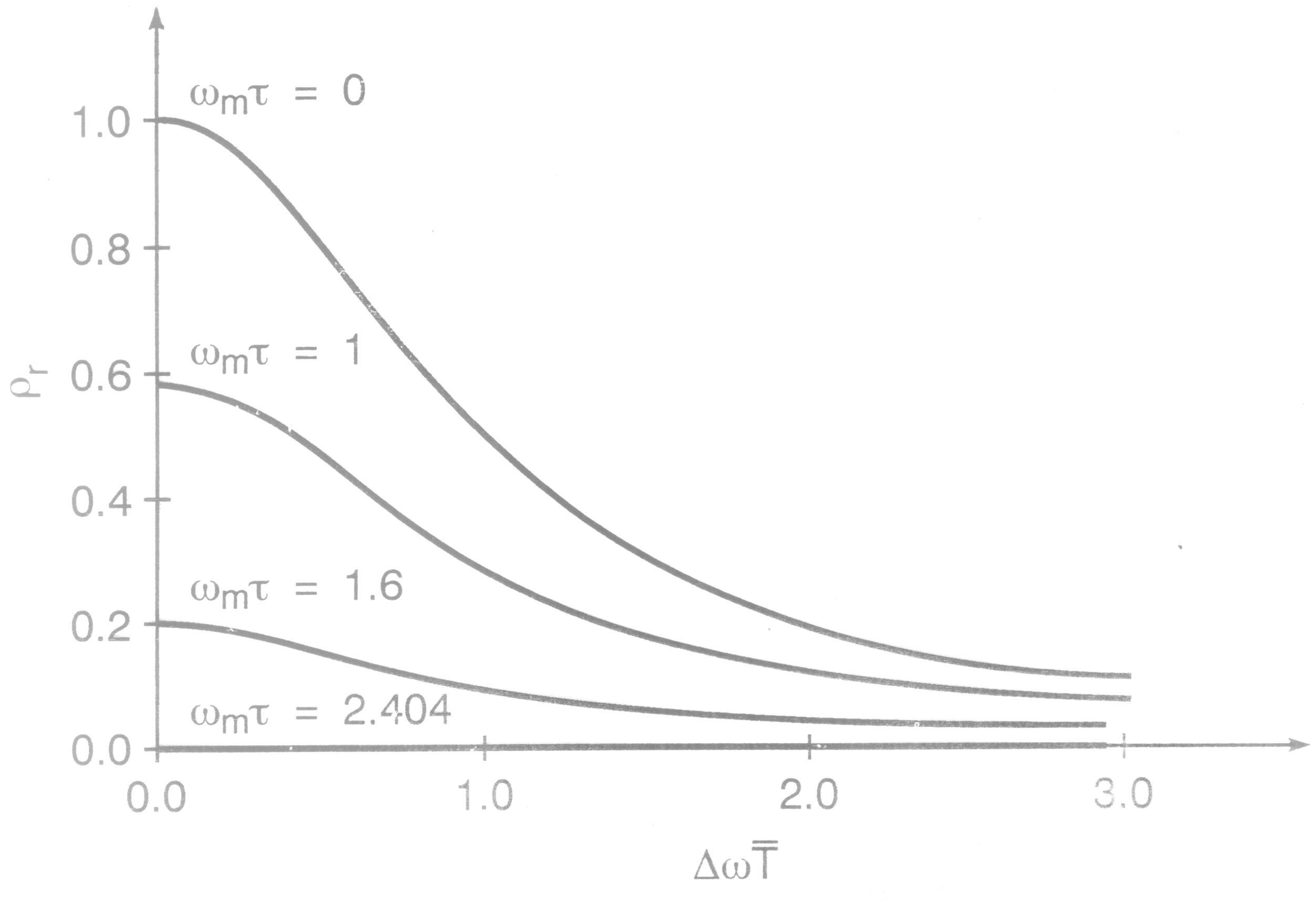
$$p(\varphi_i) = \begin{cases} 1/2\pi & ,\ 0 \le \varphi_i \le 2\pi \qquad i = 1,\ 2 \\ 0 & ,\ \text{otherwise} \end{cases}$$

$$E\left[\varphi_1\right] = E\left[\varphi_2\right] = \frac{1}{2\pi} \int_0^{2\pi} \varphi\ d\varphi = \pi$$



(b)

FIG4-3(b) 200%



(a)

FIG4-3(a) 200%

$$E\left[\varphi_1^2\right] = E\left[\varphi_2^2\right] = \frac{1}{2\pi}\int_0^{2\pi} \varphi^2 \, d\varphi = \frac{4}{3}\pi^2$$

Hence

$$\rho_\varphi = \frac{3}{\pi^2}\left(E\left[\varphi_1\varphi_2\right] - \pi^2\right)$$

The expectation $E\left[\varphi_1 \; \varphi_2\right]$ is given by

$$E\left[\varphi_1 \; \varphi_2\right] = \int_0^{2\pi}\int_0^{2\pi} \varphi_1\varphi_2 \; p(\varphi_1,\varphi_2) \; d\varphi_1 \; d\varphi_2$$

$$E\left[\varphi_1 \; \varphi_2\right] = \pi^2\left[1 + \Gamma(\rho,\phi) + 2\Gamma^2(\rho,\phi) - \frac{1}{4\pi^2}\sum_{n=1}^{\infty}\frac{\rho^{2n}}{n^2}\right]$$

$$\Gamma(\rho,\phi) = \frac{1}{2\pi}\sin^{-1}(\rho\cos\phi)$$

$$\phi = -\tan^{-1}(\Delta\omega\overline{T})$$

and

$$\rho^2 = \frac{J_0^2(\omega_m\tau)}{1 + (\Delta\omega\overline{T})^2}$$

$$\rho_\varphi = 3\,\Gamma(\rho,\phi)\left[1 + 2\Gamma(\rho,\phi)\right] - \frac{3}{4\pi^2}\sum_{n=1}^{\infty}\frac{\rho^{2n}}{n^2}$$

Coherence Bandwidth

Envelope

$$\rho_r = \frac{J_0^2(\omega_m 0)}{1 + (\Delta\omega\overline{T})^2} = 0.5$$

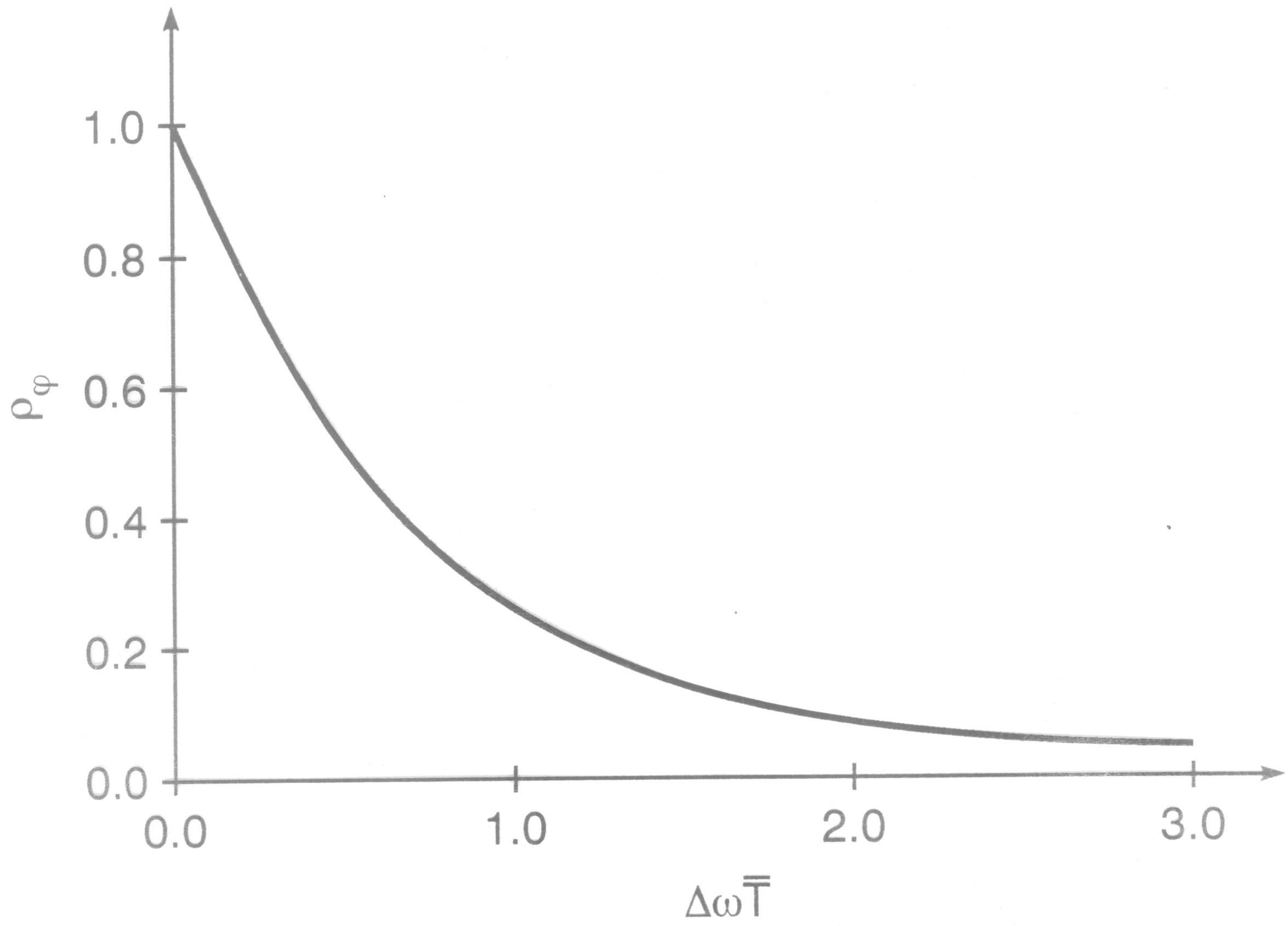


FIG4-4 200%

$$B_c \triangleq \frac{\Delta\omega}{2\pi} = \frac{1}{2\pi\overline{T}} \quad , \ \overline{T} = 5\mu s, \ B_c = 32kHz$$

