DATA AND SIGNALLING TRANSMISSION

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INTRODUCTION

DIGITAL MODULATION SCHEMES

ASK, FSK, PSK, DPSK

ERROR RATES FOR BINARY SYSTEMS

$$\text{prob}(\text{error}\,|\,\gamma_b) = \frac{1}{2}\exp(\alpha\gamma_b), \qquad \begin{cases} \alpha = 1/2 & \text{for noncoherent FSK} \\ \alpha = 1 & \text{for "noncoherent" PSK (DPSK)} \end{cases}$$

$$\text{prob}(\text{error}\,|\,\gamma_b) = \frac{1}{2}\,\text{erfc}(\sqrt{\alpha\,\gamma_b}), \qquad \begin{cases} \alpha = 1/2 & \text{for coherent FSK} \\ \alpha = 1 & \text{for ideal PSK} \end{cases}$$

The distribution of γ_b in a Rayleigh fading environment is

$$p(\gamma_b) = \frac{1}{\gamma_{b0}}\exp\left(-\frac{\gamma_b}{\gamma_{b0}}\right)$$

where γ_{b0} is the average SNR/bit.

$$\text{prob}(\text{error}) \triangleq p = \int_0^\infty \text{prob}(\text{error}\,|\,\gamma_b)\,p(\gamma_b)\,d\gamma_b$$

$$p = \frac{1}{2(1+\alpha\gamma_{b0})}, \quad \text{for noncoherent detection}$$

$$p = \frac{1}{2\left[1-\sqrt{1+\frac{1}{\alpha\gamma_{b0}}}\right]}, \quad \text{for coherent detection.}$$

PROBABILITY OF ERRORS IN A DATA STREAM

$$p(N,m) = \binom{N}{m}(1-p)^{N-m}p^m$$

where

$$\binom{N}{m} = \frac{N!}{(N-m)!\,m!}$$

$$p_{eM} = 1 - p(N,0) = 1 - (1-p)^N$$ prob. of error in message.

$$p_f = (1-p)^{N-d}p^d$$ falsely recognizable word.

IMPROVING THE PERFORMANCE OF DIGITAL TRANSMISSION

- Diversity
- Error Detecting and Correcting Codes
- Multiple Transmission
- Interleaving
- Automatic Repeat Request
- Adaptive Equalization

DIVERSITY AND DIGITAL TRANSMISSION

1) Switched Combining: Pure Selection

The distribution of γ_b is

$$p(\gamma_b) = \frac{M}{\gamma_{b0}}\left[1 - \exp\left(-\frac{\gamma_b}{\gamma_{b0}}\right)\right]^M \exp\left(-\frac{\gamma_b}{\gamma_{b0}}\right)$$

1a) *Noncoherent Detection* $p = \int_0^\infty \text{prob}(\text{error}\mid\gamma_b)\,p(\gamma_b)\,d\gamma_b$

$$\text{prob(error)} \triangleq p = \frac{M!}{2\prod_{i=1}^{M}(i + \alpha\gamma_{b0})}$$

1b) *Coherent Detection* Does NOT require coherente phase referente. It is of NO use.

2) Switched Combining: Threshold Selection

The distribution of γ_b is

$$p(\gamma_b) = \begin{cases} (1 + q)\left[1 - \exp(-\gamma_b/\gamma_{b0})\right] & , \gamma_b \geq \gamma_T \\ q\left[1 - \exp(-\gamma_b/\gamma_{b0})\right] & , \gamma_b < \gamma_T \end{cases}$$

where $q = 1 - \exp(-\gamma_T/\gamma_{b0})$ and γ_T is SNR threshold level.

2a) *Noncherent Detection*

$$\text{prob(error)} \triangleq p = \int_0^{\gamma_T} \text{prob(error}|\gamma_b)p(\gamma_b)d\gamma_b + \int_{\gamma_T}^{\infty} \text{prob(error}|\gamma_b)p(\gamma_b)d\gamma_b$$

$$p = \frac{q + (1 - q)\exp(-\alpha\gamma_T)}{2(1 + \gamma_{b0})}$$

2b) *Coherent Detection* DOES NOT REQUIRE coherent phase reference

Not applicable (see item 1b). Technique of no use.

