FADING

COUNTERACTIONS

FADING COUNTERACTIONS

Diversity Methods
Combining Methods
Diversity Branches

INTRODUCTION

LONG TERM FADING COUNTERACTION

MACROSCOPIC diversity.

SHORT TERM (FAST) FADING COUNTERACTION

MICROSCOPIC diversity.

DIVERSITY SCHEMES : Space, Polarization, Angle, Frequency, Time, Hopping.

Space Diversity

Spaced Antennas at the Base Station

Beamwidth $\Delta\varphi = 2R/D$

$$\rho_{r.} = E^2\left\{\cos\left[2\pi\frac{d}{\lambda}\cos(\theta_i - \alpha)\right]\right\} + E^2\left\{\sin\left[2\pi\frac{d}{\lambda}\cos(\theta_i - \alpha\right]\right\}$$

where $d = v\tau$

v is the speed of the vehicle

τ is the time separation between the signals

λ is the wavelength

θ_i is the incident angle of the i^{th} wave, $\alpha - \pi/2 \leq \theta_i \leq \alpha + \pi/2$

α is the mean incident angle

and $E[x]$ is the expectation of x

$$p(\theta_i) \simeq \frac{Q}{\pi}\cos^n(\theta_i - \alpha)$$

$$\int_{\alpha-\pi/2}^{\alpha+\pi/2} p(\theta_i)d\theta_i = 1$$

Horizontal space diversity is better than vertical space diversity

$$d' = \frac{850}{f}d$$

$\Delta\Omega \simeq 2Rh/D^2$. $\Delta\varphi \simeq 2R/D$, then $\Delta\Omega/\Delta\varphi \simeq h/D$

