PREDICTIVE SELF-LEARNING EQUALIZATION FOR MOBILE RADIO CHANNELS: AN LMSXRLS COMPARISON

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ABSTRACT

In this work, the applicability of blind equalizers to mobile communication systems is investigated. The equalizing scheme to be used consists of a non-linear IIR predictive structure composed of a cascade of a magnitude and a phase equalizer. The aim is to provide a comparison between the LMS and the RLS techniques implemented in this structure. Simulation results are first presented considering a stationary model of the mobile radio channel. Then, the tracking capability of both algorithms is analysed in a non-stationary context of mobile radio channels.

1. INTRODUCTION

Mobile radio channels are known to impose many impairments to digital signal transmission. In particular, very severe and non-stationary fading may be introduced, giving rise to Intersymbol Interference (ISI) [1]. From there the need for equalization techniques, which are usually based on one of the following approaches:

i. The use of a given set of data to estimate the impulse response of the channel by an ML-based technique.

ii. Conventional adaptive equalizers which usually work with some supervised learning algorithm.

In both cases, a training sequence is required, which decreases the effective bit rate of the system and, therefore, the number of possible users. For instance, in the GSM system, each 148 bits frame contains 26 training bits.

Self learning (or blind) equalization consists in retrieving the input data of an unknown channel using only some statistical information about these data, without the need for a training sequence. It is, in general, based on adaptive techniques or in high order statistics solutions. The typical real-time constraints of the communication systems lead to adaptive solutions. In this sense, the majority of the proposed algorithms is derived from the Bussgang technique and based on a FIR filter structure [2].

Predictive techniques using IIR structures was first presented by Macchi and Gu [3] and a corresponding adaptation criterion was given in [4]. The present work deals with the alternative predictive approach presented in [5], where an LMS algorithm for the structure is also derived. Based on this structure, the contribution of this paper is to present a novel RLS algorithm derived for the same structure and to provide a comparison between the two techniques. Such a study is carried out for both stationary and non-stationary models of the mobile radio channel.

The paper is organized as follows: The non-linear structure for predictive self-learning equalization is presented in the next section, together with the optimization criterion. Some aspects concerning the stability and unimodality of the scheme are briefly discussed. Section 3. is devoted to the adaptive algorithms. The LMS technique proposed in [5] is recalled and a novel RLS procedure is derived for the same stcheme. Finally, the performance of the proposed techniques is investigated through simulations, in the context of mobile communications systems. Comparisons with a Bussgang algorithms is provided, in order to enlighten the potential of different approaches in a real world case. For that, both stationary and non-stationary models are considered.

2. THE PROPOSED PREDICTIVE EQUALIZER

Suppose a zero mean independent and identically distibuted (iid) signal $\{a(n)\}$ is transmitted through a non-minimum phase channel with transfer function $F(z)$. Any non-minimum phase linear system may be decomposed in a cascade of a minimum phase system, an all-pass system and a constant real gain. Let $B(z)$ be a minimum phase system, $D(z)$ be an all-pass system and f be a gain such that:

$$F(z) = f\, B(z)\, D(z). \tag{1}$$

The ideal equalizer is the inverse of the channel transfer function, with a possible delay δ, and its transfer function $E(z)$ is given by

$$E(z) = z^{-\delta} F^{-1}(z) = g\, P(z)\, H(z), \tag{2}$$

where $P(z) = B^{-1}(z)$, $H(z) = z^{-\delta} D^{-1}(z)$, $g = f^{-1}$ and δ is such that $H(z)$ is stable and causal.

According to eq. 2, the equalizer may be seen as the cascade of three filters $\mathcal{G}$, $\mathcal{P}$ and $\mathcal{H}$, with transfer function g, $P(z)$ and $H(z)$, respectively.

The filter $\mathcal{P}$ is implemented as an IIR prediction error filter which works as a whitening filter and corrects the magnitude distortions caused by $B(z)$. Therefore, the ensemble $\mathcal{F} o \mathcal{P}$ reduces the problem to the equalization of $\mathcal{D}$, which causes only phase distortion. Since the filter $\mathcal{H}$ compensates for this phase distortion, it must also be an all-pass filter.

It is shown in [5] that this linear structure does not converge with severe channels and it becomes necessary to introduce a decision feedback structure on the phase equalizer. So, the overall proposed structure is that shown in figure 1. The corresponding adaptive algorithms are presented in the sequel.