Phase

$$B_c = \frac{1}{4\pi\overline{T}}$$

LEVEL CROSSING RATE

$$R_c = E\left[\dot{r} \mid r=R\right] = \int_0^{\infty} \dot{r} \ p(R,\dot{r}) \ d\dot{r}$$

Joint Probability Density Function

$$\dot{X} = E_0\beta v \sum_{i=1}^{n} -a_i \sin(\omega_i t - \omega T_i)\cos\theta_i$$

$$\dot{Y} = E_0\beta v \sum_{i=1}^{n} a_i \cos(\omega_i t - \omega T_i)\cos\theta_i$$

$$E\left[\dot{X}\right] = E\left[\dot{Y}\right] = E\left[XY\right] = E\left[X\dot{X}\right] = E\left[X\dot{Y}\right] = E\left[Y\dot{X}\right] = E\left[Y\dot{Y}\right] = E\left[\dot{X}\dot{Y}\right] = 0$$

$$E\left[X^2\right] = E\left[Y^2\right] = E_0^2/2 = \sigma^2$$

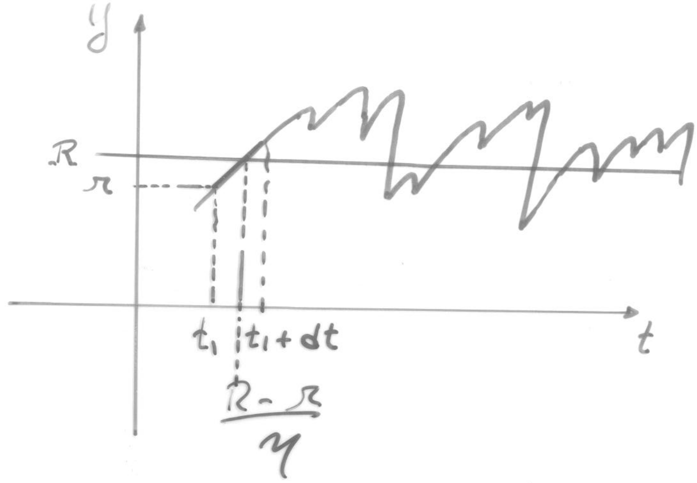
$$E\left[\dot{X}^2\right] = E\left[\dot{Y}^2\right] = (E_0\beta v/2)^2 \triangleq \dot{\sigma}^2$$

$$p(X,Y,\dot{X},\dot{Y}) = \frac{1}{4\pi^2\sigma^2\dot{\sigma}^2} \exp\left[-\frac{1}{2}\left(\frac{X^2 + Y^2}{\sigma^2} + \frac{\dot{X}^2 + \dot{Y}^2}{\dot{\sigma}^2}\right)\right]$$

$$p(r,\dot{r},\varphi,\dot{\varphi}) = |J| \ p(X,Y,\dot{X},\dot{Y})$$

$$\dot{X} = \dot{r} \cos\varphi - r\dot{\varphi} \sin\varphi$$

$$\dot{Y} = \dot{r} \sin\varphi + r\dot{\varphi} \cos\varphi$$



$y = F(a_1, a_2, \ldots, a_n; t)$; a_i are random variables

Choose dx small : portions of all but a negligible fraction of possible random curves lying in strip (t_1, t_1+dt) may regarded as straight lines. If $y = r$ at t_1 and passes through R for $t_1 < t < t_1+dt$, then its intercepts on $y = R$ is $\frac{R-r}{\eta} + t_1$ where $\eta = \left.\frac{\partial F}{\partial t}\right|_{t=t_1}$

Then

$$t_1 < t_1 - \frac{r}{\eta} < t_1 + dt$$

$$R - \eta dt < r < R$$

r and η satisfying the inequality, then the curve goes through R.

The probability of such an event is

$$ob = \int_0^\infty d\eta \int_{R-\eta\, dt}^{R} dr\, p(r, \eta; t_1)$$

But dx is chosen to be very small

$$rob = \int_0^\infty [R\, p(R, \eta; t_1) - (R - \eta dt) p(R, \eta, t_1)] d\eta$$

$$= dt \int_0^\infty \eta\, p(r = R, \eta; t_1) d\eta$$

$$\text{prob}(1\,\text{crossing}) = dt \int_0^\infty \dot{r}\, p(r = R, \dot{r})\, d\dot{r}$$

Expected number of crossings within $0 \leq dt \leq T$

$$N = T \int_0^\infty \dot{r}\, p(r = R, \dot{r})\, d\dot{r}$$

$$\boxed{N_c = \frac{N}{T} = \int_0^\infty \dot{r}\, p(r = R, \dot{r})\, d\dot{r}}$$

$$p(r,\dot{r},\varphi,\dot{\varphi}) = \frac{r^2}{4\pi^2\sigma^2\dot{\sigma}^2} \exp\left[-\frac{1}{2}\left(\frac{r^2}{\sigma^2} + \frac{\dot{r}^2 + r^2\dot{\varphi}^2}{\dot{\sigma}^2}\right)\right]$$

$$p(r,\dot{r}) = \frac{r}{\sigma^2\sqrt{2\pi\dot{\sigma}^2}} \exp\left[-\frac{1}{2}\left(\frac{r^2}{\sigma^2} + \frac{\dot{r}^2}{\dot{\sigma}^2}\right)\right]$$

$$R_c = \sqrt{2\pi}\, f_m \frac{R}{\sqrt{2}\,\sigma} \exp\left[-\left(\frac{R}{\sqrt{2}\,\sigma}\right)^2\right] ,$$

At $R = \sqrt{2}\,\sigma$, $v = 72km/h$, $f = 900MHz$, $R_c = 55$ cross/sec.

AVERAGE DURATION OF FADES

$$\tau = \frac{\sum \tau_i}{R_c T}$$

$$\tau = \frac{1}{\sqrt{2\pi}\, f_m\, (R/\sqrt{2}\,\sigma)} \left[\exp\left(\frac{R}{\sqrt{2}\,\sigma}\right)^2 - 1\right]$$

At $R = \sqrt{2}\,\sigma$, $v = 72km/h$, $f = 900MHz$, $\tau = 11ms$.

RANDOM FREQUENCY MODULATION

Probability Distribution

$$p(\dot{\varphi}) = \int_0^{\infty}\int_{-\infty}^{\infty}\int_0^{2\pi} p(r,\dot{r},\varphi,\dot{\varphi})\, dr\, d\dot{r}\, d\varphi$$

Hence

$$P(\dot{\Phi}) = prob(\dot{\varphi} \le \dot{\Phi}) = \int_{-\infty}^{\dot{\Phi}} p(\dot{\varphi})d\dot{\varphi}$$

Then

$$p(\dot{\varphi}) = \frac{1}{\sqrt{2}\,\beta v}\left[1 + \frac{2}{(\beta v)^2}\dot{\varphi}^2\right]^{-3/2}$$