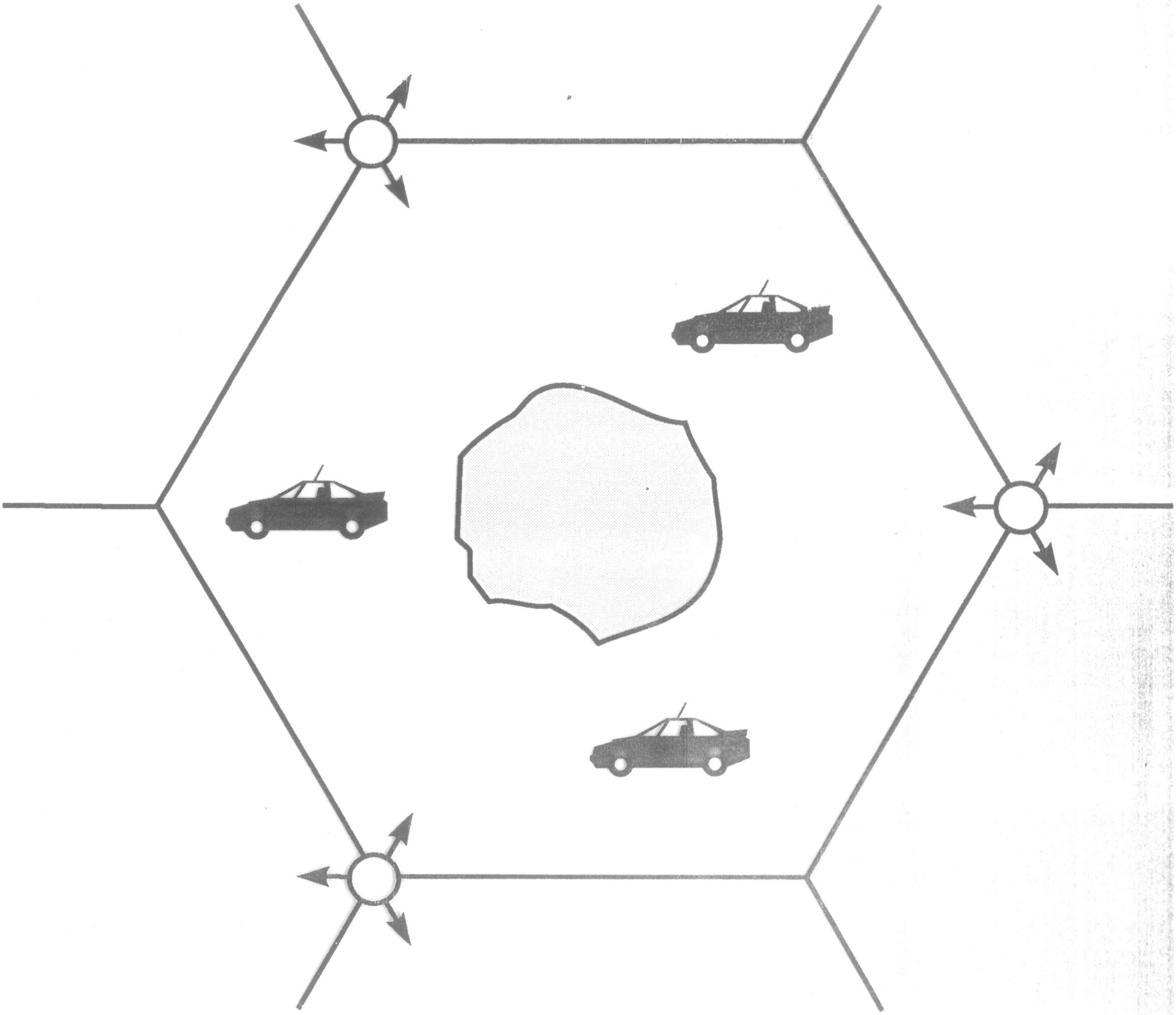


FIG5-1 300%

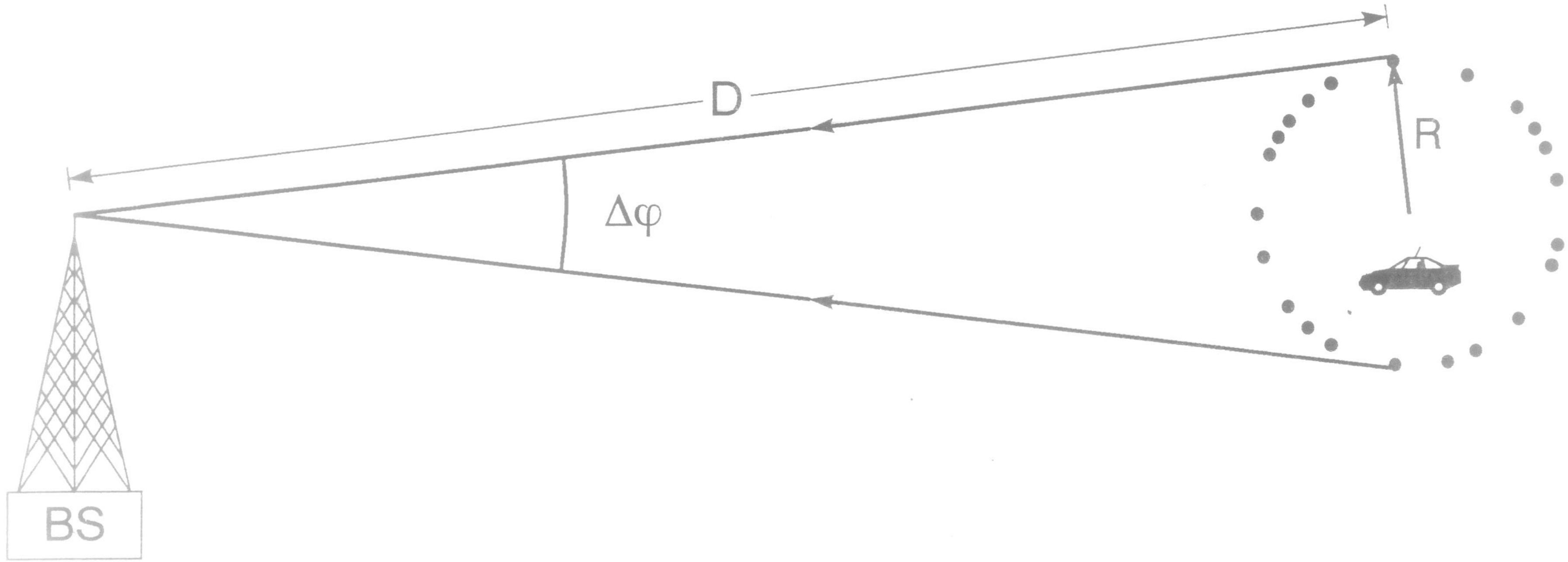


FIG5-3 200%

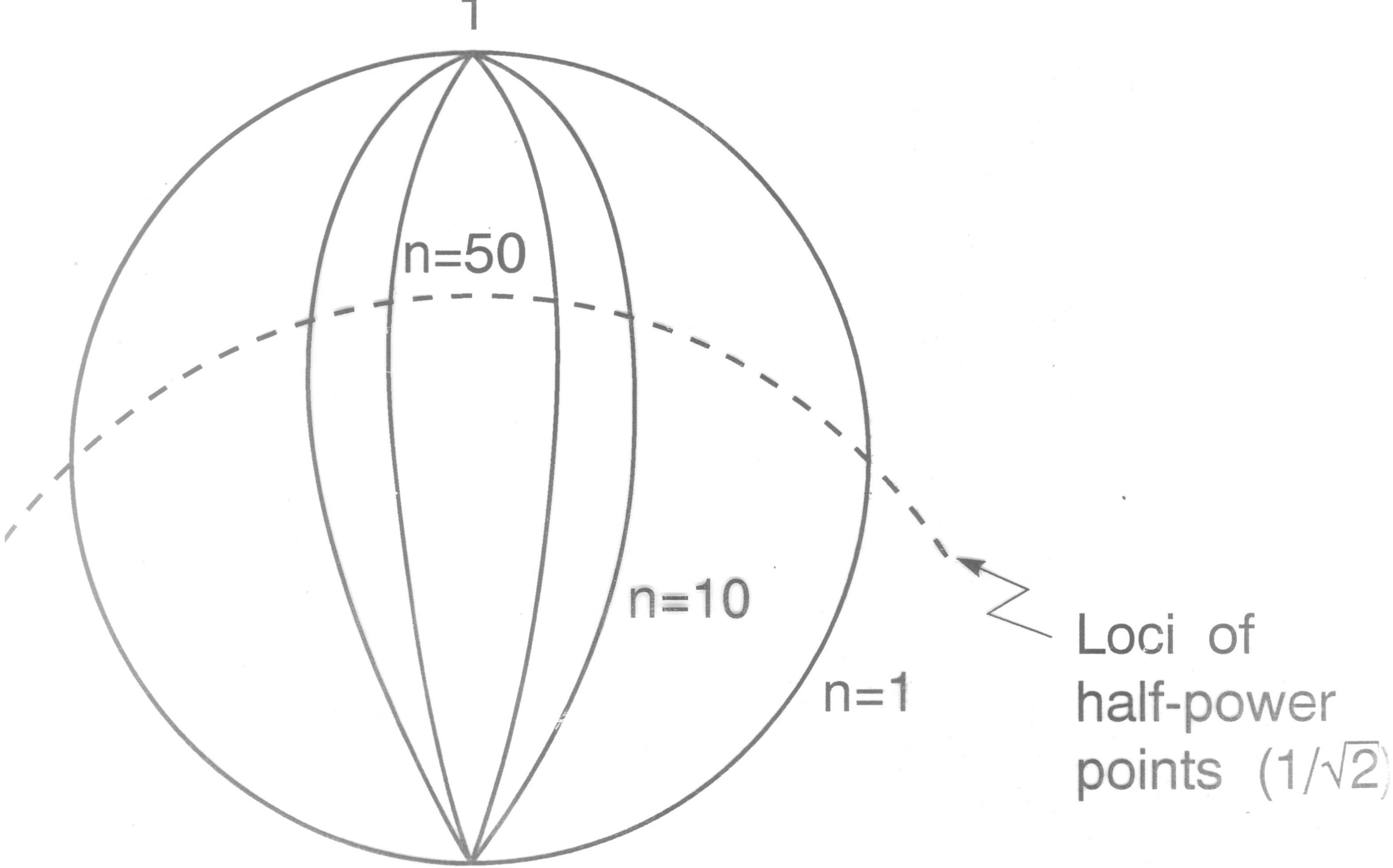


FIG5A-1 300%

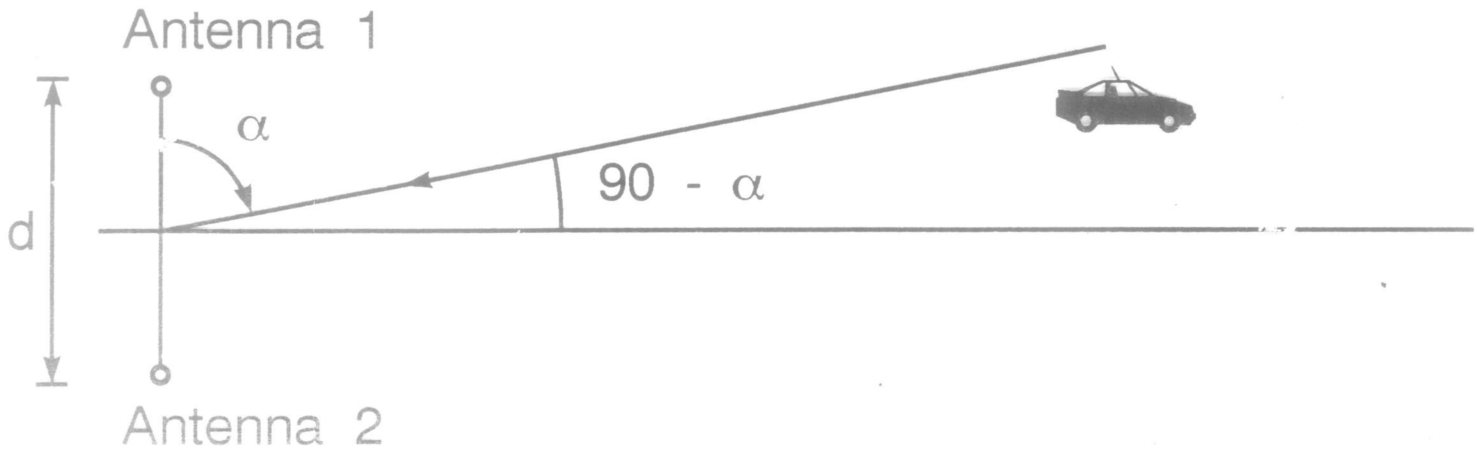
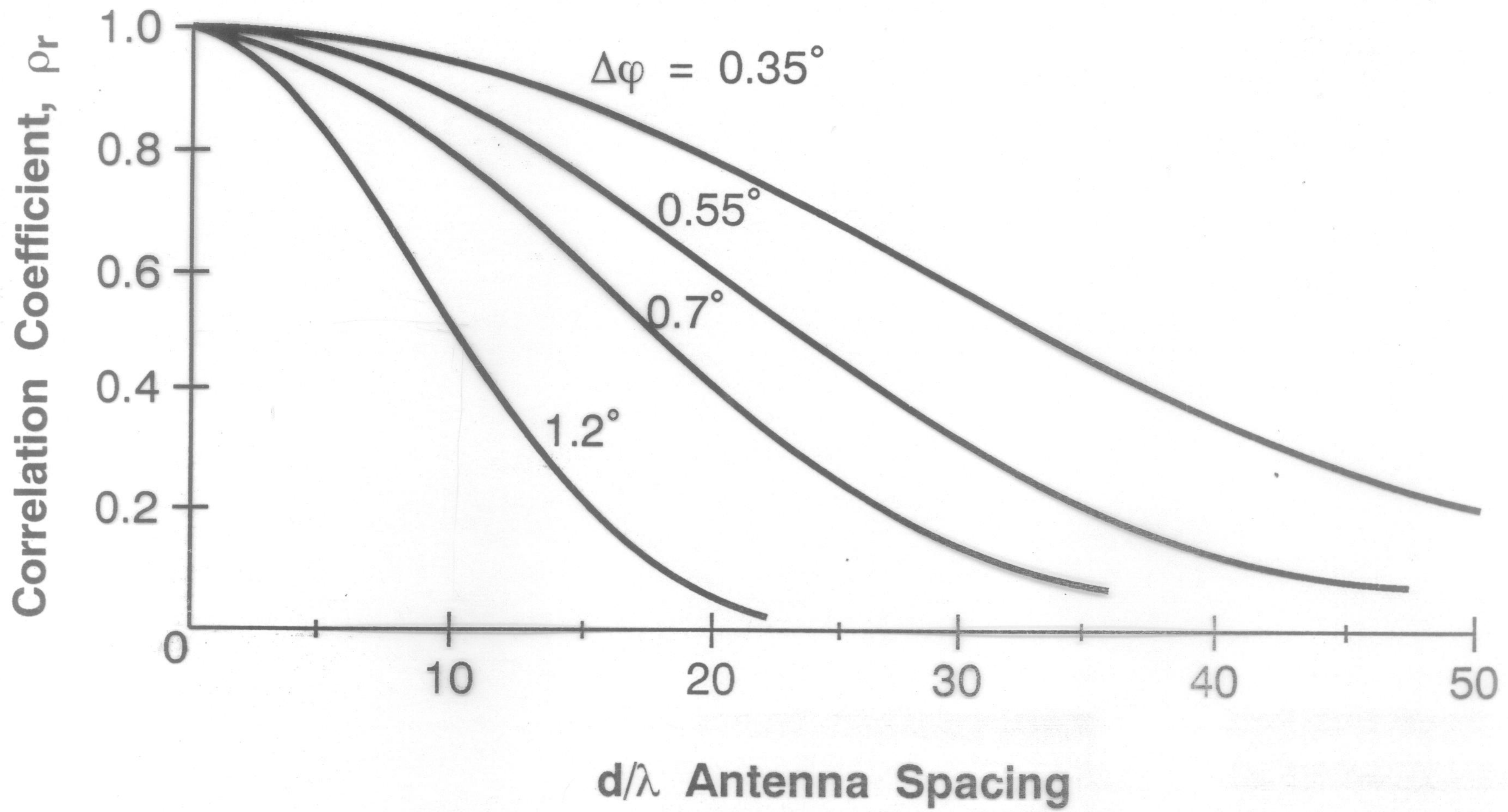
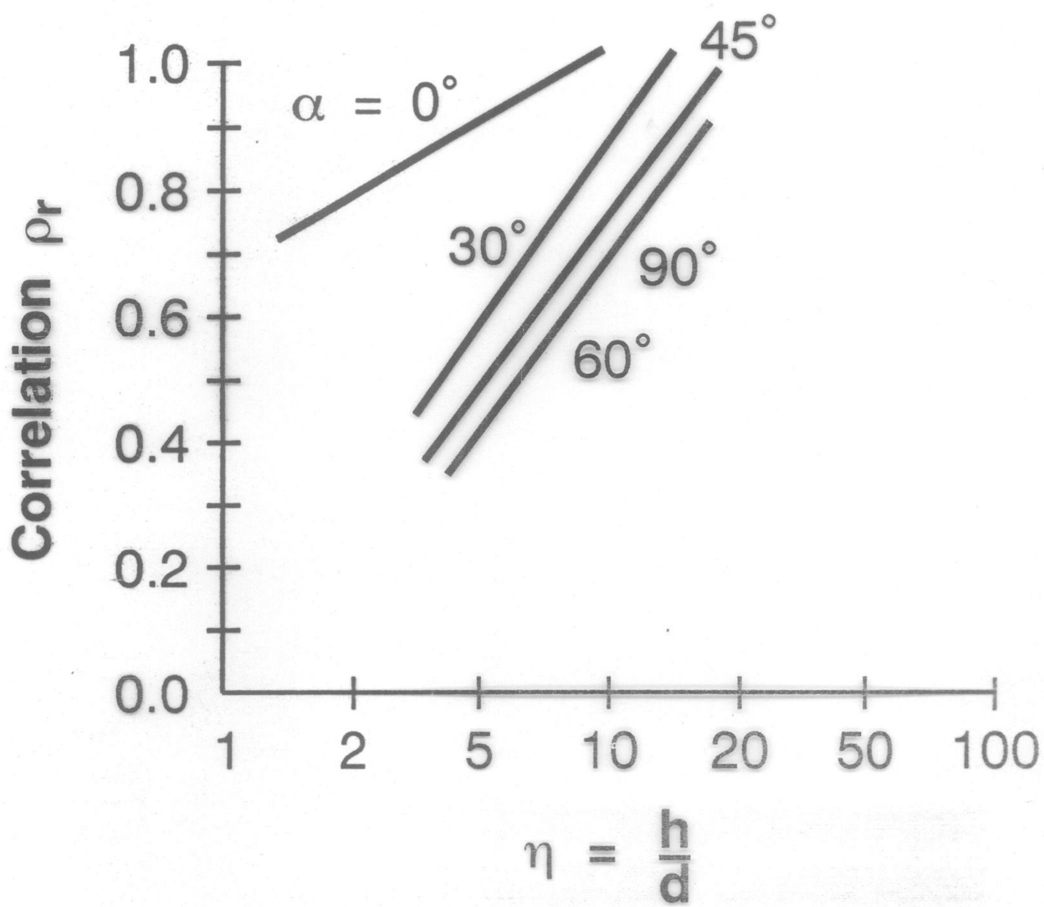


FIG5-4 200%


Correlation Coefficient, ρr
1.0
0.8
0.6
0.4
0.2
0
10
20
30
40
50
Δφ = 0.35°
0.55°
0.7°
1.2°
d/λ Antenna Spacing

(a)

FIG5-5(a) 200%


Correlation ρr
1.0
0.8
0.6
0.4
0.2
0.0
α = 0°
30°
45°
90°
60°
1
2
5
10
20
50
100
η = h/d

(b)