3. ADAPTIVE ALGORITHMS

3.1. LMS Approach

Just for the sake of illustration, the LMS algorithms derived in [5] and [6] are presented in this section. According to figure 1, $\mathcal{P}$ is an IIR linear prediction error filter, such that

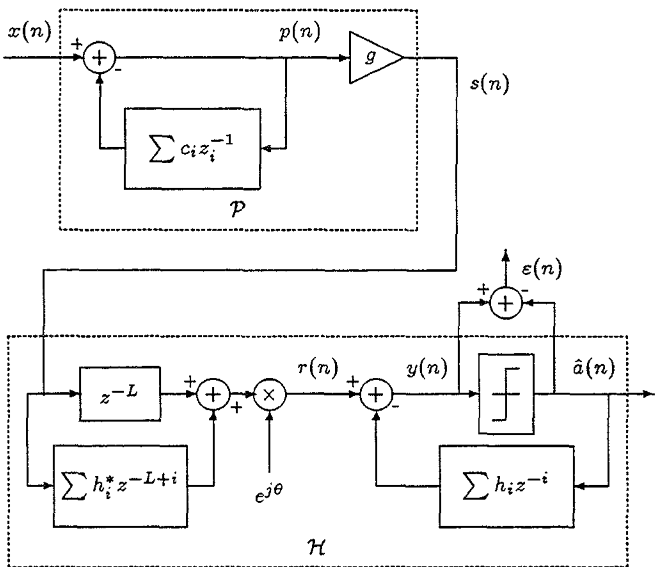

Figure 1. Nonlinear Blind Equalizer.

its output be an uncorrelated sequence. Such a filter is globally stable [5] when updated by the following algorithm:

$$C(n) = C(n-1) + \lambda\, p(n) P^*(n-1), \tag{3}$$

where $C(n) = (c_1(n), c_2(n), \ldots, c_N(n))^T$ is the coefficient vector of $\mathcal{P}$, $P(n-1) = (p(n-1), p(n-2), \ldots, p(n-N))^T$ is a vector containing the output data and $\lambda > 0$ is the step-size.

The output power of $\mathcal{P}$ must also be equalized to that of the input sequence $a(n)$. This is accomplished by the AGC $\mathcal{G}$, whose parameter is real and is updated by

$$\begin{aligned} G_k &= G_{k-1} + \eta \left[E\left(|a(n)^2|\right) - |s(n)|^2 \right] & (4)\\ g &= \sqrt{|G_k|}, & (5) \end{aligned}$$

with a step-size $\eta > 0$. The output $s(n)$ of the ACG has a flat spectrum identical to that of $a(n)$, and it remains unchanged when applied to the all-pass filter $\mathcal{H}$, which aims at correcting the phase distortions.

Since the phase equalizer $\mathcal{H}$ in figure 1 presents a feedback structure, it seems suitable to use an adaptation criterion based on the Decision-Directed (DD) error, whose unimodality proporties have been discussed in [6]. Hence, considering the cost function $J_\varepsilon(H) = E\left(|\varepsilon(n)|^2\right)$ and an estimative of its gradient given by:

$$\hat{\nabla} J_\varepsilon(H) = \Xi(n) = (\xi^1(n), \xi^2(n), \ldots, \xi^L(n))^T, \tag{6}$$

we obtain

$$\xi^j(n) = \varepsilon(n)^*\, e^{j\theta(n)}\, s(n-L+j) - \varepsilon(n)\, \hat{a}^*(n-j). \tag{7}$$

Finally, the adaptive algorithm becomes

$$H(n+1) = H(n) - \kappa\Xi(n), \quad \kappa > 0, \tag{8}$$

where $H(n) = [h_1(n), \ldots, h_L(N)]^T$ is the coefficient vector of $\mathcal{H}$

This algorithm is completed by the phase recovering procedure given by:

$$\theta(n+1) = \theta(n) + \nu \Im(\varepsilon(n)^* r(n)), \quad \nu > 0, \tag{9}$$

where $\Im$ denotes the imaginary part.

3.2. RLS Approach

The RLS algorithm for an IIR predictor has been derived in [8] and is just recalled here:

$$\begin{aligned} &\phi(n) = p(n) + C(n)^H \Phi(n-1)\\ &U_{\mathcal{P}}(n) = R_{\mathcal{P}}(n-1)\Phi(n)\\ &k_{\mathcal{P}}(n) = \left(w_{\mathcal{P}} + \Phi^H(n) U_{\mathcal{P}}(n)\right)^{-1}\\ &R_{\mathcal{P}}(n) = \tfrac{1}{w_{\mathcal{P}}}\left(R_{\mathcal{P}}(n-1) - U_{\mathcal{P}}(n) k_{\mathcal{P}}(n) U_{\mathcal{P}}^T(n)\right)\\ &C(n+1) = C(n) - R_{\mathcal{P}}(n)\Phi(n) p^*(n), \end{aligned}$$

where $\Phi(n) = (\phi(n), \phi(n-1), \ldots, \phi(n-N+1)$ is the estimate of the derivate of the output with respect to the past N outputs and $w_{\mathcal{P}}$ is the forgetting factor of the algorithm.