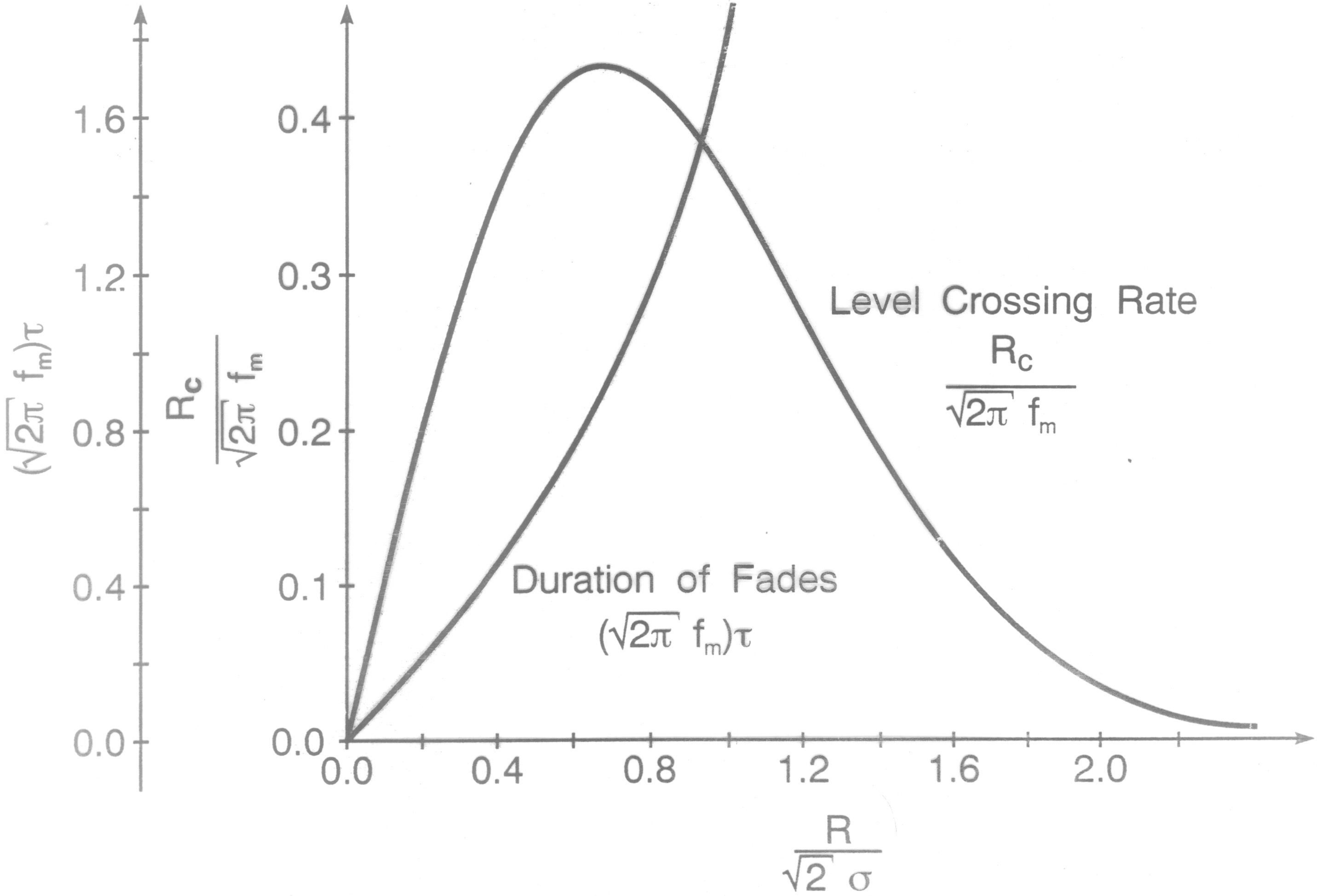
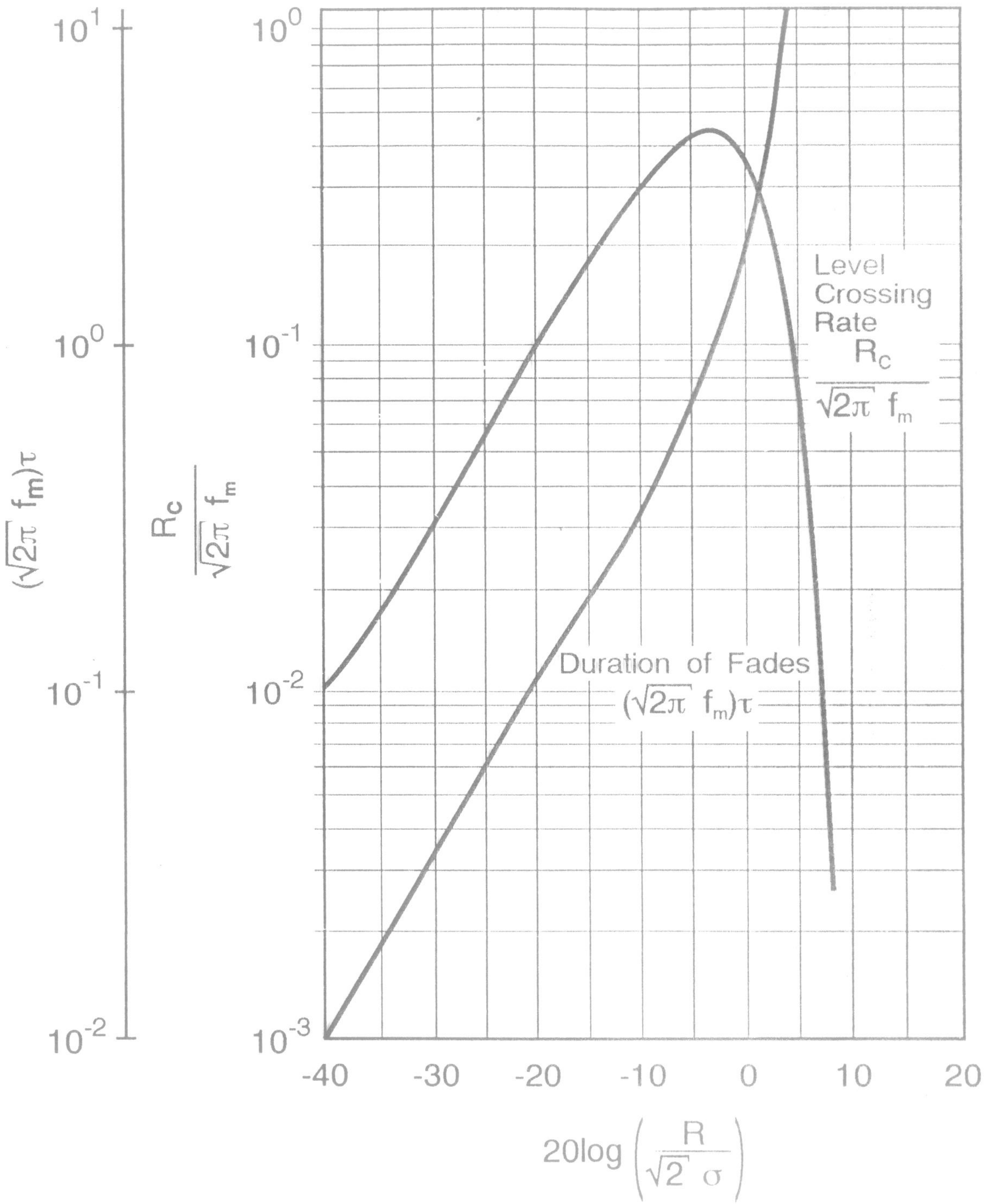

Level Crossing Rate
Rc / √2π fm
Duration of Fades
(√2π fm)τ
R / √2 σ
Rc / √2π fm
(√2π fm)τ
0.0
0.4
0.8
1.2
1.6
2.0
0.1
0.2
0.3

FIG4-5 200%



10^1
10^0
10^{-1}
10^{-2}
$(\sqrt{2\pi}\ f_m)\tau$
$\frac{R_c}{\sqrt{2\pi}\ f_m}$
10^0
10^{-1}
10^{-2}
10^{-3}
Level Crossing Rate
$\frac{R_c}{\sqrt{2\pi}\ f_m}$
Duration of Fades
$(\sqrt{2\pi}\ f_m)\tau$
-40
-30
-20
-10
0
10
20
$20\log\left(\frac{R}{\sqrt{2}\ \sigma}\right)$

FIG4.6 150%

and

$$P(\dot{\Phi}) = \frac{1}{2} + \frac{1}{\sqrt{2}\,\beta v}\,\dot{\Phi}\left[1 + \frac{2}{(\beta v)^2}\,\dot{\Phi}^2\right]^{-3/2}$$