FIG5-5(b) 200%

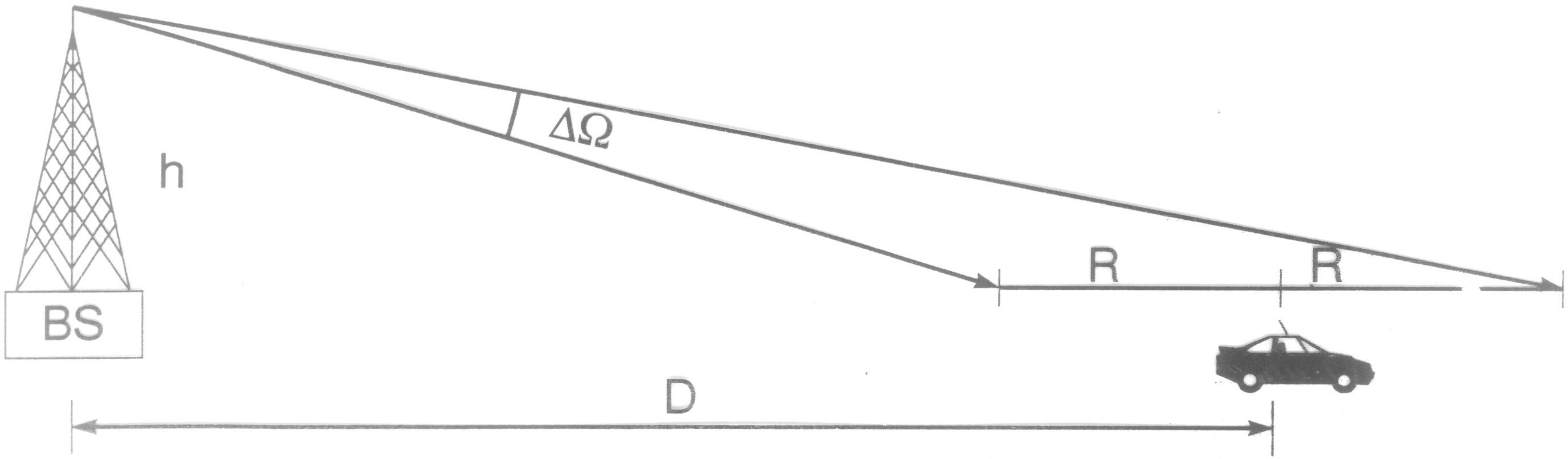


FIG5-6 200%

Spaced Antennas at the Mobile

$$\rho_r = J_o^2(\omega_m \tau)$$

$$\omega_m \tau = 2\pi \frac{v\tau}{\lambda} = 2\pi \frac{d}{\lambda}$$

$J_o(\omega_m \tau) = 0$, then $\omega_m \tau = 2.404$, $d/\lambda \simeq 0{,}38$ ($d = 0{,}5\lambda$) $\rho_r = 0.16$ for $d = 0.16$

$f = 900$ MHz $\Rightarrow d = 17$ cm

Polarization Diversity

Angle Diversity

Frequency Diversity

$B_c = \frac{1}{2\pi \overline{T}}$

In suburban areas, $\overline{T} \simeq 0.5\ \mu s$,

$B_c = 320$ kHz or $B_c = 160$ kHz for the envelope or phase correlations

Frequency separation of 2 MHz ($\simeq 6 \times 320$ kHz).

1:N protection switching

Time Diversity

M branches

delay $\frac{M}{2f_m}$

$\omega_m \tau = 2.404$, or $d \simeq 0.38\ \lambda, \tau \simeq 0.38\ \lambda/v = 0.38/f_m$

$$\tau \geq 1/(2 f_m)$$

At $v = 72$ km/h, $f = 900$ MHz, $\tau \geq 8.3$ ms.

Diversity by Hopping

COMBINING SCHEMES

Switched $\quad r = \text{One out of } \{r_1, r_2, \ldots, r_n\}$

Gain $\quad r = \sum_{i=1}^{M} a_i r_i$

Switched Combining: Pure Selection

Switched Combining: Threshold Selection { SWITCH AND EXAMINE / SWITCH AND STAY

Gain Combining: Maximal Ratio

Gain Combining: Equal Gain

STATISTICAL PROPERTIES AND PERFORMANCE MEASURE

$$\gamma_i \triangleq \frac{\text{local mean signal power}}{\text{mean noise power}}$$

$$\gamma_i = \frac{r_i^2}{2N_i} = \frac{r_i^2}{2N} \quad ; \quad p(r_i) = \frac{r_i}{\sigma_i^2} \exp\left(-\frac{r_i^2}{2\sigma^2}\right)$$

$$p(\gamma_i)\,|d\gamma_i| = p(r_i)\,|dr_i| \qquad \gamma_0 \triangleq \frac{E_0^2/2}{N} = \frac{\sigma_i^2}{N}$$

$$p(\gamma_i) = \frac{1}{\gamma_0} \exp\left(-\frac{\gamma_i}{\gamma_0}\right)$$

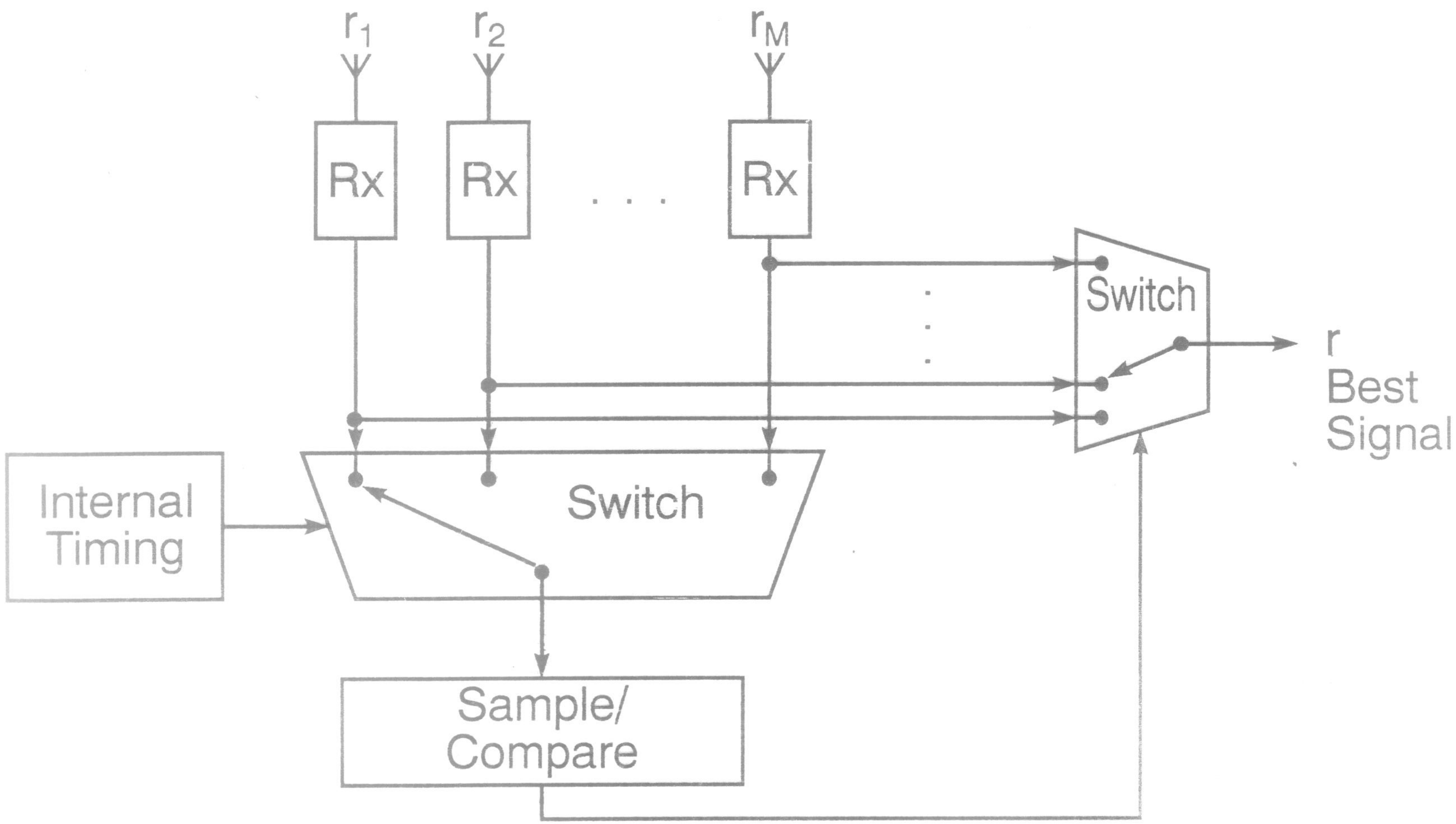
$$P(\Gamma) = \text{prob}(\gamma_i \le \Gamma) = \int_0^{\Gamma} p(\gamma_i)\, d\gamma_i = 1 - \exp\left(-\frac{\Gamma}{\gamma_0}\right)$$

Switched Combining: Pure Selection

$$P_{SEL}(\Gamma_s) = \text{prob}(\gamma_1, \ldots, \gamma_M \le \Gamma_s) = \left[1 - \exp\left(-\frac{\Gamma_s}{\gamma_0}\right)\right]^M$$

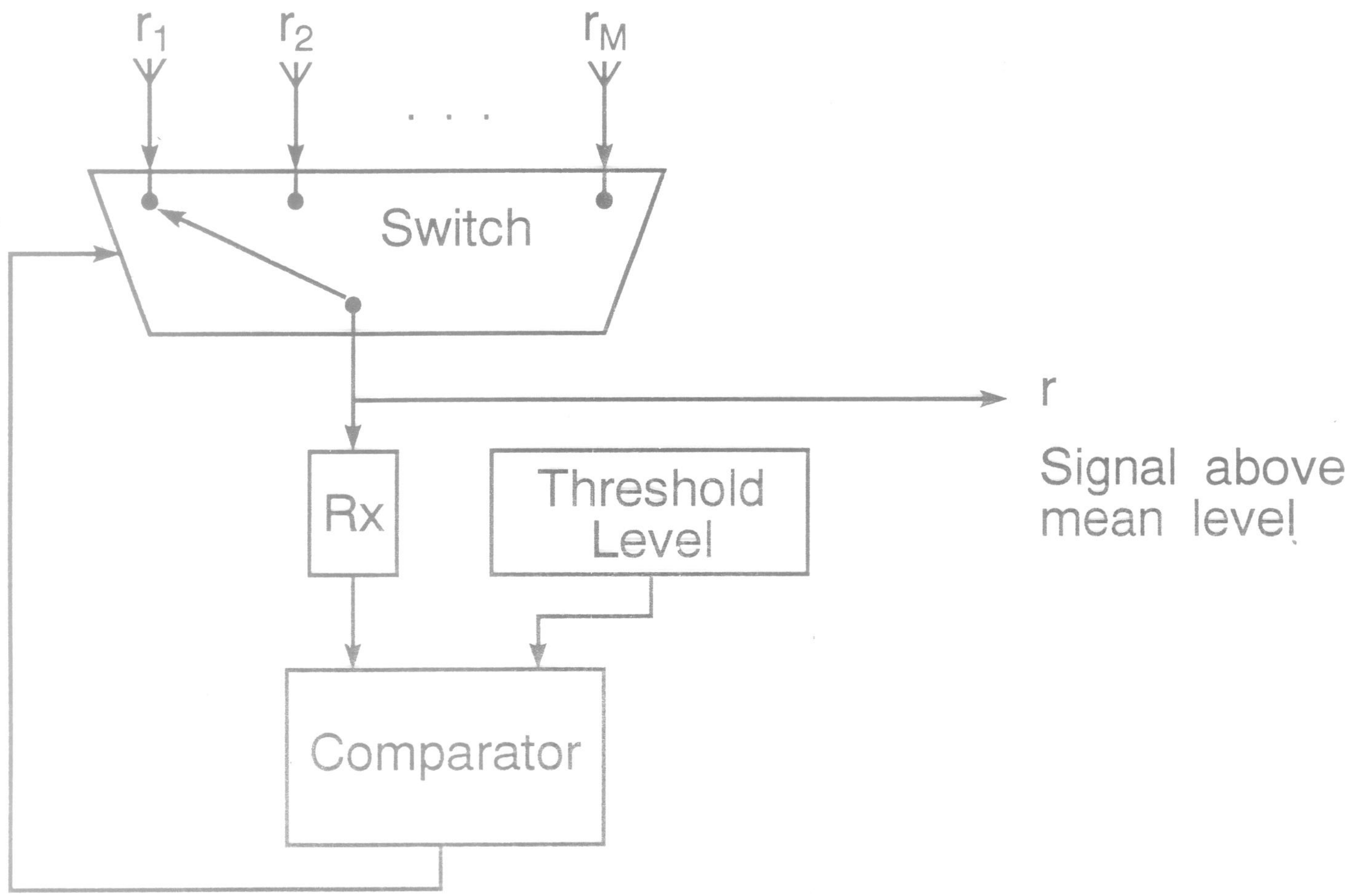
$$\bar{\Gamma}_s = \int_0^{\infty} \Gamma_s\, p_{SEL}(\Gamma_s)\, d\Gamma_s$$