The RLS algortithm aims at minimizing the sum of the weighted squared errors. In the case of the phase equalizer, it was shown that cost functions based on the DD error yield suitable adaptation criteria. Hence, it seems natural to define the RLS algorithm as the one that seeks the minimum of the sum of the weighted squared DD errors, that is:

$$\mathcal{E}(n) = \sum_{i=1}^{n} w^{n-i} |\varepsilon(i,n)|^2, \tag{10}$$

where $\varepsilon(i,n)$ is the DD error at time i using the parameters calculated at time n.

The derivation of the algorithm for the phase equalizer becomes very similar to that of the traditional RLS algorithm if the equations are written in a proper manner. Then, let us pose:

$$\begin{aligned} A(i) &= [\hat{a}(i-1), \ldots, \hat{a}(i-L)]^T & (11)\\ S(i) &= e^{j\theta_i}[s(i-L+1), s(i-L+2), \ldots, s(i)]^T \end{aligned}$$

So, the decision-directed error may be written as

$$\begin{aligned} \varepsilon(i,n) &= s(i-L)e^{j\theta_i} + \left(H_{\Re}^T(n) - jH_{\Im}^T(n)\right) S(i)\\ &\quad - \left(H_{\Re}^T(n) + jH_{\Im}^T(n)\right) A(i) - \hat{a}(i), \qquad (12) \end{aligned}$$

where the indexes $\Re$ and $\Im$ denote, respectively, the real and imaginary parts of the variable. In a more compact form, we have:

$$\varepsilon(i,n) = \mathcal{H}^T(n)\mathcal{Y}(i) - d(i) = \mathcal{Y}(i)^T \mathcal{H}(n) - d(i), \tag{13}$$

where $d(i) = \hat{a}(i) - s(i-L)e^{j\theta_i}$ involves all the terms that are independent of the equalizer parameters and

$$\mathcal{H}(n) = \begin{bmatrix} H_{\Re}(n) \\ H_{\Im}(n) \end{bmatrix} \quad \text{and} \quad \mathcal{Y}(i) = \begin{bmatrix} S(i) - A(i) \\ -jS(i) - jA(i) \end{bmatrix}. \tag{14}$$

The find the minimum of $\mathcal{E}(n)$, is obtained by setting to zero its derivative with respect to $\mathcal{H}(n)$. Therefore, we obtain

$$R(n)\mathcal{H}(n) = b(n), \tag{15}$$

where

$$\begin{aligned} R(n) &= \sum_{i=1}^{n} w^{n-i} Y(i) Y^T(i) & (16)\\ b(n) &= \sum_{i=1}^{n} w^{n-i} Re\{d^*(i)\mathcal{Y}(i)\}. & (17) \end{aligned}$$

Now we are ready to follow the same procedure used in the derivation of the traditional RLS algorithm. The resulting algorithm is as follows:

$$
\begin{aligned}
\varepsilon(n) &= \mathcal{H}^T(n-1)\mathcal{Y}(n) - d(n) \\
U(n) &= P(n-1)Y(n) \\
k(n) &= \left(wI_2 + Y^T(n)U(n)\right)^{-1} \\
P(n) &= \tfrac{1}{w}\left(P(n-1) - U(n)k(n)U^T(n)\right) \\
\mathcal{H}(n) &= \mathcal{H}(n-1) - U(n)k(n)\begin{bmatrix} \varepsilon_{\Re}(n) \\ \varepsilon_{\Im}(n) \end{bmatrix}
\end{aligned}
$$

To derive an RLS algorithm for the phase recovery, the derivative of $\mathcal{E}(n)$ with respect to $\theta(n)$ is equated to zero. Then, straightforward calculations yield the following algorithm

$$
\begin{aligned}
z(n) &= wz(n-1) + H^T(n)A(n) + \hat{a}(n) \\
\theta_n &= \angle z(n)
\end{aligned}
$$

4. SIMULATION RESULTS

Simulation results concern the transmission of 1000 symbols using 4-PSK modulation. This sequence was transmitted at a bit rate of 271 kbits/s through the channel described in [7], which is supposed stationary for sequences of this length. This simulation environment is in accordance with the GSM European Standards for testing of equalizing devices. Figure 2 shows the location of the zeros of this channel.