Power Spectrum

$$R_{\dot{\varphi}}(\tau) = E\left[\dot{\varphi}(t)\,\dot{\varphi}(t-\tau)\right]$$

$$R_{\dot{\varphi}}(\tau) = -\frac{1}{2}\left\{\left[\frac{R'_x(\tau)}{R_x(\tau)}\right]^2 - \left[\frac{R''_x(\tau)}{R_x(\tau)}\right]^2\right\}\ln\left\{1 - \left[\frac{R_x(\tau)}{R_x(0)}\right]^2\right\}$$

$$R_x(\tau) = \sigma^2 J_0(\omega_m \tau)$$

Then

$$\frac{R_x(\tau)}{R_x(0)} = J_0(\omega_m \tau)$$

$$\frac{R'_x(\tau)}{R_x(\tau)} = \frac{1}{R_x(\tau)}\,\frac{dR_x(\tau)}{d\tau} = -\omega_m\,\frac{J_1(\omega_m \tau)}{J_0(\omega_m \tau)}$$

$$\frac{R''_x(\tau)}{R_x(\tau)} = \frac{1}{R_x(\tau)}\,\frac{d^2R_x(\tau)}{d\tau^2} = \omega_m^2\left[\frac{J_1(\omega_m \tau)}{\omega_m \tau\, J_0(\omega_m \tau)} - 1\right]$$

$$S_{\dot{\varphi}}(f) = \int_{-\infty}^{\infty} R_{\dot{\varphi}}(\tau)\,\exp(-j\omega\tau)\,d\tau = 2\int_{0}^{\infty} R_{\dot{\varphi}}(\tau)\,\cos(\omega\tau)\,d\tau$$

$$\lim_{f\to\infty} S_{\dot{\varphi}}(f) = \left[(\dot{\sigma}/\sigma)^2 - (\dot{\sigma}_{xy}/\sigma)^4\right] f^{-1}$$

where

$$\dot{\sigma}_{xy} = E\left[X_1\dot{Y}_2\right] = -E\left[X_2\dot{Y}_1\right] = 0$$

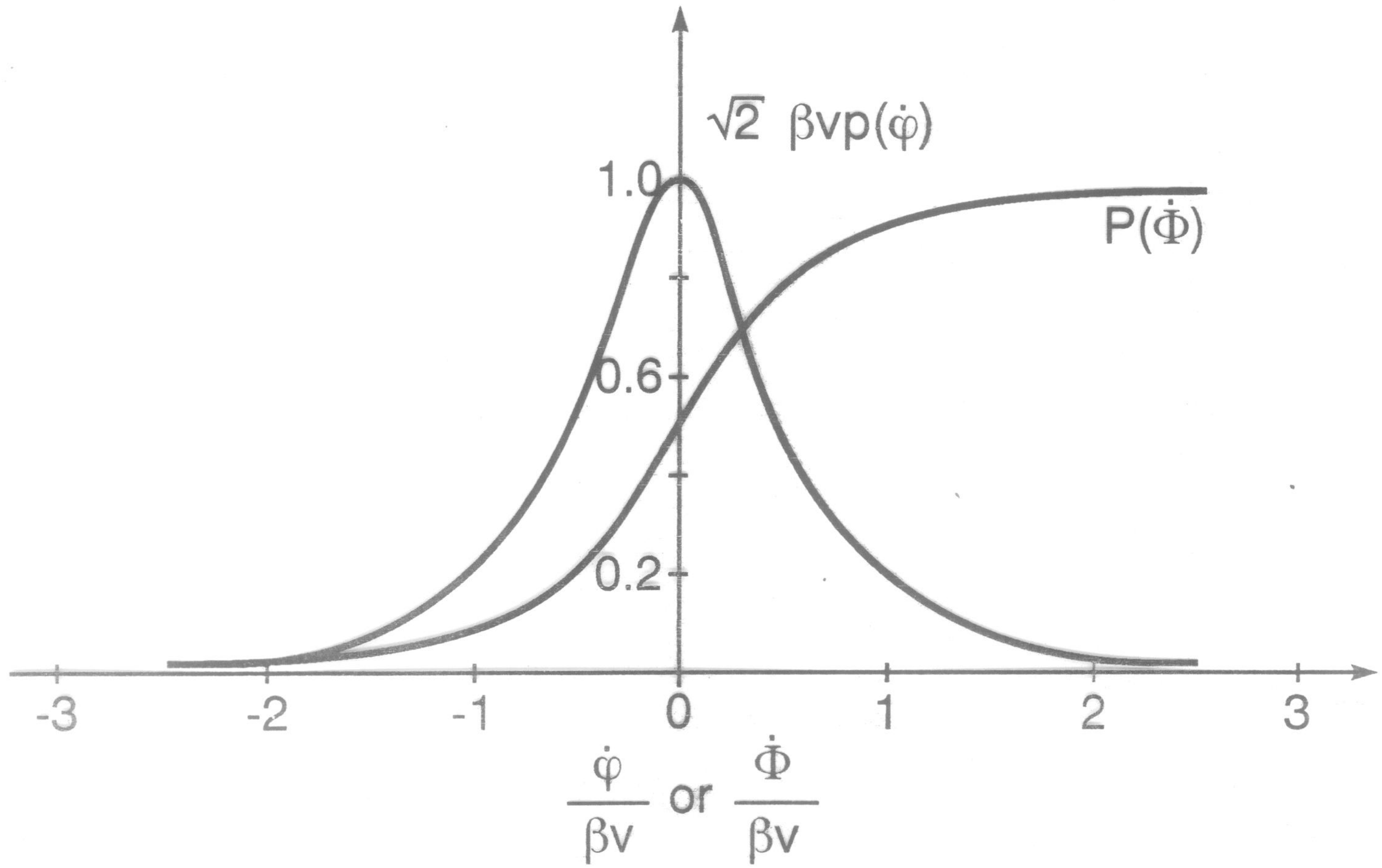


FIG4-7 200%

$$\frac{1}{\beta v} S_{\dot{\varphi}}(f) \simeq \frac{\beta v}{2f}$$

$$N = \int_{\omega_1}^{\omega_2} S_{\dot{\varphi}}(f)\, df = \frac{(\beta v)^2}{2} \ln\left(\frac{\omega_2}{\omega_1}\right)$$

ω_2 = 3400 Hz, ω_1 = 300 Hz, v = 72 km/h, f = 900 MHz, N = 52 dB

$$\frac{N_2}{N_1} = \left(\frac{2v}{v}\right)^2 = 4 = 6 \text{ dB}$$

POWER SPECTRA OF THE RECEIVED SIGNAL

$$\omega(\theta) = \omega_c + \omega_m \cos\theta$$

$$S(\omega)|d\omega| = \frac{W_0}{2\pi} \left[G(\theta)\ p(\theta) + G(-\theta)\ p(-\theta)\right] |d\theta|$$

$$|d\omega| = |-\beta v \sin\theta|\,|d\theta| = \left[(\beta v)^2 - (\omega - \omega_c)^2\right]^{1/2} |d\theta|$$

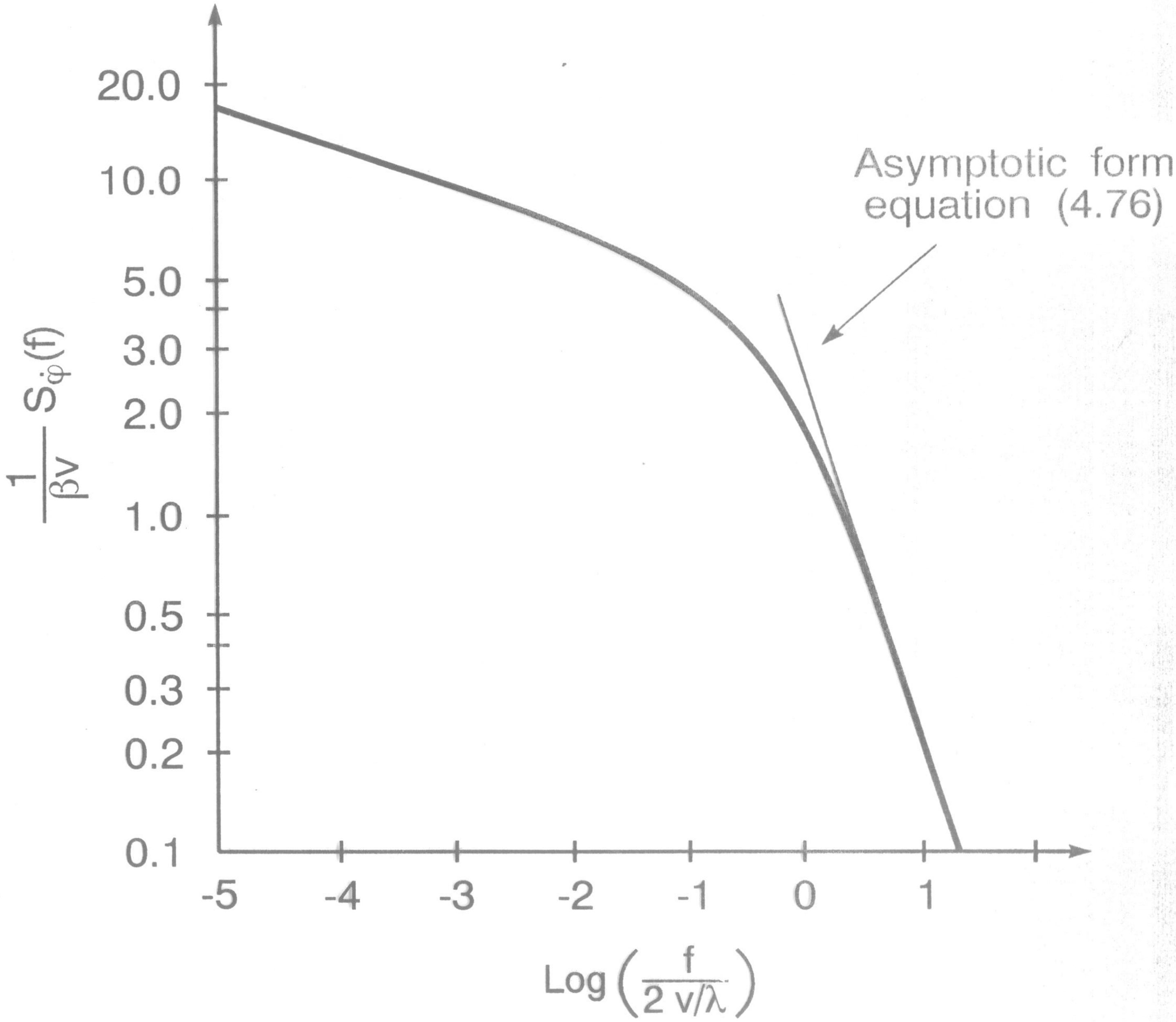
$$p(\theta) = \begin{cases} 1/2\pi & , -\pi \le \theta \le \pi \\ 0 & , \text{otherwise} \end{cases}$$