$$p_{SEL}(\Gamma_s) = \frac{dP_{SEL}(\Gamma_s)}{d\Gamma_s} = \frac{M}{\gamma_0}\left[1 - \exp\left(-\frac{\Gamma_s}{\gamma_0}\right)\right]^{M-1} \exp\left(-\frac{\Gamma_s}{\gamma_0}\right)$$


r1
r2
rM
Rx
Rx
Rx
Switch
Switch
r
Best
Signal
Internal
Timing
Sample/
Compare

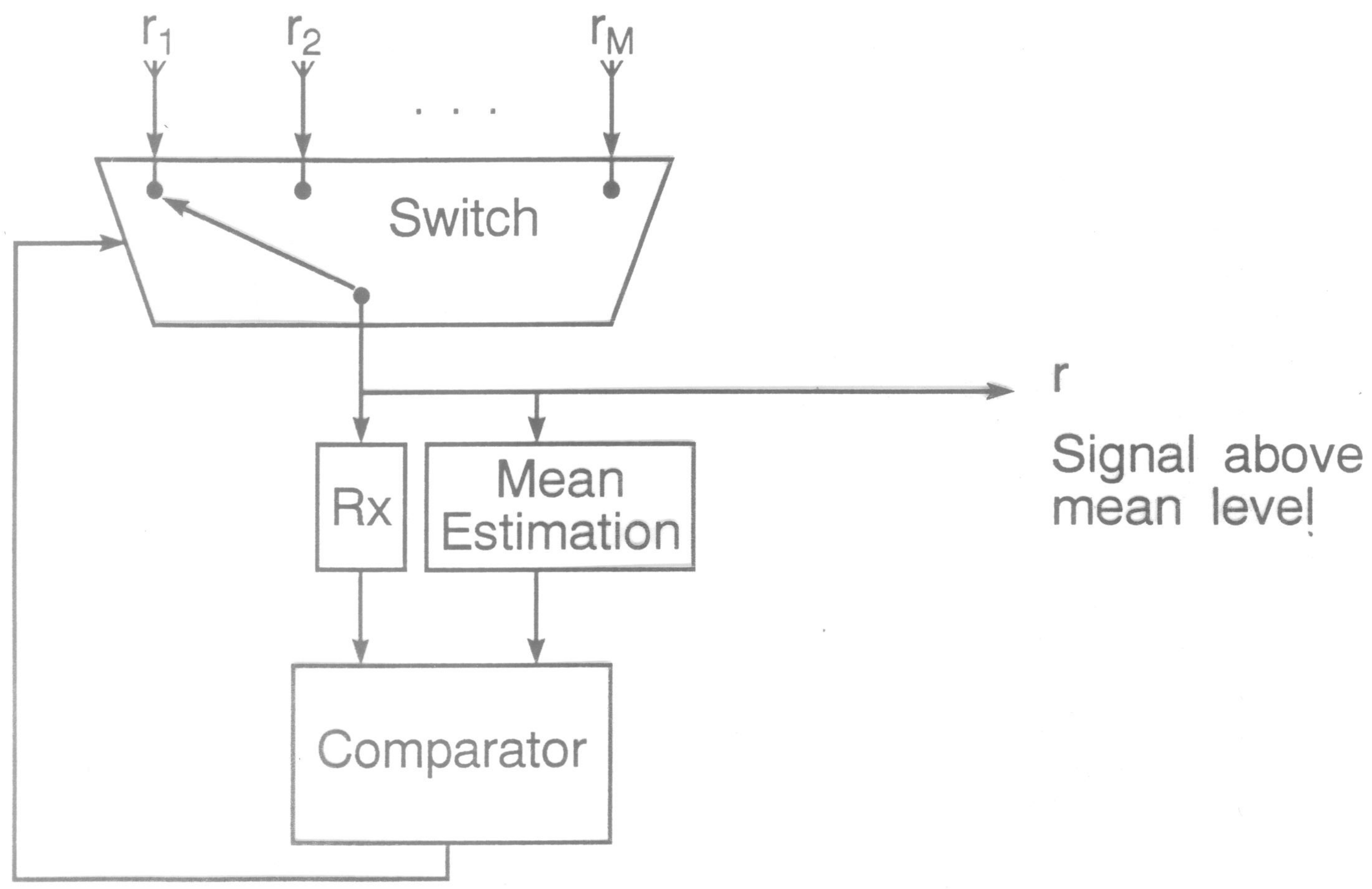
(a)

FIG5-7(a) 200%


r1
r2
rM
Switch
r
Signal above
mean level
Rx
Threshold
Level
Comparator

(b)

FIG5-7(b) 200%


r1
r2
rM
Switch
r
Signal above mean level
Rx
Mean Estimation
Comparator

(c)