3) Gain Combining: Maximal Ratio

The distribution of γ_b is

$$p(\gamma_b) = \frac{\gamma_b^{M-1} \exp(-\gamma_b/\gamma_{b0})}{\gamma_{b0}^{M} (M - 1)!}$$

3a) *Noncoherent Detection* $p = \int_0^{\infty} \text{prob(error}/\gamma_b)\, p(\gamma_b)\, d\gamma_b$

$$\text{prob(error)} \triangleq p = \frac{1}{2(1 + \alpha\gamma_{b0})^M}$$

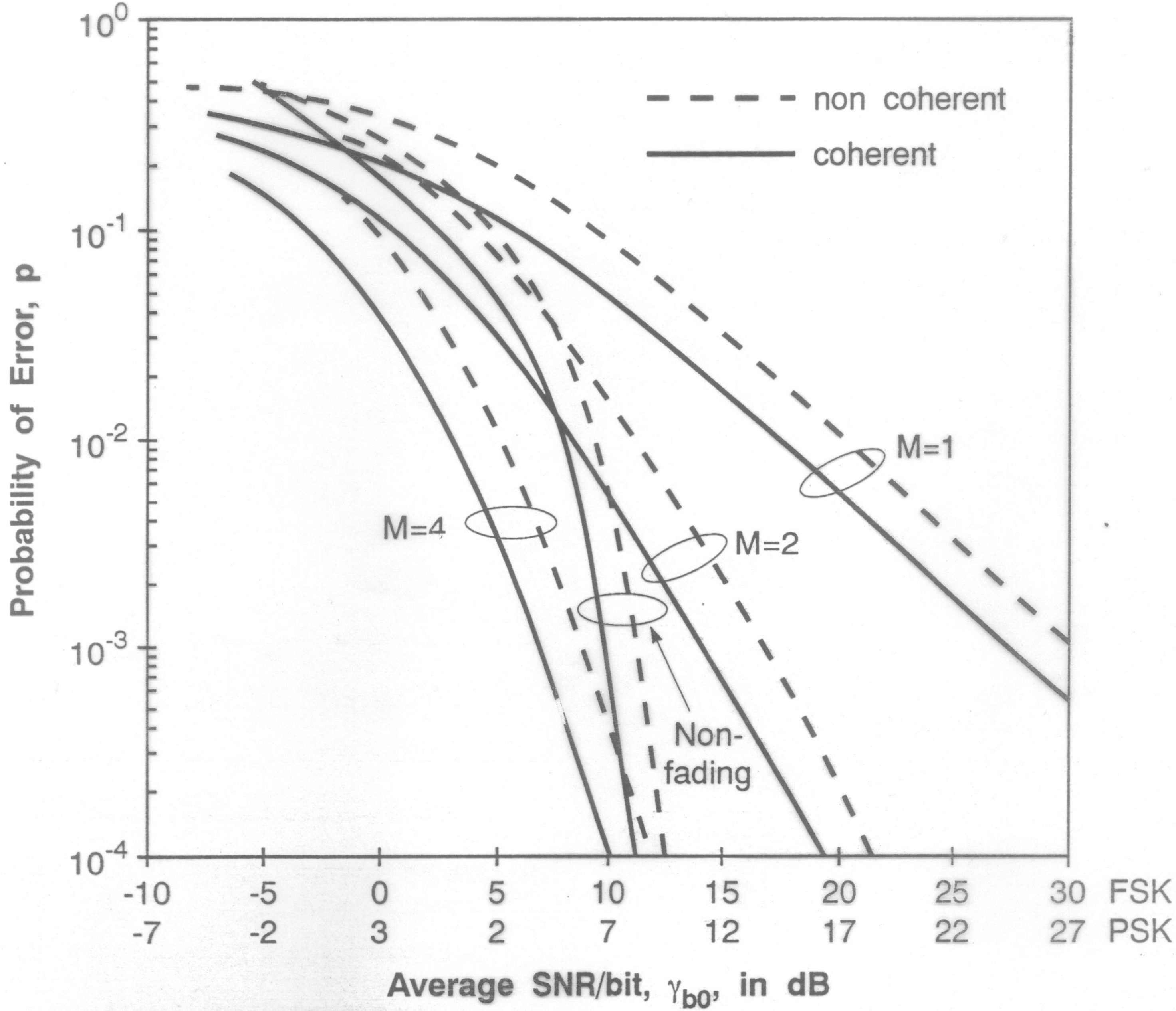
3b) *Coherent Detection* Using the leading term (low SNR, low BER).