$$S(\omega) = \frac{W_0\left[G(\theta) + G(-\theta)\right]}{\omega_m \sqrt{1 - \left(\dfrac{\omega - \omega_c}{\omega_m}\right)^2}}$$

1. *Power spectrum of the electric field E_z:* vertical monopole (whip) antenna,

$G(\theta) = G(-\theta) = 3/2$

$$S(\omega) = \frac{3W_0}{\omega_m} \left[1 - \left(\frac{\omega - \omega_c}{\omega_m}\right)^2\right]^{-1/2}$$



20.0
10.0
5.0
3.0
2.0
1.0
0.5
0.3
0.2
0.1
$\frac{1}{\beta v} S_{\dot{\varphi}}(f)$
Asymptotic form equation (4.76)
-5
-4
-3
-2
-1
0
1
$\mathrm{Log}\left(\frac{f}{2\,v/\lambda}\right)$

FIG4-8 200%

2. *Power spectrum of the magnetic field* H_x*:* small loop antenna along the y-axis, $G(\theta) = G(-\theta) = (3/2)\sin^2\theta$

$$S(\omega) = \frac{3W_0}{\omega_m}\left[1 - \left(\frac{\omega - \omega_c}{\omega_m}\right)^2\right]^{1/2}$$

3. *Power spectrum of the magnetic field* H_y*:* small loop antenna along the x-axis, $G(\theta) = G(-\theta) = (3/2)\cos^2\theta$

$$S(\omega) = \frac{3W_0}{\omega_m}\left(\frac{\omega - \omega_c}{\omega_m}\right)^2\left[1 - \left(\frac{\omega - \omega_c}{\omega_m}\right)^2\right]^{-1/2}$$

FIELD MEASUREMENT

$d = vt$; $f_s = \frac{1}{t} = \frac{v}{d}$

$\therefore \omega_s = 2\pi f_s = 2\pi v/d$ rad/s

1. *Sampling Interval*

$$\omega_s = 2\pi\ v/d \qquad \text{rad/s}$$

$$\omega_s = 2\pi\ v/d \geq 2(\text{Bandwidth}) = 2(2\beta v)$$

Hence

$$d \leq \lambda/4$$

Let $f = 900$ MHz $\therefore \lambda = 1/3$ m

$d \leq 8.3$ cm

2. *Separation of Slow and Fast Fading*

$$S(t) = M(t) + R(t)$$

$$\hat{M}_i = \frac{1}{2k+1}\sum_{j=-k}^{k} S_{i+j}$$

$$\hat{R}_i = S_i - \hat{M}_i$$

$$f_{co} = \frac{f_s}{2k+1}$$

$$f_s = 4\gamma f_m = 4\gamma v/\lambda = 4\gamma f_m$$

$d \leq \lambda/4 \therefore 1/d \geq 4/\lambda$

$v/d \geq 4v/\lambda$

$f_s \geq 4v/\lambda = 4f_m$

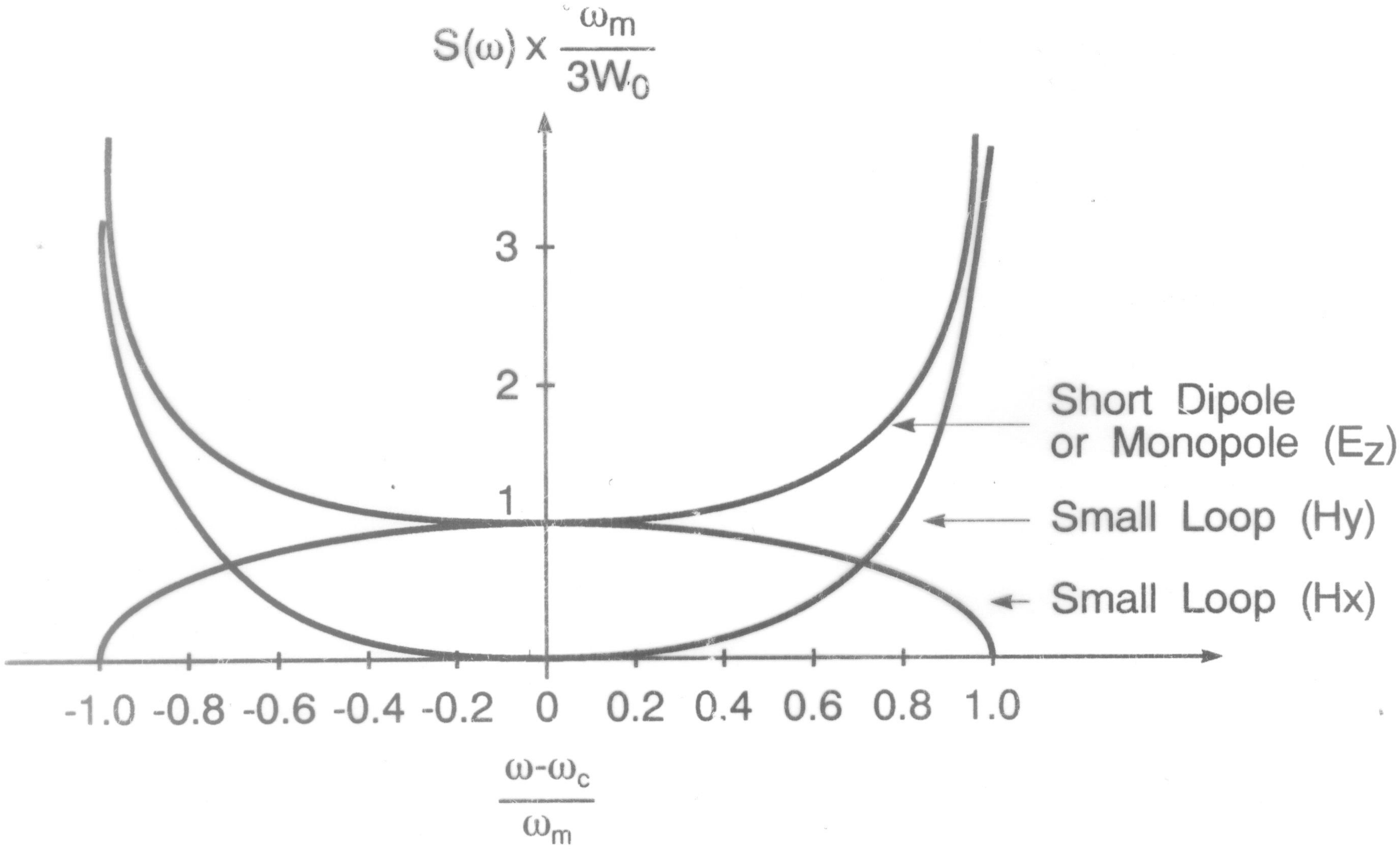

S(ω) x ωm / 3W0
3
2
1
Short Dipole
or Monopole (EZ)
Small Loop (Hy)
Small Loop (Hx)
-1.0 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1.0
ω-ωc / ωm

FIG4-9 200%

where $\gamma \geq 1$.

$$f_{co} = \alpha f_m$$ But $f_{co} = \frac{f_s}{2k+1} = \frac{4\gamma f_m}{2k+1}$

where $0 \leq \alpha \leq 1$.

$$2k + 1 = 4\gamma/\alpha$$

$$L = \frac{2k+1}{f_s} v$$

$$L = \lambda/\alpha$$

f_c = 900 MHz, $\gamma = 2$, (sampling frequency = 4 x bandwidth = 8 x Doppler

$\alpha = 0.04$ (cut off frequency = 4% of the maximum Doppler)

Then 2k + 1 (number of samples = filter's length) = 200

$L = 25\ \lambda = 8.33$ m.