FIG5-7(c) 200%

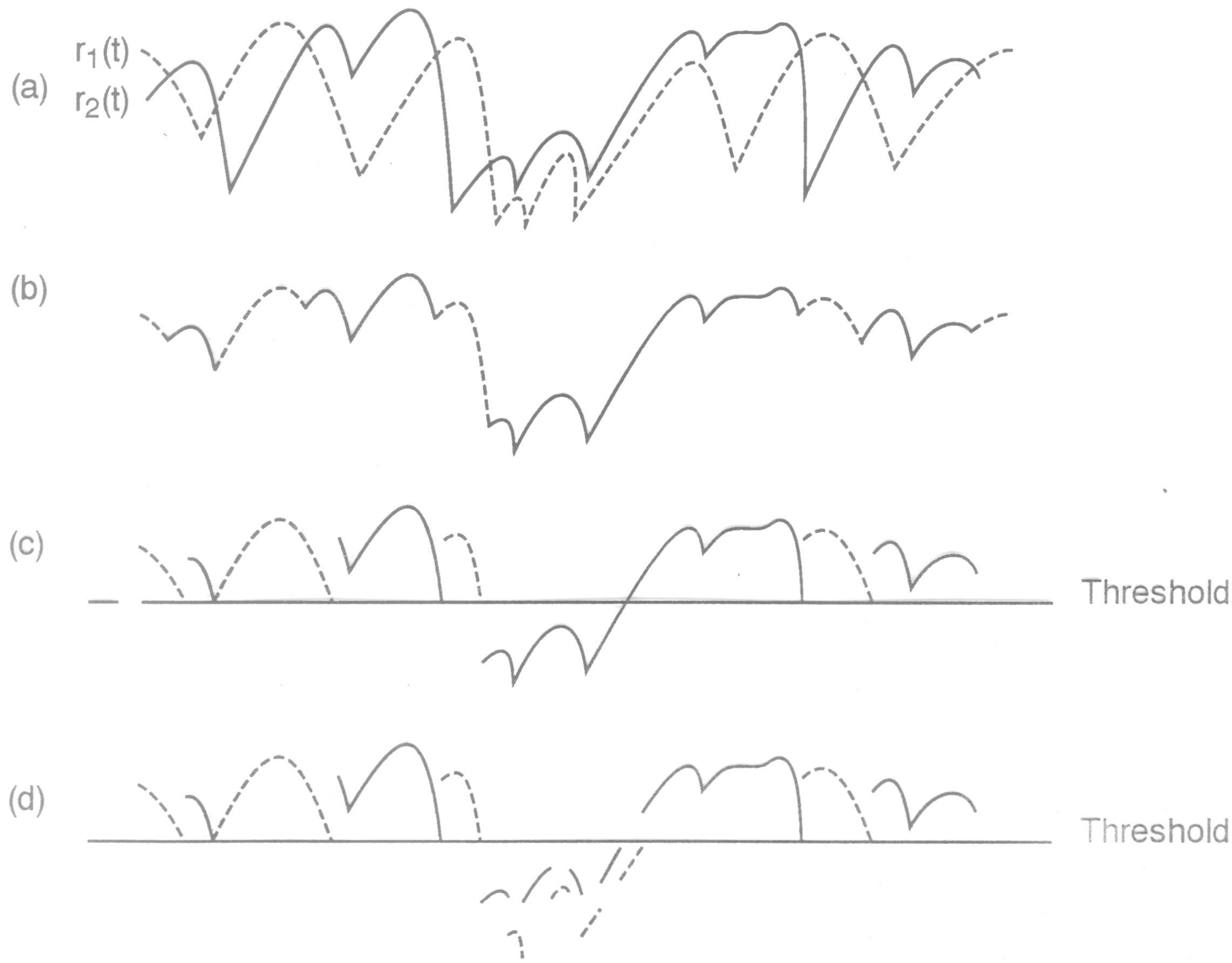


FIG5-8 150%

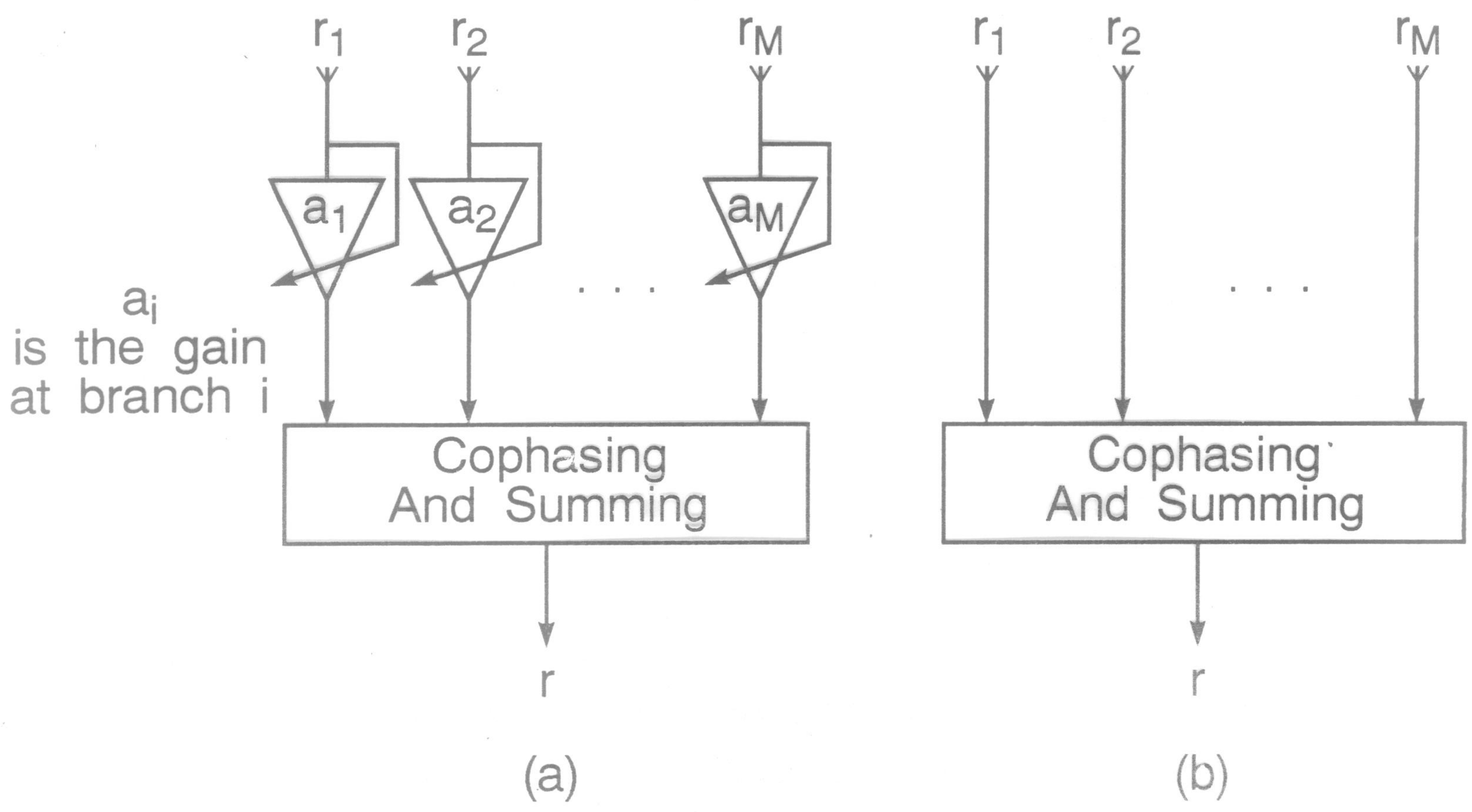
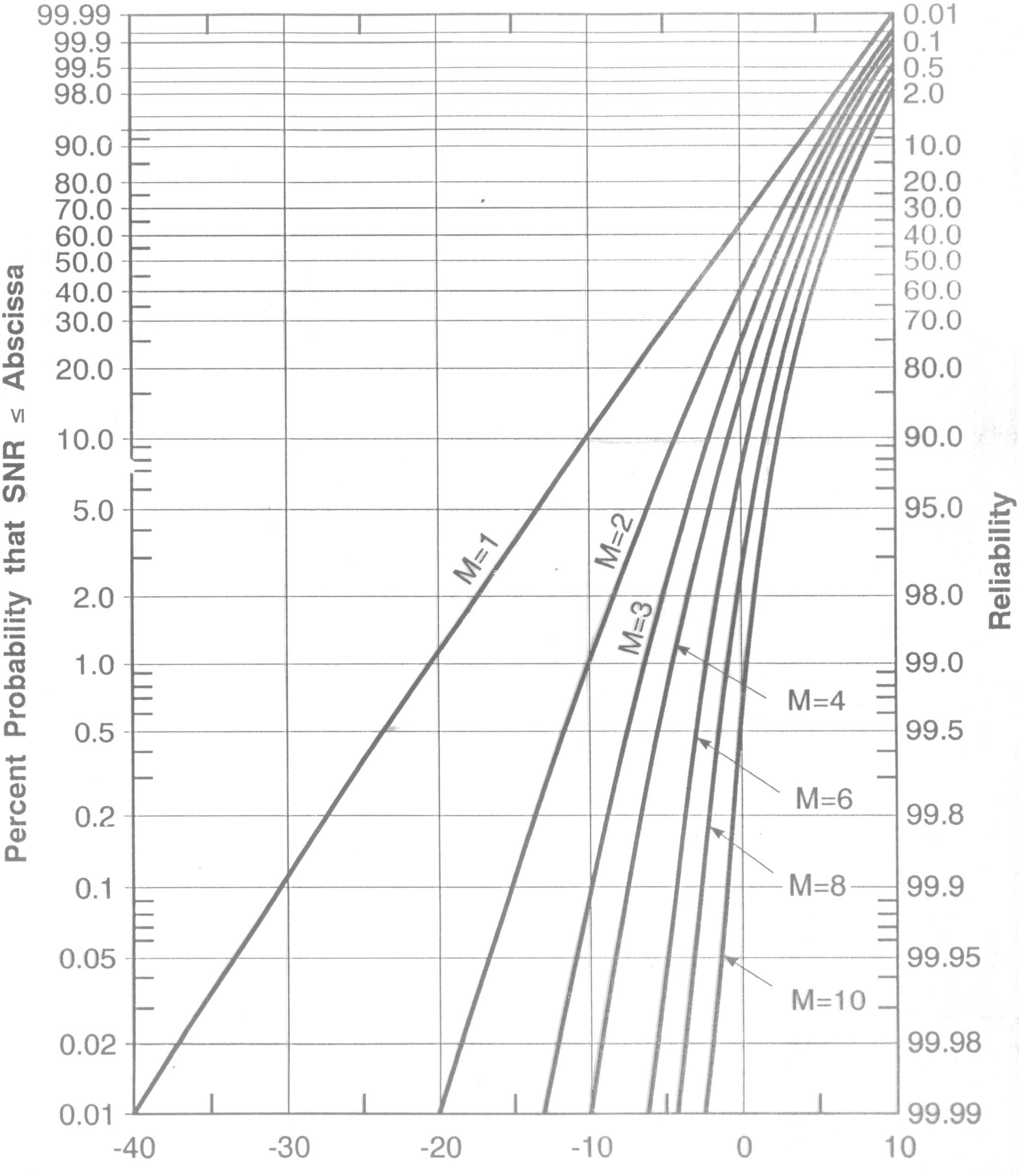

r1
r2
rM
a1
a2
aM
ai
is the gain
at branch i
Cophasing
And Summing
r
(a)
r1
r2
rM
Cophasing
And Summing
r
(b)

FIG5-9 200%



$$10\log\left(\frac{\Gamma_s}{\gamma_0}\right)$$

FIG5-10 150%

$$\text{prob}(\gamma \le \Gamma_s) = \text{prob}(\gamma \le \Gamma_s \mid \gamma = \gamma_i)$$

$$= \text{prob}(\gamma \le \Gamma_s \mid \gamma > \Gamma_T)\,\text{prob}(\gamma > \Gamma_T \mid \gamma = \gamma_x + \gamma_y)$$

$$+ \text{prob}(\gamma \le \Gamma_s \mid \gamma \le \Gamma_T)\,\text{prob}(\gamma \le \Gamma_T \mid \gamma = \gamma_x + \gamma_y)$$

$\gamma = \gamma_x + \gamma_y$; γ_x above Γ_T
γ_y below Γ_T

But $\text{prob}(\gamma > \Gamma_T \mid \gamma = \gamma_x + \gamma_y) =$ prop. time $\gamma > \Gamma_T$
$\text{prob}(\gamma \le \Gamma_T \mid \gamma = \gamma_x + \gamma_y) =$ prop. time $\gamma \le \Gamma_T$

unsuccessful transition

$$q = P(\Gamma_T) = \text{prob}(\gamma \le \Gamma_T) = 1 - \exp\left(-\frac{\Gamma_T}{\gamma_0}\right)$$

Successful transition

$$p = 1 - q = \exp\left(-\frac{\Gamma_T}{\gamma_0}\right)$$

Define τ_a as time $\gamma_i > \Gamma_T$
τ_b " " $\gamma_i < \Gamma_T$

Given a successful transition $\tau_a/2$ $\gamma_i > \Gamma_T$

Given a unsuccessful transition $\tau_b/2$ $\gamma_i < \Gamma_T$

Time between unsuccessful and the next is $\tau_b/2 + \tau_a$

Time between a successful and the next is $\tau_a/2$

$1 > \Gamma_T \Rightarrow \sigma_x = (1-q)\frac{\sigma_a}{2} + q\,\sigma_a = (1+q)\frac{\sigma_a}{2}$

$1 < \Gamma_T \Rightarrow \quad \sigma_Y = q\frac{\sigma_b}{2}$

$$\frac{\sigma_a}{\sigma_b} = \frac{p}{q} = \frac{1-q}{q}$$

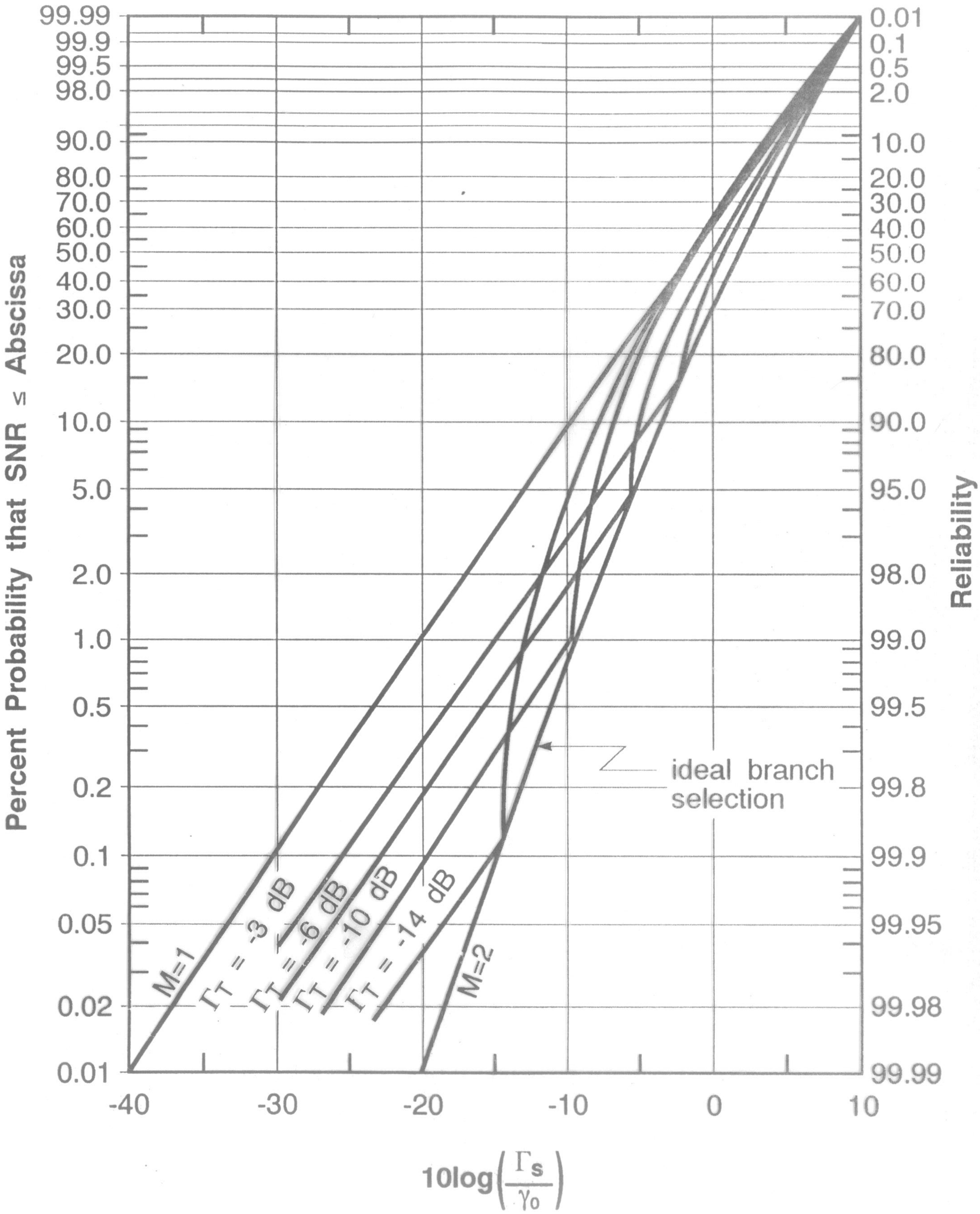
$$\frac{\text{prob}(\gamma > \Gamma_T \mid \gamma = \gamma_x + \gamma_Y)}{\text{prob}(\gamma \le \Gamma_T \mid \gamma = \gamma_x + \gamma_Y)} = \frac{\sigma_x}{\sigma_Y} = \frac{1-q^2}{q^2}$$