$$p(\gamma_b) \simeq \frac{\gamma^{M-1}}{\gamma_{b0}^{M}(M - 1)!}$$

$p = \int_0^{\infty} \text{prob(error}/\gamma_b)\, p(\gamma_b)\, d\gamma_b$

$$\text{prob(error)} \triangleq p = \frac{(M - 1/2)!}{2\sqrt{\pi}\, (\alpha\gamma_{b0})^M\, M!}$$

where $(\cdot)!$ is the gamma function . In this particular case



Probability of Error, p
10^0
10^-1
10^-2
10^-3
10^-4
non coherent
coherent
M=1
M=2
M=4
Non-
fading
-10 -5 0 5 10 15 20 25 30 FSK
-7 -2 3 2 7 12 17 22 27 PSK
Average SNR/bit, γb0, in dB

FIG6-1 150%

$$\left(M - \frac{1}{2}\right)! = \frac{1.3.5.7...(2M - 1)}{2^M} \sqrt{\pi}$$

4) Gain Combining: Equal Gain Differ from Max. Ratio by a factor $(M/2)^M \sqrt{\pi} / (M - \frac{1}{2})!$

4a) *Noncoherent Detection*

$$\text{prob(error)} \triangleq p = \frac{(M/2)^M \sqrt{\pi}}{2(M - 1/2)!} \frac{1}{(1 + \alpha\gamma_{b0})^M}$$

4b) *Coherent Detection* Same reasoning.

$$\text{prob(error)} \triangleq p = \frac{(M/2)^M}{2(\alpha\gamma_{b0})^M \; M!}$$

ERROR DETECTING AND CORRECTING CODES

Code $A(n, k)$
n : codeword
k: information
$n-k$: redundancy
$r = k/n$
$t = \frac{d-1}{2}$; $\gamma_{b0} \frac{k}{n}$

$$p_{eM} = 1 - \sum_{m=0}^{t} p(N.m)$$

MULTIPLE TRANSMISSION

$$p' = \sum_{i = \frac{s+1}{2}}^{s} \binom{s}{i} p^i (1 - p)^{s-i}$$

$$p(N,m) = \binom{N}{m} (1 - p')^{N-m} p'^m$$

$$p_{eM} = 1 - \sum_{m=0}^{t} p(N,m)$$

INTERLEAVING

Its basic principle is to spread the code word, conveniently positioning the bits one away from another, so that they experience independent fading.

Error Detecting and Correcting Codes

$A(n, k)$

n: code word

k: information

$n-k$: redundancy

$r = k/n$

$t = \frac{d_{min}-1}{2}$

Ex: Golay $(23, 12)$, $d_{min} = 7$

To keep same power $SNR = r\, SNR_0$

$\therefore \quad \gamma_{bo\,coded} = \frac{k}{n} \gamma_{bo}$

Examples:

Information 12 bits

Shortened Hamming $(17, 12)$ $d_{min} = 3$

$N = 17$

$t = 1$

$r = 12/17$

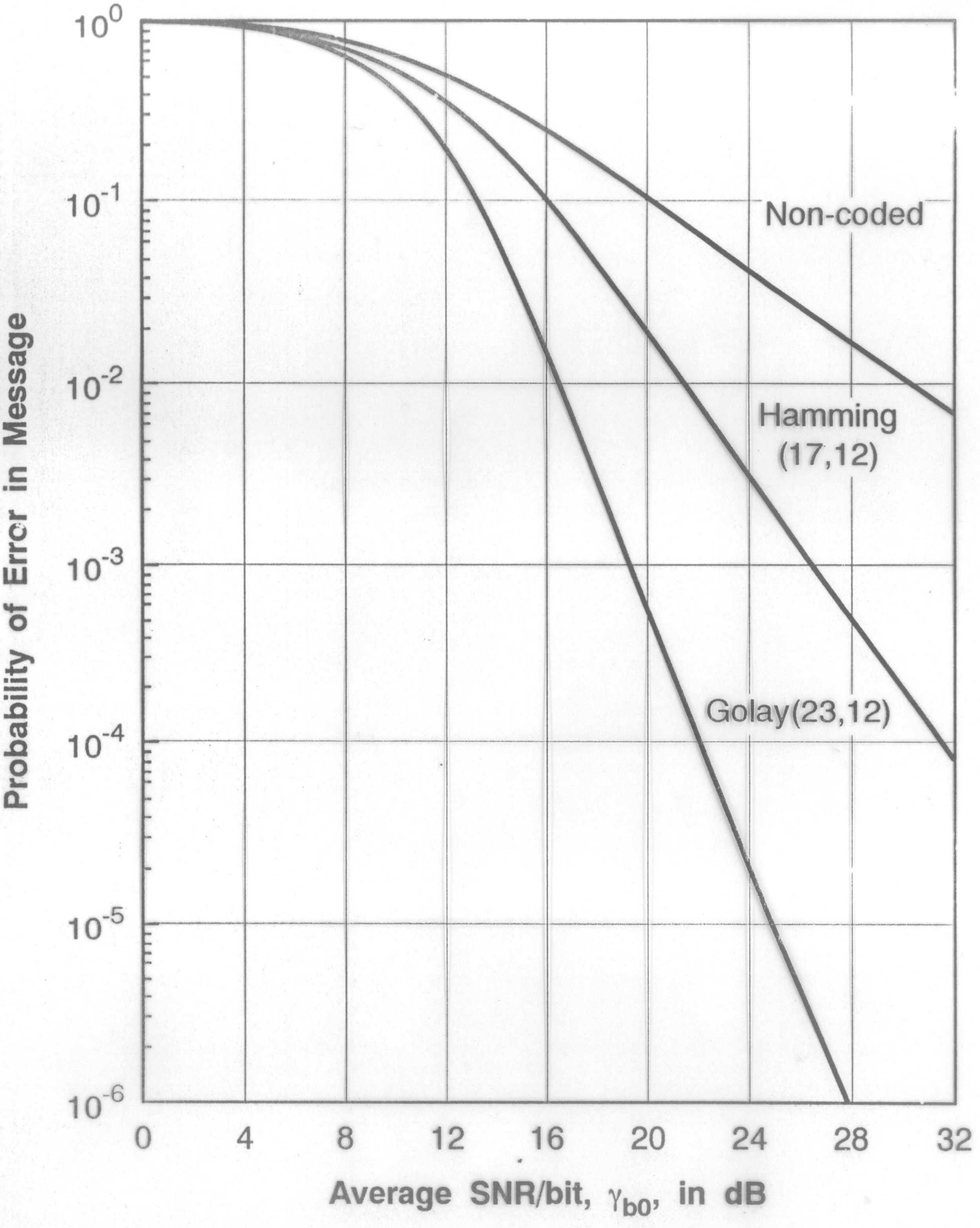
Golay $(23, 12)$ $d_{min} = 7$

$N = 17$

$t = 3$

$r = 12/23$

$$P_{em} = 1 - \sum_{m=0}^{t} p(N, m) \; ; \; p(N, m) = \binom{N}{m} (1-p)^{N-m} p^{m}$$



Probability of Error in Message
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
Non-coded
Hamming (17,12)
Golay(23,12)
0
4
8
12
16
20
24
28
32
Average SNR/bit, γ_{b0}, in dB