At v = 72 km/h ,sampling rate = 480 samples/s (4cm/sample interval).

3. *Validation of the Measurements*

RADIO CHANNEL SIMULATION

$$s(t) = Xx(t) - Y\hat{x}(t)$$

where

$$X = E_0 \sum_{i=1}^{M} a_i \cos(\omega_i t + \psi_i)$$

$$Y = E_0 \sum_{i=1}^{M} a_i \sin(\omega_i t + \psi_i)$$

$$a_i^2 = p(\theta_i)d\theta p(\psi_i)d\psi$$

$$\omega_i = \beta v \cos\theta_i$$

$\hat{x}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau$ is the Hilbert transform* of x(t)

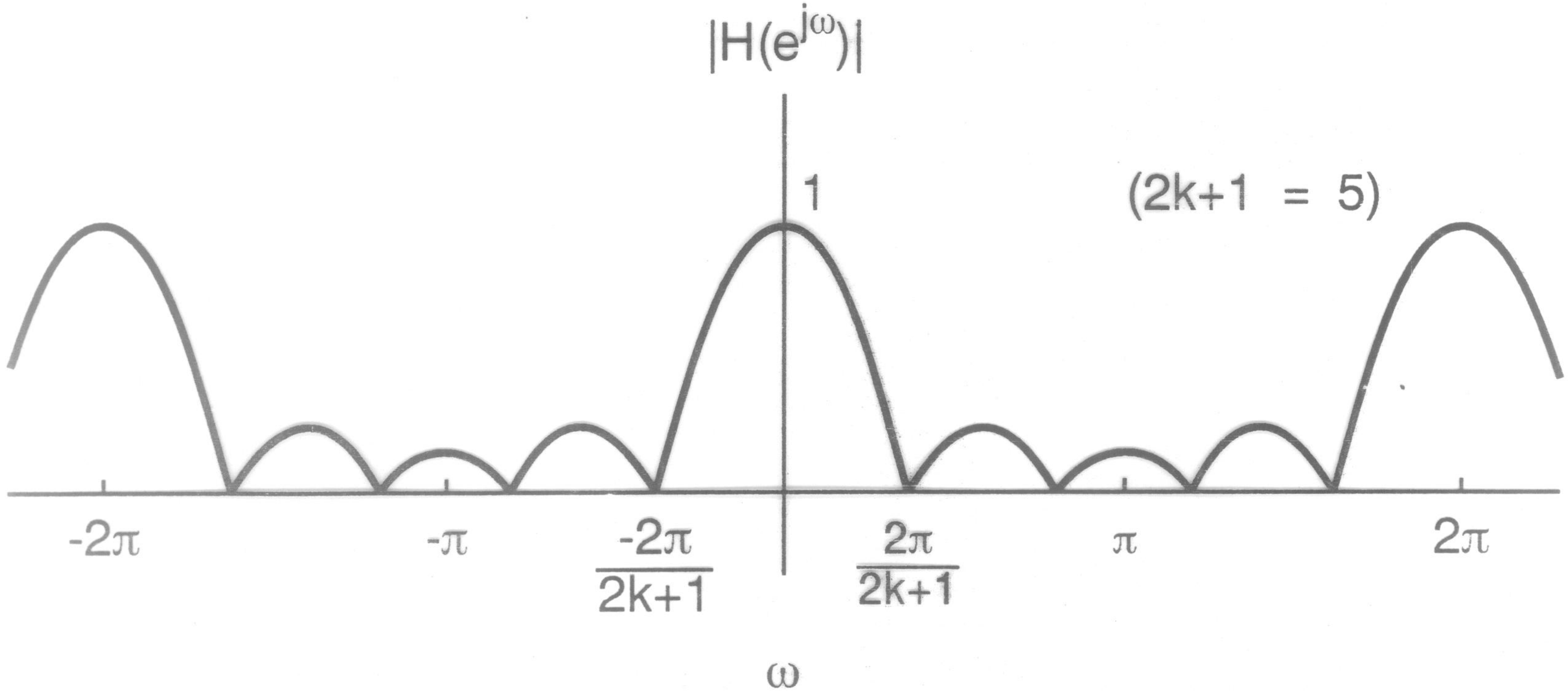


FIG4B-1 200%

Analog Solution

1. *Waves Sum Solution*

2. *Gaussian Noise Filtering Solution*

DSP Solution

$$\tau = \frac{1}{R_c} \text{ prob } [r \le R] = \frac{1}{R_c} \int_0^R p(r)\, dr$$

Waves Sum Solution

$$s(t) = X\, x(t) - Y\, \hat{x}(t)$$

$$X = E_0 \sum_{i=1}^{m} a_i \cos(\omega_i t + \psi_i)$$

$$Y = E_0 \sum_{i=1}^{m} a_i \sin(\omega_i t + \psi_i)$$

$$a_i^2 = p(\theta_i)\, d\theta\, p(\psi_i)\, d\psi$$

$$\omega_i = \beta v \cos \theta_i$$

$$\hat{x} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau$$

$$p(\theta_i) = \frac{1}{2\pi}, \quad d\theta = \frac{2\pi}{m}, \quad \theta_i = \frac{2\pi i}{m} \quad i = 1, \dots, m$$

$$p(\psi_i) = \frac{1}{2\pi}, \quad d\psi = \frac{2\pi}{m}, \quad \psi_i = \frac{2\pi i}{m}, \quad i = 1, \dots, m$$

$$\therefore \; a_i^2 = p(\theta_i)\, d\theta \; p(\psi_i)\, d\psi$$

$$= \frac{1}{2\pi} \frac{2\pi}{m} \frac{1}{2\pi} \frac{2\pi}{m} = \frac{1}{m^2}$$

$$\therefore \; \boxed{a_i = \frac{1}{m}}$$

$$X = E_0 \sum a_i \cos(\omega_i t + \psi_i) \qquad \text{Seja } E_0 = 1$$

$$X = \frac{1}{m} \left(\cos\omega_i t \cos\psi_i - \sin\omega_i t \sin\psi_i \right)$$

e

$$Y = \frac{1}{m} \left(\sin\omega_i t \cos\psi_i + \cos\omega_i t \sin\psi_i \right)$$