$$\text{prob}(\gamma \le \Gamma_s \mid \gamma > \Gamma_T) = \frac{\text{prob}(\Gamma_T < \gamma \le \Gamma_s)}{\text{prob}(\gamma > \Gamma_T)} = \frac{\int_{\Gamma_T}^{\Gamma_s} p(\gamma)\,d\gamma}{\int_{\Gamma_T}^{\infty} p(\gamma)\,d\gamma}$$

$$= \begin{cases} \dfrac{P(\Gamma_s) - P(\Gamma_T)}{1 - P(\Gamma_T)} = \dfrac{P(\Gamma_s) - q}{1-q} & , \Gamma_s \ge \Gamma_T \\[2ex] 0 & , \Gamma_s < \Gamma_T \end{cases}$$

$$\text{prob}(\gamma \le \Gamma_S \mid \gamma \le \Gamma_T) = \frac{\text{prob}(\gamma \le \Gamma_S \text{ and } \gamma \le \Gamma_T)}{\text{prob}(\gamma \le \Gamma_T)}$$

$$= \begin{cases} \dfrac{P(\Gamma_T)}{P(\Gamma_T)} = 1 , & \Gamma_S \geq \Gamma_T \\[2ex] \dfrac{P(\Gamma_S)}{P(\Gamma_T)} = \dfrac{P(\Gamma_S)}{1-q} , & \Gamma_S < \Gamma_T \end{cases}$$



Percent Probability that SNR ≤ Abscissa
Reliability
99.99
99.9
99.5
98.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
5.0
2.0
1.0
0.5
0.2
0.1
0.05
0.02
0.01
0.01
0.1
0.5
2.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
95.0
98.0
99.0
99.5
99.8
99.9
99.95
99.98
99.99
-40
-30
-20
-10
0
10
ideal branch selection
M=1
Γ_T = -3 dB
Γ_T = -6 dB
Γ_T = -10 dB
Γ_T = -14 dB
M=2
$10\log\left(\frac{\Gamma_s}{\gamma_0}\right)$

FIG5-11 150%

$$\bar{\Gamma}_s = \gamma_0 \sum_{i=1} 1/i$$

Switched Combining: Threshold Selection

$$P_{SCN}(\Gamma_s) = \text{prob}(\gamma \le \Gamma_s)$$

SWITCH AND STAY

$$P_{SCN}(\Gamma_s) = \text{prob}(\gamma \le \Gamma_s) = \text{prob}(\gamma \le \Gamma_s \mid \gamma = \gamma_1) = \text{prob}(\gamma \le \Gamma_s \mid \gamma = \gamma_2)$$

→

$$q = P(\Gamma_T) = \text{prob}(\gamma \le \Gamma_T) = 1 - \exp\left(- \frac{\Gamma_T}{\gamma_0}\right)$$

$$p = 1 - q = \exp\left(- \frac{\Gamma_T}{\gamma_0}\right)$$

$$P_{SCN}(\Gamma_S) = \begin{cases} (1 + q)\ P(\Gamma_S) - q & , \Gamma_S \ge \Gamma_T \\ qP(\Gamma_S) & , \Gamma_S < \Gamma_T \end{cases}$$

$$p_{SCN}(\Gamma_S) = \frac{dP_{SCN}(\Gamma_S)}{d\Gamma_S} = \begin{cases} (1 + q)\ p(\Gamma_S) & , \Gamma_S \ge \Gamma_T \\ qp(\Gamma_S) & , \Gamma_S < \Gamma_T \end{cases}$$

$$\bar{\Gamma}_S = \int_0^\infty \Gamma_S p_{SCN}(\Gamma_S)\ d\Gamma_S = \int_0^{\Gamma_T} \Gamma_S p_{SCN}(\Gamma_S)\ d\Gamma_S + \int_{\Gamma_T}^\infty \Gamma_S p_{SCN}(\Gamma_S)\ d\Gamma_S$$

$$\bar{\Gamma}_S = \gamma_0 + \Gamma_T \exp\left(- \frac{\Gamma_T}{\gamma_0}\right)$$

Gain Combining: Maximal Ratio

$$r = \sum_{i=1}^{M} a_i r_i$$

$$\mathcal{N} = N \sum_{i=1}^{M} a_i^2$$

$$\gamma = \frac{r^2/2}{\mathcal{N}} = \frac{1}{2} \frac{\left(\sum_{i=1}^{M} a_i r_i\right)^2}{N \sum_{i=1}^{M} a_i^2} \leq \frac{1}{2} \frac{\sum a_i^2 \sum r_i^2}{N \sum a_i^2}$$

$$a_i = r_i/N \qquad \therefore \gamma \leq \frac{\sum r_i^3}{2N} \Rightarrow a_i = \frac{r_i}{N}$$

$$\gamma = \frac{1}{2} \frac{\left(\sum_{i=1}^{M} r_i^2/N\right)^2}{N \sum_{i=1}^{M} (r_i/N)^2} = \sum_{i=1}^{M} \frac{r_i^2}{2N} = \sum_{i=1}^{M} \gamma_i$$

$$\gamma = \sum_{i=1}^{M} \gamma_i = \sum_{i=1}^{M} \frac{1}{2N} r_i^2 = \sum_{i=1}^{M} \frac{x_i^2}{2N} + \sum_{i=1}^{M} \frac{y_i^2}{2N}$$

$$p_{MAX}(\gamma) = \frac{\gamma^{M-1} \exp(-\gamma/\gamma_0)}{\gamma_0^M (M-1)!} \qquad , \gamma \geq 0$$

$$P_{MAX}(\Gamma_S) = 1 - \exp\left(-\Gamma_S/\gamma_0\right) \sum_{i=1}^{M} \frac{(\Gamma_S/\gamma_0)^{i-1}}{(i-1)!}$$

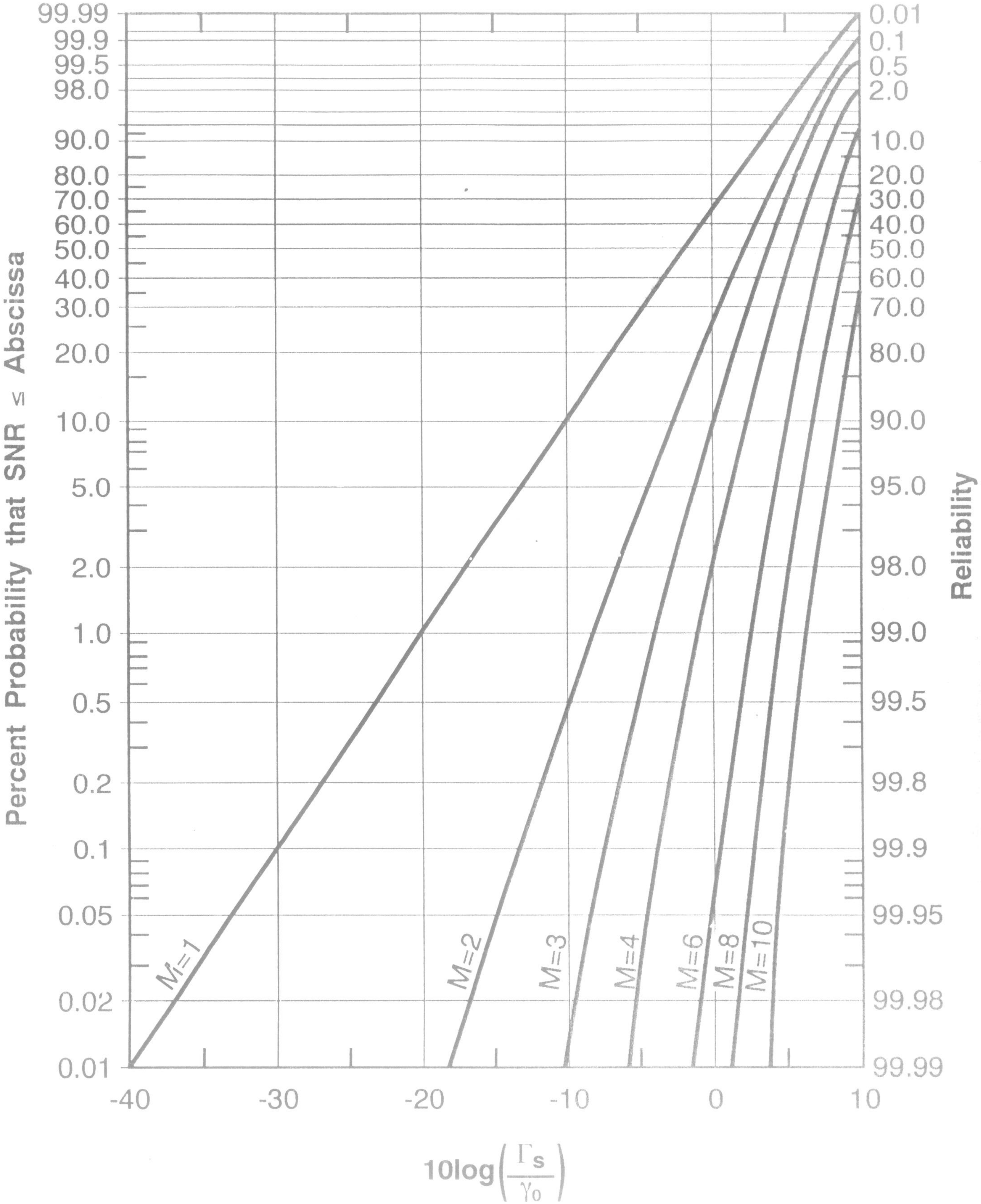
$$\overline{\Gamma}_S = \langle \gamma \rangle = \langle \sum_{i=1}^{M} \gamma_i \rangle = \sum_{i=1}^{M} \langle \gamma_i \rangle = M \gamma_0$$

Gain Combining: Equal Gain

$$r = \sum_{i=1}^{M} r_i$$

$$\gamma = \frac{r^2/2}{\mathcal{N}} = \frac{\left(\sum_{i=1}^{M} r_i\right)^2}{2NM} = \frac{r^2}{2NM}$$

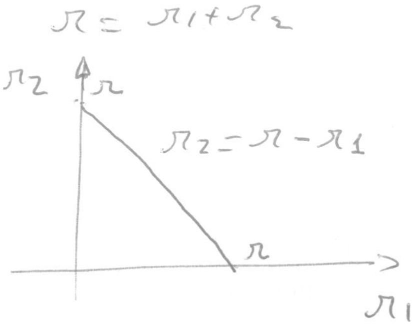
$$p_{EQU}(\gamma) \; |d\gamma| = p(r) \; |dr|$$