FIG6-2 150%

MULTIPLE TRANSMISSION

Repetition Code $(s,1)$; $d_{min} = s$; $t = \frac{s-1}{2}$

rate $= 1/s$

$(s+1)/2$ -out-of-s

Example $(3,1)$: 2 out of 3 corrects 1

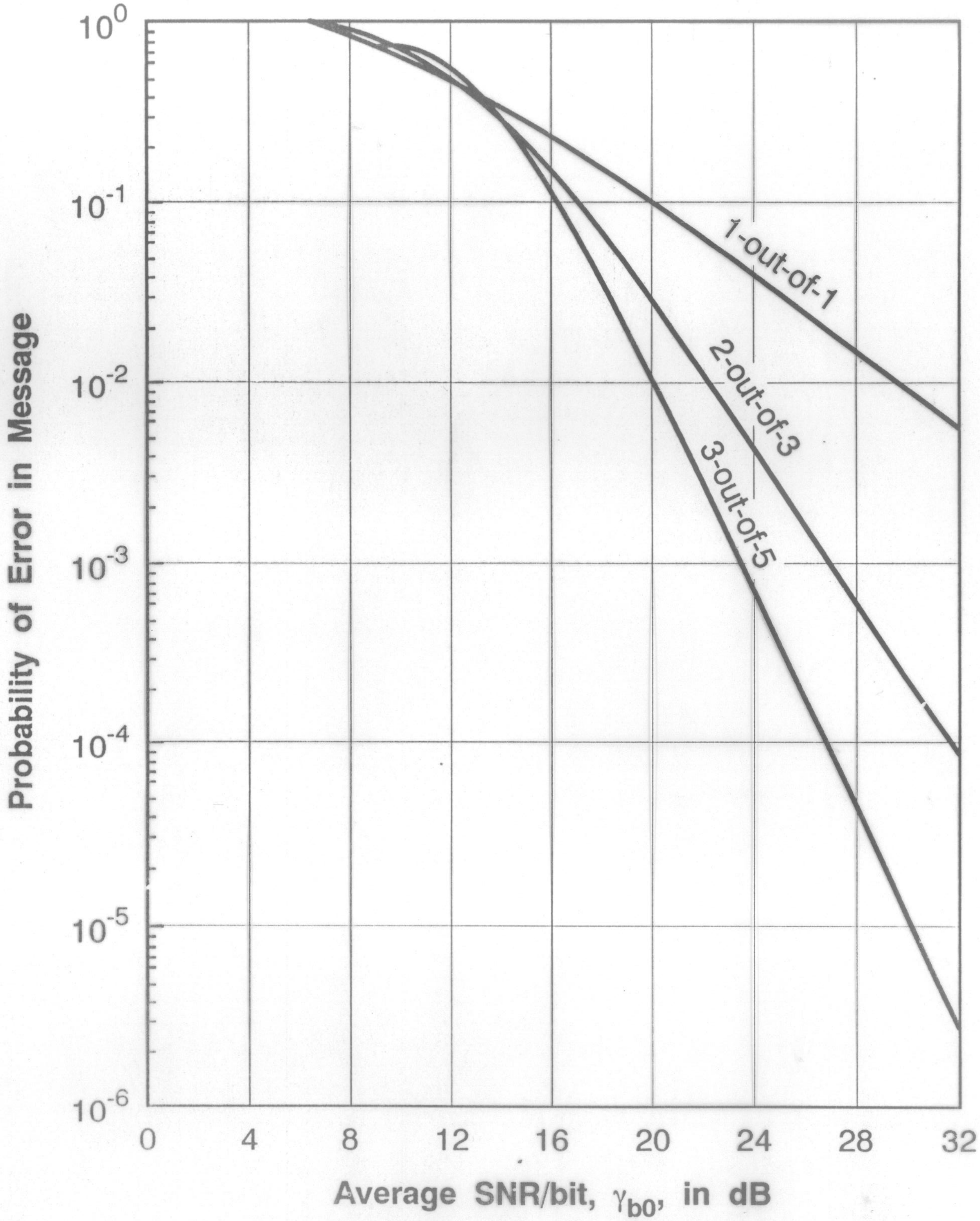
$(5,1)$: 3 out of 5 corrects 2

$$p' = \sum_{i=\frac{s+1}{2}}^{s} \binom{s}{i} p^{i} (1-p)^{s-i}$$

$$p(N,m) = \binom{N}{m} (1-p')^{N-m} p'^{m}$$

$$\gamma_{bo\,coded} = \frac{1}{s} \gamma_{bo}$$

$$p_{em} = 1 - p(N,0)$$



Probability of Error in Message
Average SNR/bit, γ_{b0}, in dB
1-out-of-1
2-out-of-3
3-out-of-5

FIG6-3 150%

AUTOMATIC REPEAT REQUEST

Let p_{eM} be the probability of error in message. Then the time to transmit a set of messages is increased by $1/(1 - p_{eM})$, whereas the throughput is reduced by $(1 - p_{eM})$.

ADAPTIVE EQUALIZATION

Equalization is the compensation for phase and amplitude distortions of the signals using the telephone channel.

Stationary environment

Non stationary (real time) : combat fading, intersymbol interference (ISI)

COMPARATIVE PERFORMANCE AND COMBINED TECHNIQUES

1) *Diversity and Coding*
2) *Diversity and Majority Voting*
3) *Coding and Majority Voting*
4) *Diversity and Coding and Majority Voting*

CHOICE OF CODE

Capture effect

$$20\log(R/\sqrt{2}\,\sigma) \geq T$$

$$R_c = \sqrt{2\pi}\; f_m(R/\sqrt{2}\,\sigma)\; \exp\left[-(R/\sqrt{2}\,\sigma)^2\right]$$

$$\tau = \frac{1}{\sqrt{2\pi}\; f_m(R/\sqrt{2}\,\sigma)} \left\{\exp\left[(R/\sqrt{2}\,\sigma)^2\right] - 1\right\}$$

Transmission rate : z bits/s.

Mean time between crossings : $1/R_c$

Mean number bits : z/R_c.