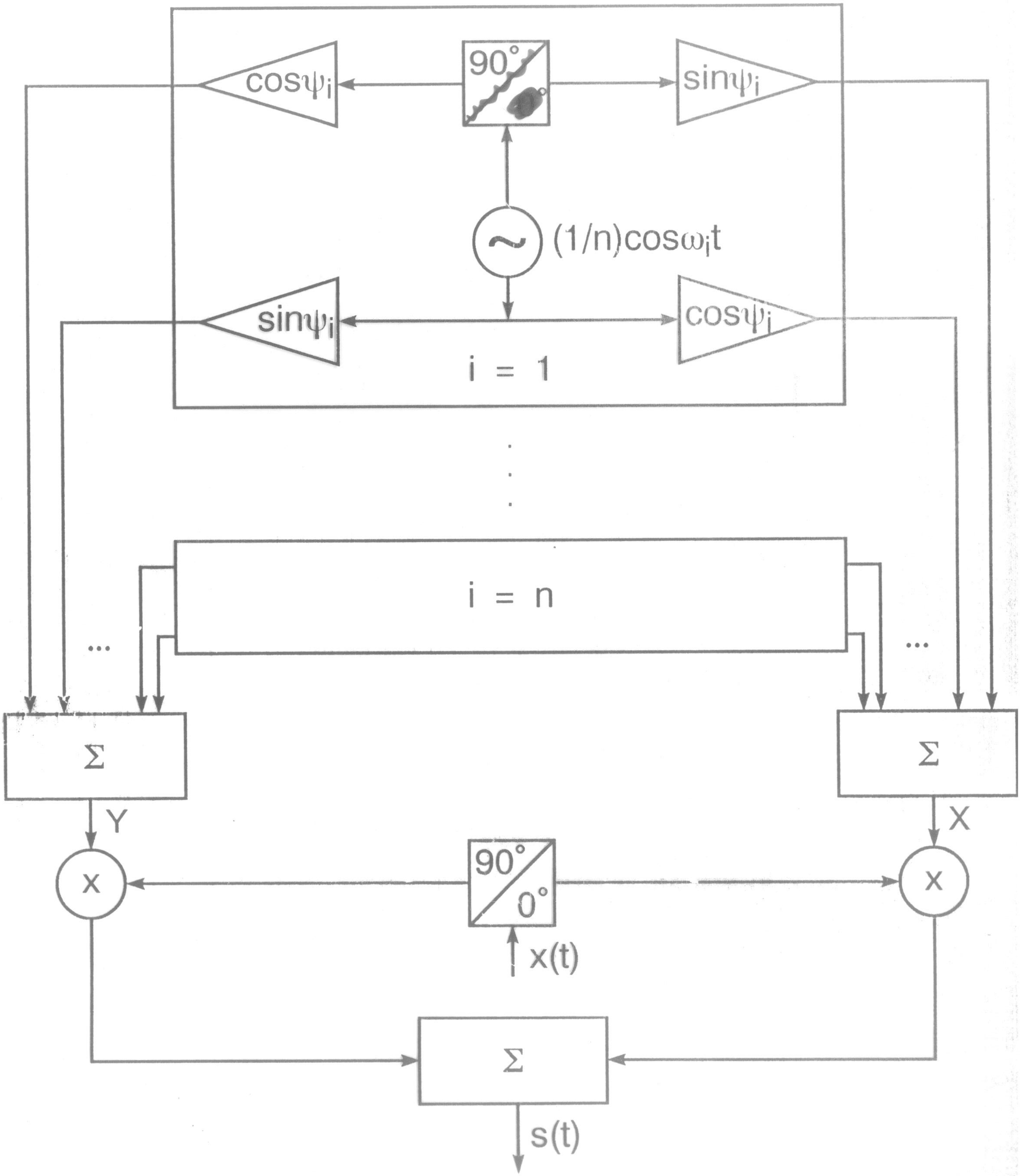

$\cos\psi_i$
90°
$\sin\psi_i$
$(1/n)\cos\omega_i t$
$\sin\psi_i$
$\cos\psi_i$
i = 1
i = n
...
...
Σ
Σ
Y
X
x
90°
0°
x
x(t)
Σ
s(t)

FIG4-10 200%

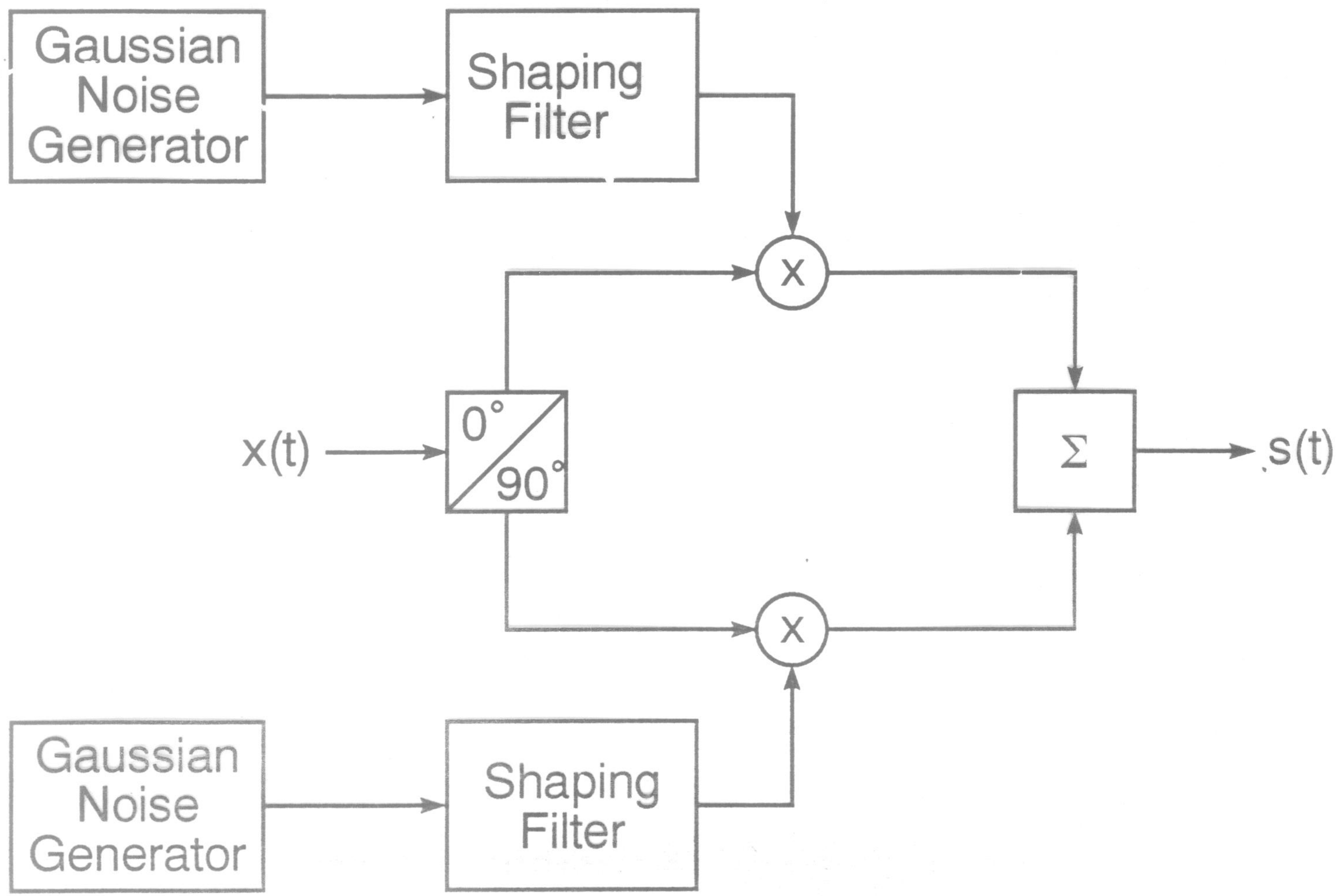

Gaussian Noise Generator
Shaping Filter
x(t)
0°
90°
X
X
Σ
s(t)
Gaussian Noise Generator
Shaping Filter

FIG4-11 200%

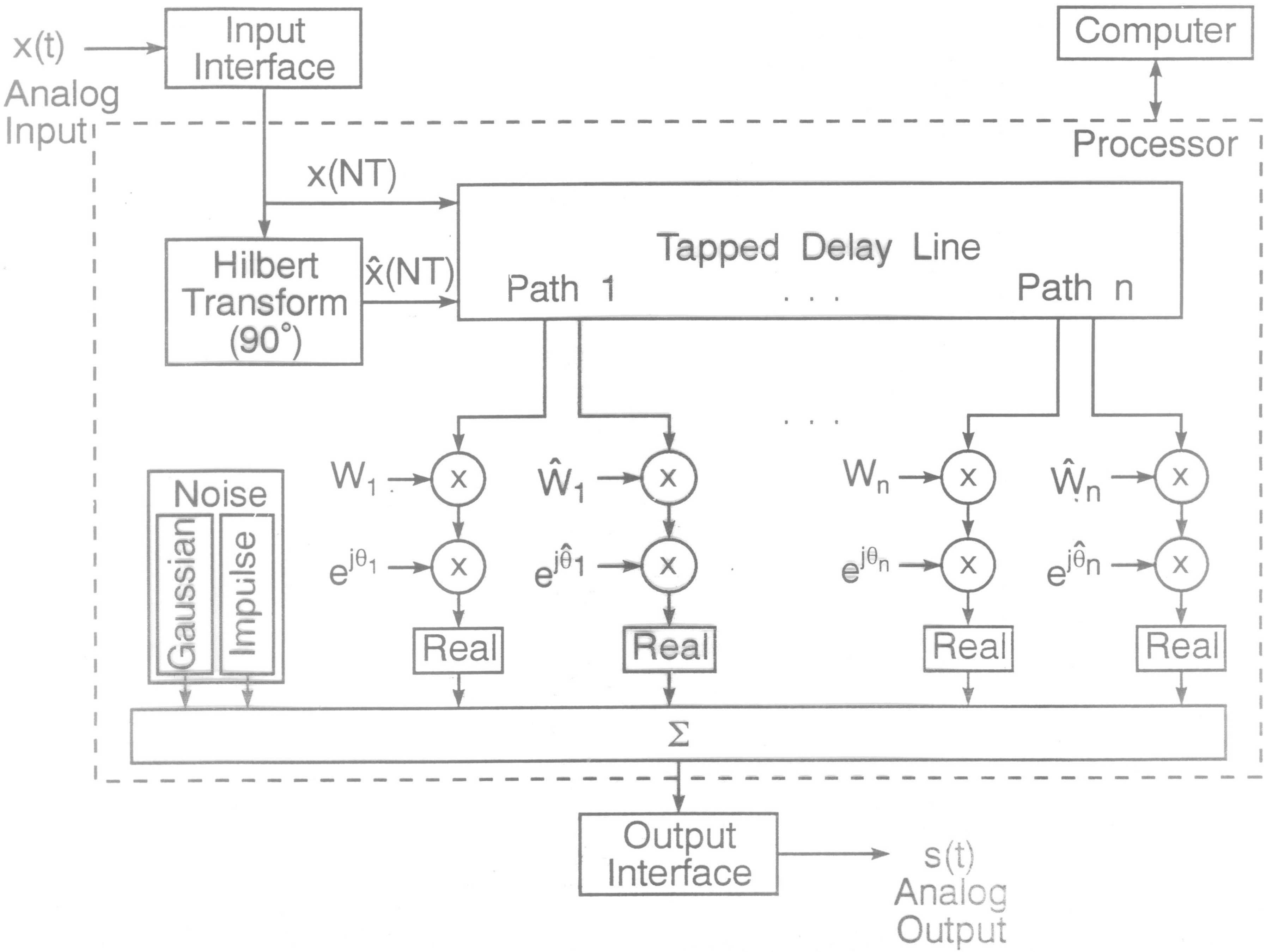

x(t)
Analog
Input
Input
Interface
Computer
Processor
x(NT)
Hilbert
Transform
(90°)
x̂(NT)
Tapped Delay Line
Path 1
Path n
Noise
Gaussian
Impulse
W_1
$\hat{W}_1$
W_n
$\hat{W}_n$
$e^{j\theta_1}$
$e^{j\hat{\theta}_1}$
$e^{j\theta_n}$
$e^{j\hat{\theta}_n}$
Real
Real
Real
Real
Σ
Output
Interface
s(t)
Analog
Output