Percent Probability that SNR ≤ Abscissa
Reliability
99.99
99.9
99.5
98.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
5.0
2.0
1.0
0.5
0.2
0.1
0.05
0.02
0.01
0.01
0.1
0.5
2.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
95.0
98.0
99.0
99.5
99.8
99.9
99.95
99.98
99.99
M=1
M=2
M=3
M=4
M=6
M=8
M=10
-40
-30
-20
-10
0
10
$10\log\left(\frac{\Gamma_s}{\gamma_0}\right)$

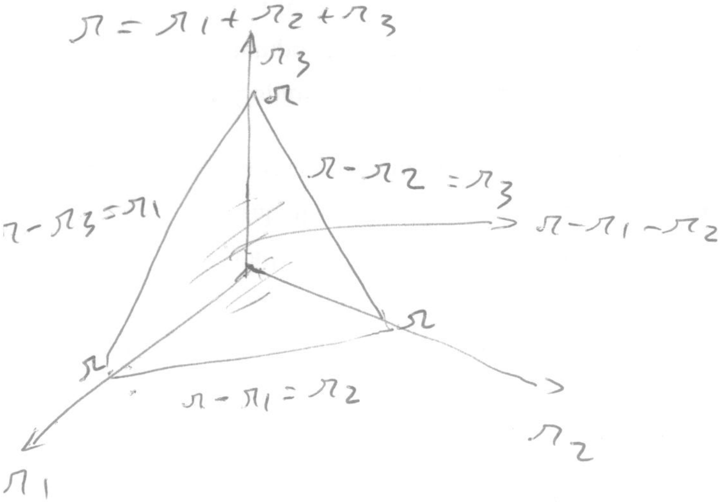
FIG5.12 150%

Equal Gain



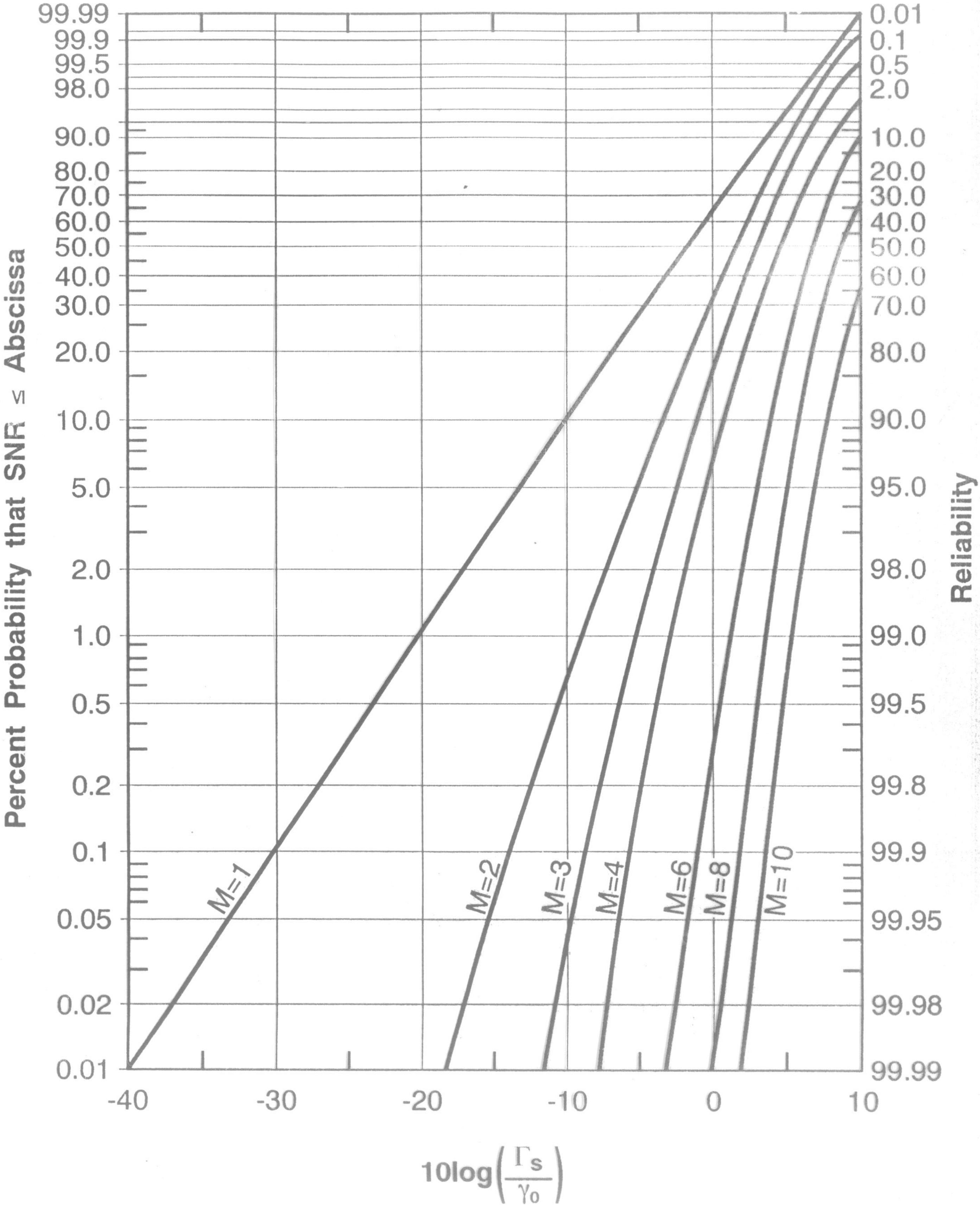
$$P(r) = \int_0^{r}\int_0^{r-r_1} p(r_1, r_2)\, dr_2\, dr_1$$

$$p(r) = \int_0^{r} p(r_1, r-r_1)\, dr_1$$



$$P(r) = \int_0^{r}\int_0^{r-r_1}\int_0^{r-r_1-r_2} p(r_1, r_2, r_3)\, dr_3\, dr_2\, dr_1$$

$$p(r) = \int_0^{r}\int_0^{r-r_1} p(r_1, r_2, r-r_1-r_2)\, dr_2\, dr_1$$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5.0
2.0
1.0
0.5
0.2
0.1
0.05
0.02
0.01
99.8
99.9
99.95
99.98
-40
-30
-20
-10
0
10
M=1
M=2
M=3
M=4
M=6
M=8
M=10
$10\log\left(\frac{\Gamma_s}{\gamma_0}\right)$

FIG5-13 150%

$$p_{EQU}(\gamma) = NM \frac{p(r)}{r} = NM \frac{p(\sqrt{2\gamma NM})}{\sqrt{2\gamma NM}}$$

For $M = 2$, $r = r_1 + r_2$ or $r_2 = r - r_1$

Then

$$p(r) = \int_0^r p(r_1, r_2)\Big|_{r_2 = r - r_1} dr_1 = \int_0^r p(r_1, r - r_1) dr_1$$

$$P_{EQU}(\Gamma_S) = 1 - \exp(-2\Gamma_S) - \sqrt{\pi\Gamma_S}\ \exp(-\Gamma_S)\ \mathrm{erf}(\sqrt{\Gamma_S})$$

$$p_{EQU}(\gamma) = \frac{2^{M-1}\ M^M}{(2M-1)!} \frac{\gamma^{M-1}}{\gamma^M}$$

$$P_{EQU}(\Gamma_S) \simeq \frac{(M/2)^M \sqrt{\pi}}{(M - 1/2)!} \frac{1}{M!} \left(\frac{\Gamma_S}{\gamma_0}\right)^M$$

where $(\circ)!$ is the gamma function. In particular

$$\left(M - \frac{1}{2}\right)! = \frac{1.3.5.7...(2M-1)!}{2^M}$$

$$\bar{\Gamma}_S = \langle \gamma \rangle = \frac{1}{2NM} \left\langle \left(\sum_{i=1}^{M} r_i\right)^2 \right\rangle = \frac{1}{2NM} \sum_{i,j}^{M} \langle r_i r_j \rangle$$

$$\bar{\Gamma}_S = \frac{1}{2NM}\left[2M\sigma^2 + M(M-1)\pi \frac{\sigma^2}{2}\right]$$

$$\bar{\Gamma}_S = \gamma_0 \left[1 + (M-1)\frac{\pi}{4}\right]$$

where $\gamma_0 = \sigma^2/N$ (see equation (5.16)).

COMPARATIVE PERFORMANCE OF COMBINING TECHNIQUES

$$\frac{\bar{\Gamma}_S}{\gamma_0} = \sum_{i=1}^{M} 1/i \qquad \text{, for pure selection}$$

$$\frac{\bar{\Gamma}_S}{\gamma_0} = 1 + \frac{\Gamma_T}{\gamma_0} \exp\left(-\frac{\Gamma_T}{\gamma_0}\right) \quad \text{, for threshold scanning}$$

$$\frac{\bar{\Gamma}_S}{\gamma_0} = M \quad \text{, for maximal ratio}$$

$$\frac{\bar{\Gamma}_S}{\gamma_0} = 1 + (M - 1)\frac{\pi}{4} \quad \text{, for equal gain}$$