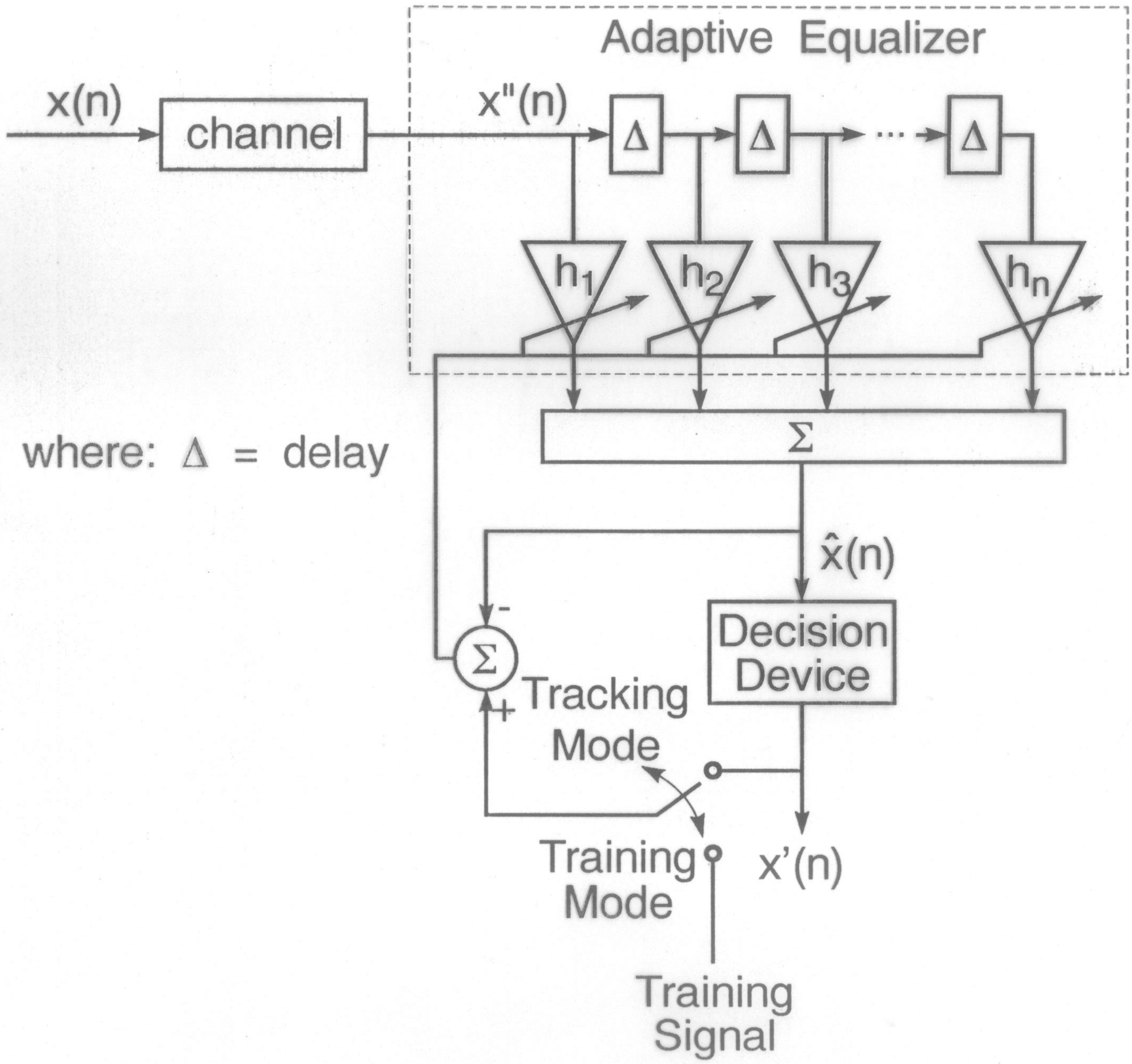

Adaptive Equalizer
x(n)
channel
x"(n)
Δ
Δ
...
Δ
h1
h2
h3
hn
Σ
where: Δ = delay
x̂(n)
-
Σ
+
Decision Device
Tracking Mode
Training Mode
x'(n)
Training Signal

FIG6-4 200%

Diversity + Coding

$$P_{em} = 1 - \sum_{m=0}^{t} p(N, m)$$

$$p(N, m) = \binom{N}{m} (1-p)^{N-m} p^{m}$$

$$p = \int_0^\infty prob(error \mid \gamma_b)\, p(\gamma_b)\, d\gamma_b$$

$p(\gamma_b)$ given by diversity combining method

Diversity + Majority Voting

$$p' = \sum_{i=\frac{s+1}{2}}^{s} \binom{s}{i} p^{i} (1-p)^{s-i}$$

$$p_{em} = 1 - p(N, 0)$$

$$p(N, m) = \binom{N}{m} (1-p')^{N-m} p'^{m}$$

$$p = \int_0^\infty prob(error \mid \gamma_b)\, p(\gamma_b)\, d\gamma_b$$

$p(\gamma_b)$ given by Div-combining.