FIG4-12 200%

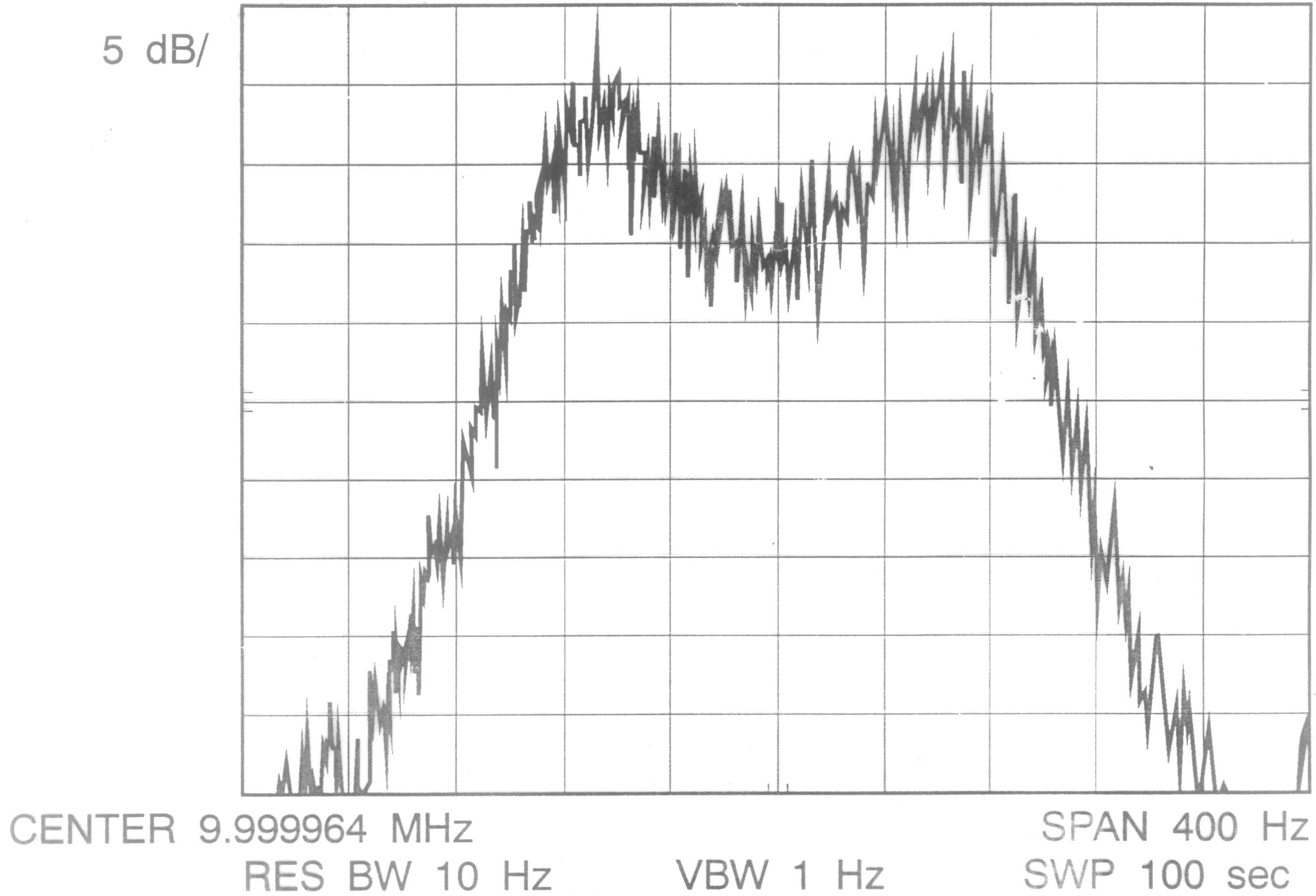

5 dB/
CENTER 9.999964 MHz
SPAN 400 Hz
RES BW 10 Hz
VBW 1 Hz
SWP 100 sec

FIG4-13 200%

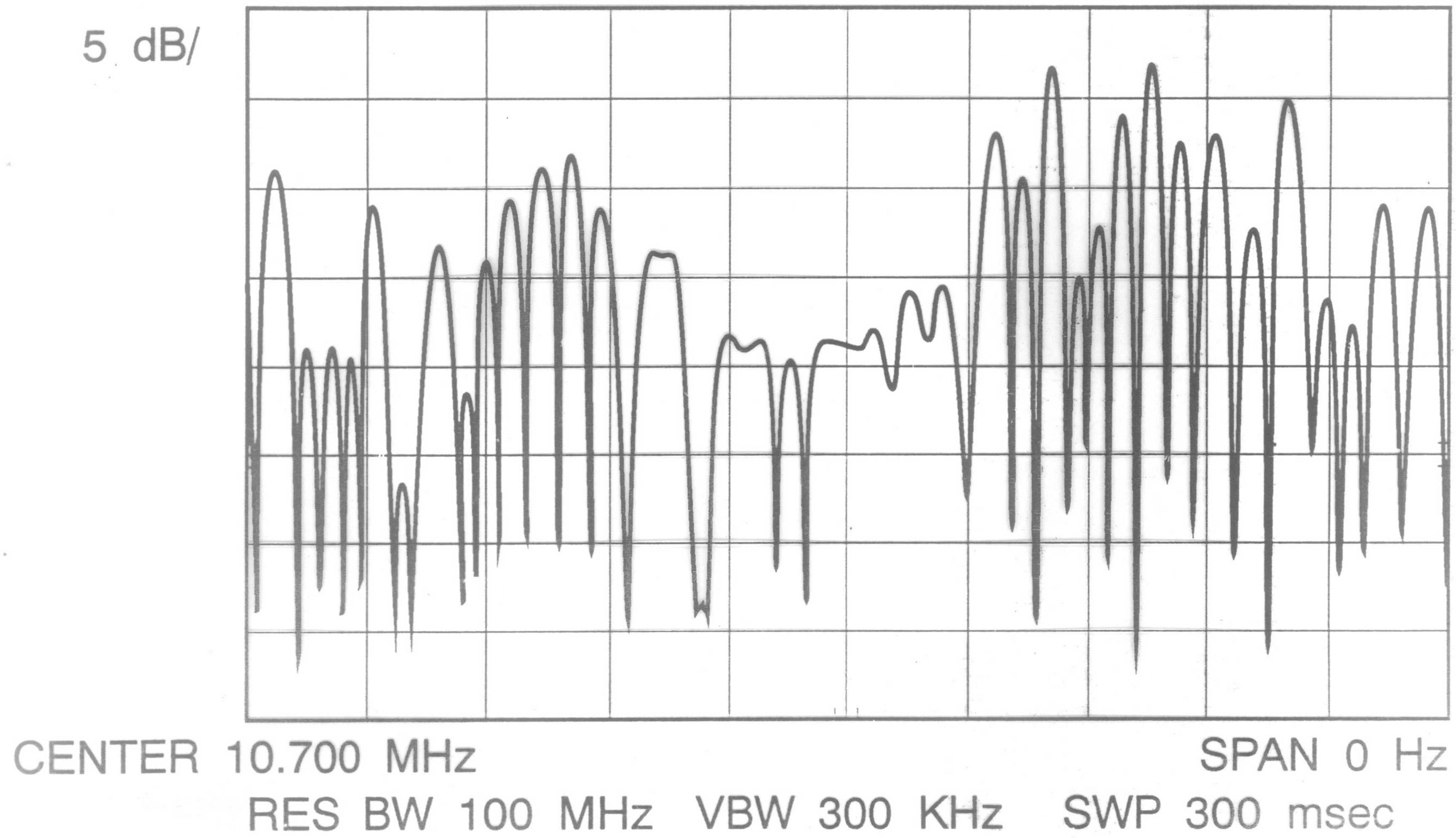

5 dB/
CENTER 10.700 MHz
SPAN 0 Hz
RES BW 100 MHz VBW 300 KHz SWP 300 msec

FIG4-14 200%