OTHER RELEVANT POINTS

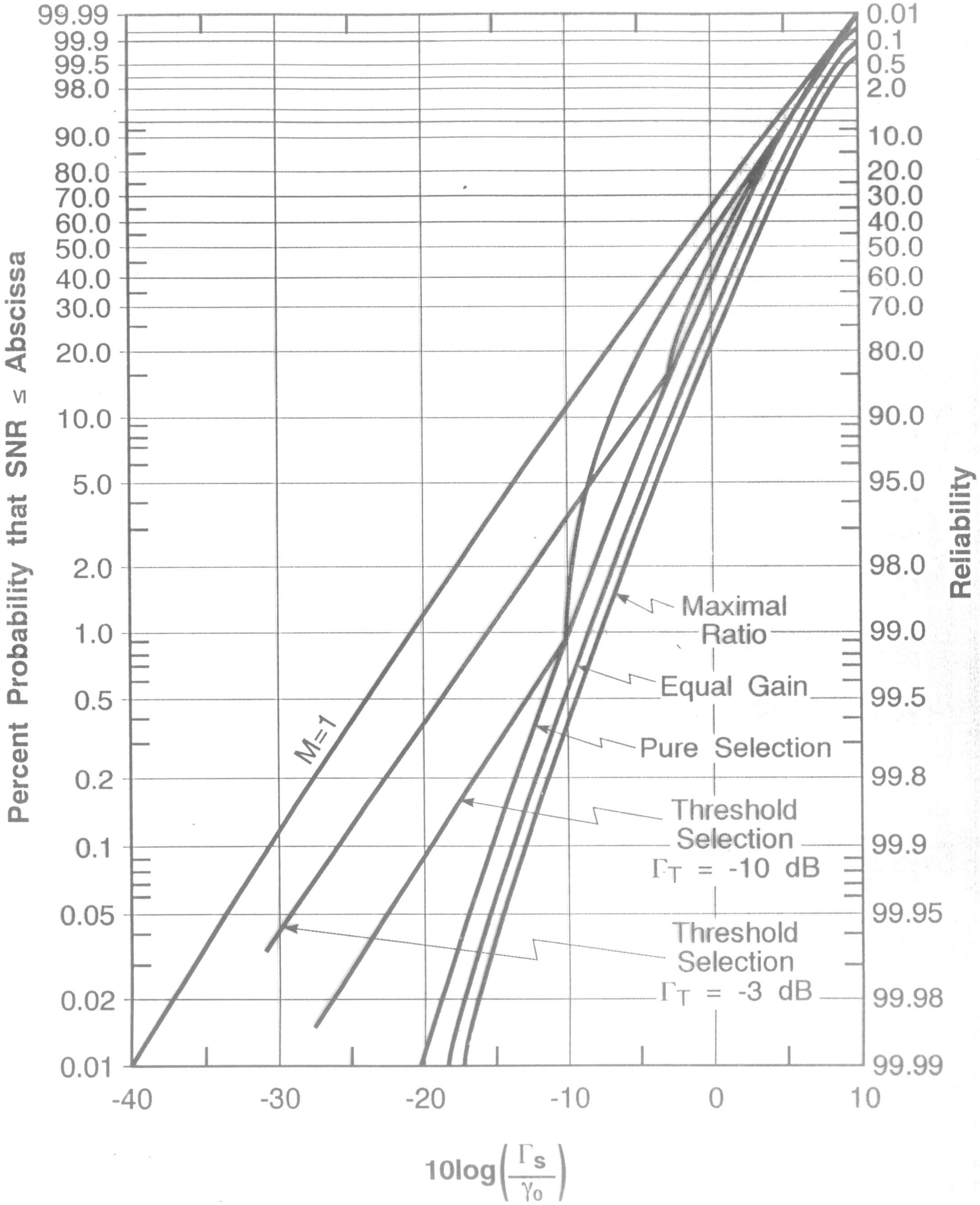
Combining Correlated Signals

Level Crossing Rate (LCR)

Average Duration of Fades

Random FM

Predetection and Postdetection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1.0
0.5
0.2
0.1
0.05
0.02
0.01
0.01
0.1
0.5
2.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
95.0
98.0
99.0
99.5
99.8
99.9
99.95
99.98
99.99
-40
-30
-20
-10
0
10
M=1
Maximal Ratio
Equal Gain
Pure Selection
Threshold Selection Γ_T = -10 dB
Threshold Selection Γ_T = -3 dB
$10\log\left(\frac{\Gamma_s}{\gamma_0}\right)$

FIG5-14 150%

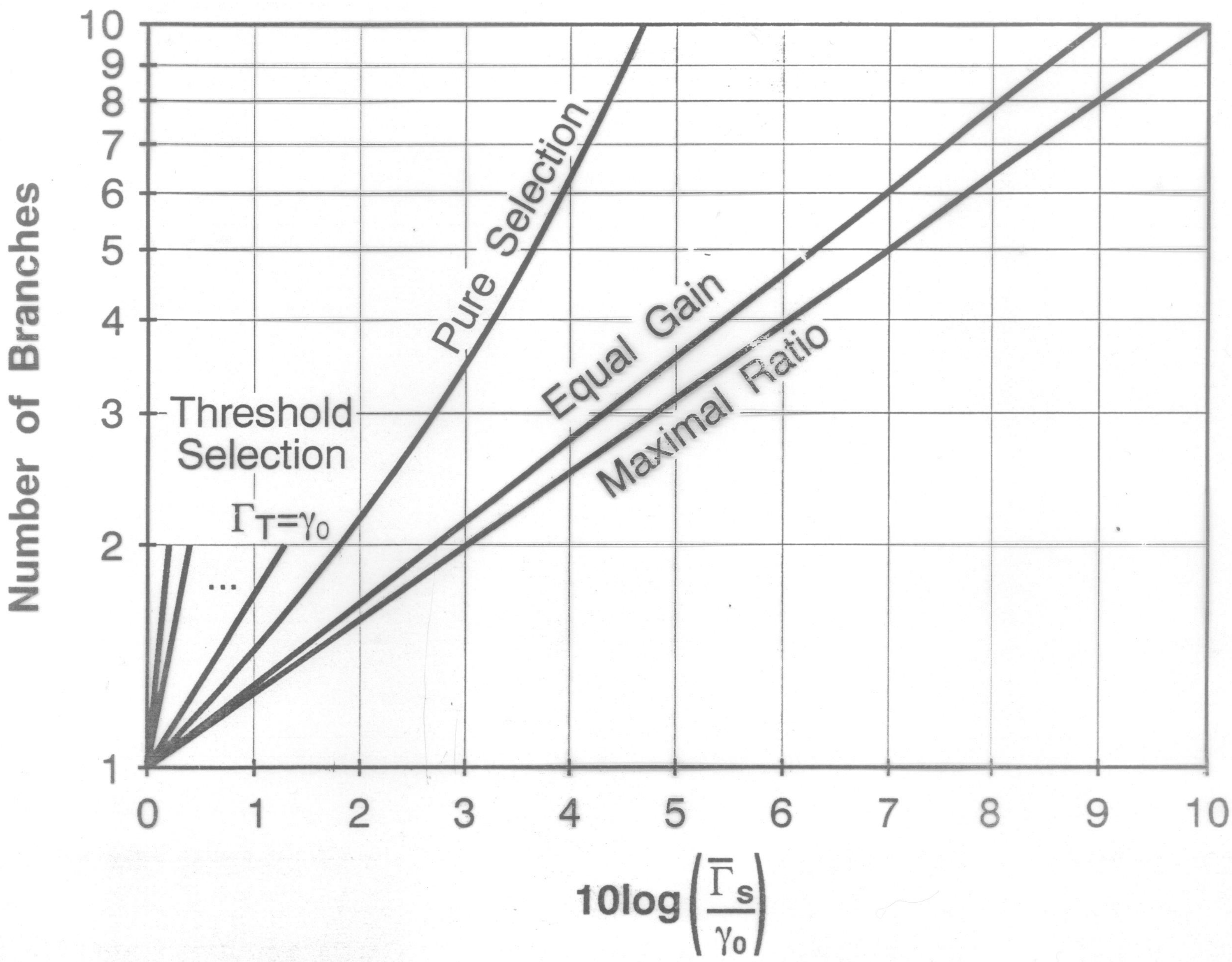
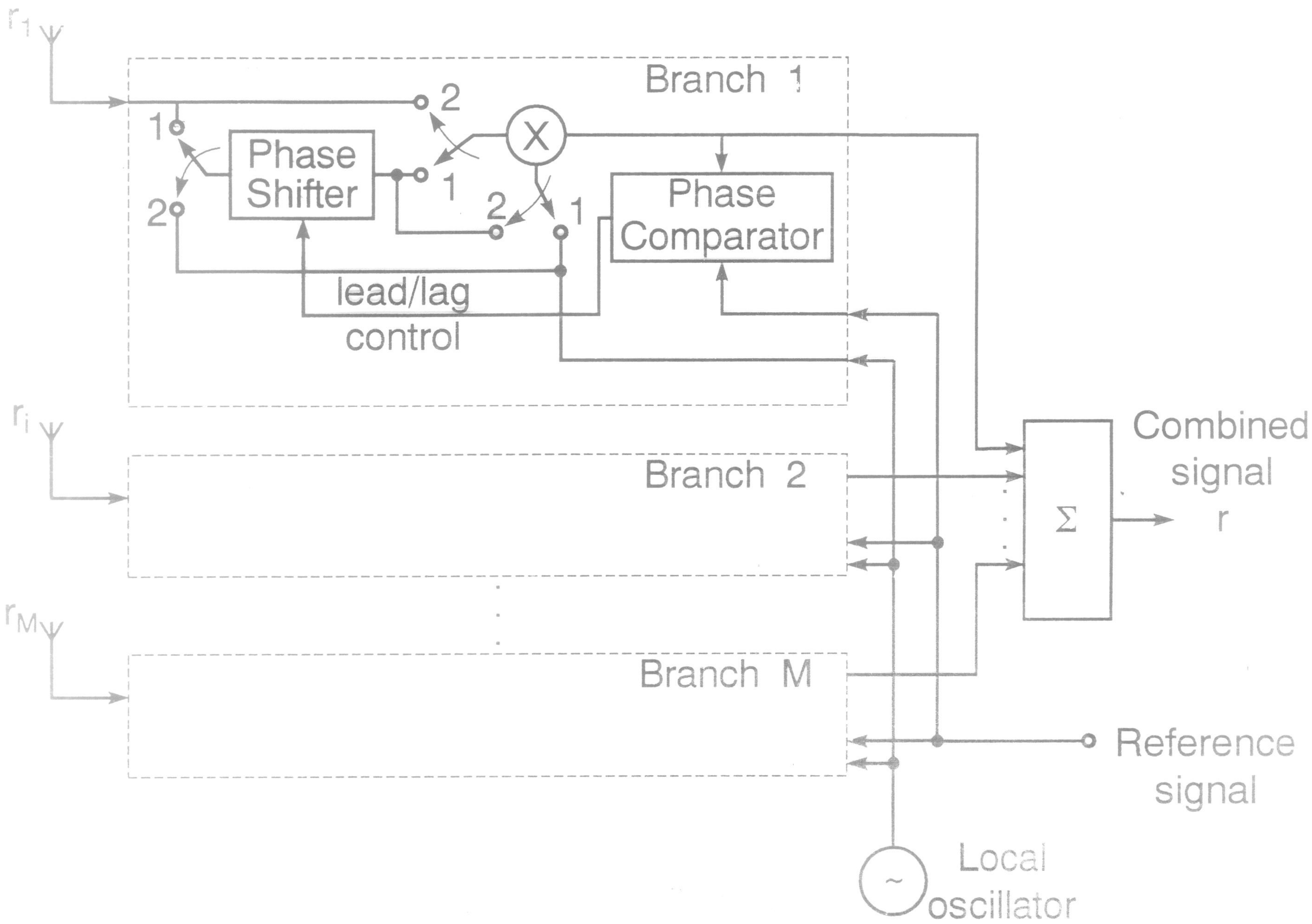

Number of Branches
1
2
3
4
5
6
7
8
9
10
0
1
2
3
4
5
6
7
8
9
10
$10\log\left(\frac{\overline{\Gamma}_s}{\gamma_0}\right)$
Threshold
Selection
$\Gamma_T=\gamma_0$
...
Pure Selection
Equal Gain
Maximal Ratio

FIG5-15 200%



r1
ri
rM
Branch 1
Branch 2
Branch M
Phase Shifter
Phase Comparator
X
1
2
lead/lag control
Σ
Combined signal r
Reference signal
Local oscillator
~

FIG5-16 200%

Mean number of faded (corrupted) bits : $z\tau$.

Mean proportion of corrupted bits : $z\tau/(z/R_c) = \tau R_c$

$$\tau R_c = 1 - \exp\left[-(R/\sqrt{2}\,\sigma)^2\right]$$

$$t/n \geq 1 - \exp\left[-(R/\sqrt{2}\,\sigma)^2\right] = \tau R_c$$

$$t/n \geq 1 - \exp(-10^{T/10})$$

As an example, let capture occur for $T = -9$ dB. Then using (6.30) we obtain $t/n \geq 0.118$. The Golay (23,12) corrects up to $t = 3$ bits in an $n = 23$ bit-message, corresponding to a proportion of $3/23 \simeq 0.13$. Therefore, this code satisfies the minimum bit error correction requirement.