Coding + Majority Voting

$$p' = \sum_{i=\frac{s+1}{2}}^{s} \binom{s}{i} p^{i} (1-p)^{s-i}$$

$$p(N,m) = \binom{N}{m} (1-p')^{N-m} \; p'^{m}$$

$$P_{em} = 1 - \sum_{m=0}^{t} p(N,m) \quad ; \; rate = \frac{k}{ns}$$

Coding + Majority Voting + Diversity

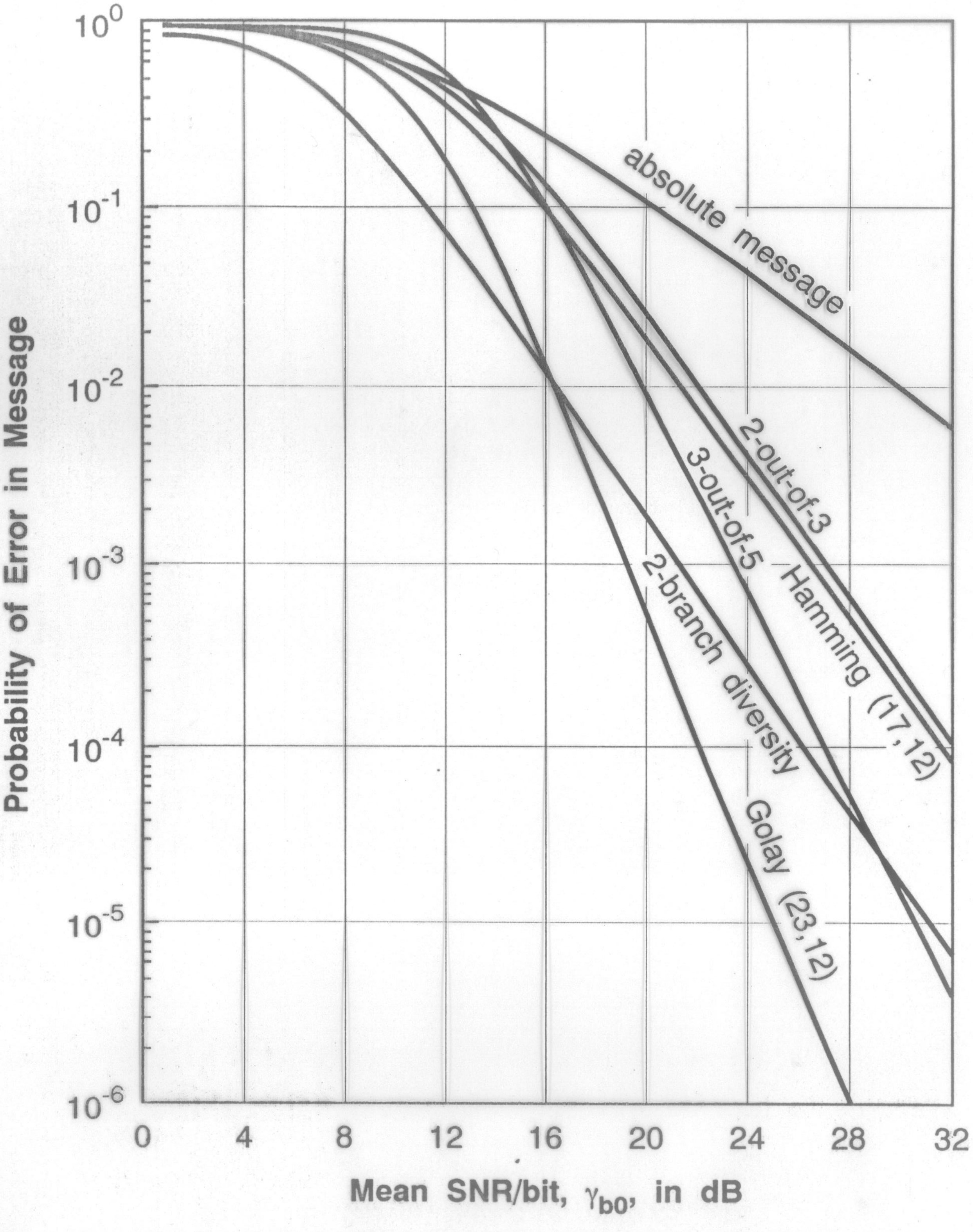
$p' = \dots$

$p(N,m) = \dots$

$$p = \int_0^{\infty} prob(error \mid \gamma_b) \, p(\gamma_b) \, d\gamma_b$$

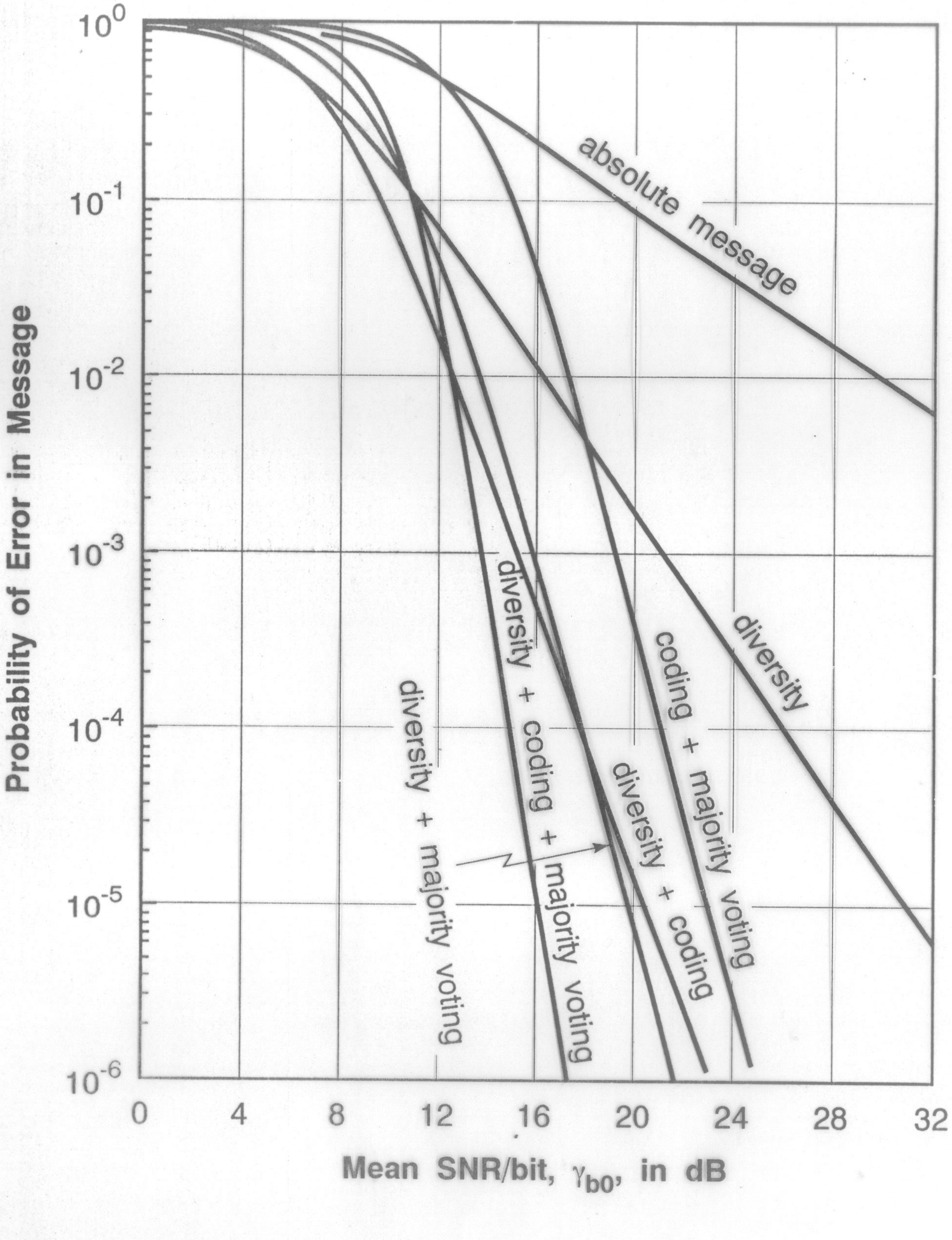
$p(\gamma_b)$ given by diversity-combing.

$rate : \frac{k}{ns}$



Probability of Error in Message
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
4
8
12
16
20
24
28
32
Mean SNR/bit, γ_{b0}, in dB
absolute message
2-out-of-3
3-out-of-5
Hamming (17,12)
2-branch diversity
Golay (23,12)

FIG6-5 150%



Probability of Error in Message
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
0
4
8
12
16
20
24
28
32
Mean SNR/bit, γb0, in dB
absolute message
diversity
coding + majority voting
diversity + coding
diversity + coding + majority voting
diversity + majority voting

FIG6-6 150%