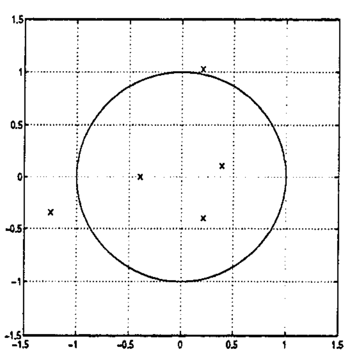

Figure 2. Zero-Pole Plot of the GSM Channel

Figure 3 shows the convergence behavior of the proposed equalizer using LMS techniques, while figure 4 presents the results of the RLS approach. In order to compare these results with some of the most used blind equalizing schemes we have also considered the Bussgang-type algorithm proposed by Godard, whose convergence behavior is presented in figure 5.

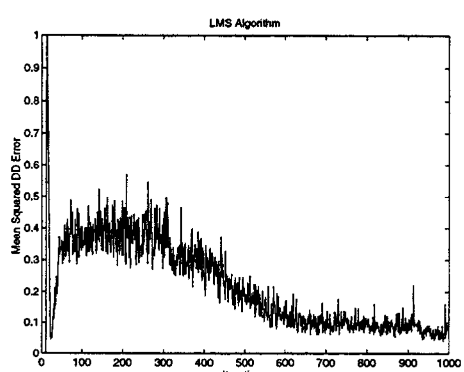

Figure 3. Convergence Behavior of the LMS Algorithm - Stationary Environment

In these figures, it is seen that the two algorithms based on predictive techniques present a similar performance, which is about 5 times as fast as that of the Godard technique.

To simulate a non-stationary environment, the software developped in [9] was used. It simulates the transmission of

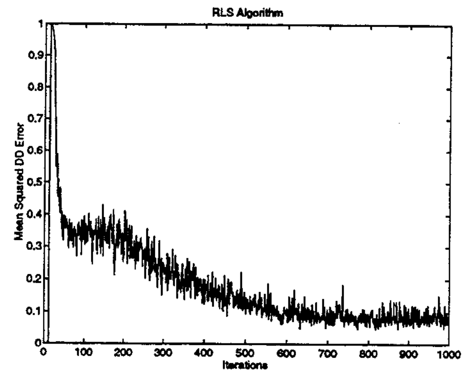

Figure 4. Convergence Behavior of the RLS Algorithm - Stationary Environment

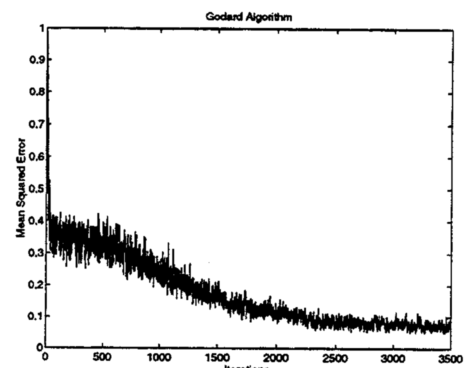

Figure 5. Convergence Behavior of the Godard Algorithm - Stationary Environment

digital data through a D-AMPS channel. Figure 6 shows its output power, supposing that the mobile unit was moving at 4 km/h.

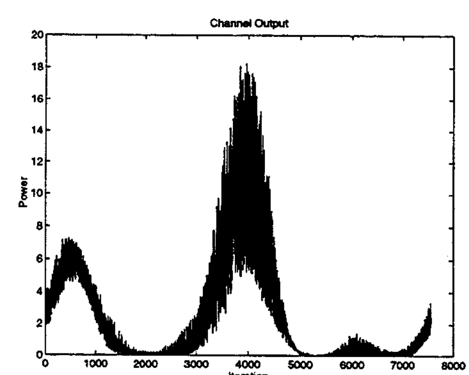

Figure 6. Power Profile of the Mobile Channel Ouput

Figures 7 and 8 show the convergence behavior of the LMS and the RLS techniques, respectively. Once again, these two techniques present a very similar performance.

Just for the sake of illustration, figures 9 and 10 present the constelation at the output of the equalizers, using the LMS and the RLS techniques, respectively. Only the open-eye periods have been considered, which corresponds to the following iterations range: $\{[150, 1100]; [2000, 2300]; [5200, 5600]\}$ for the LMS algorithm and $\{[60, 1250]; [3500, 4900]; [5200, 5550]\}$ for the RLS algorithm. Again, their performances are similar, even though the RLS technique keeps the eye open for a longer period.

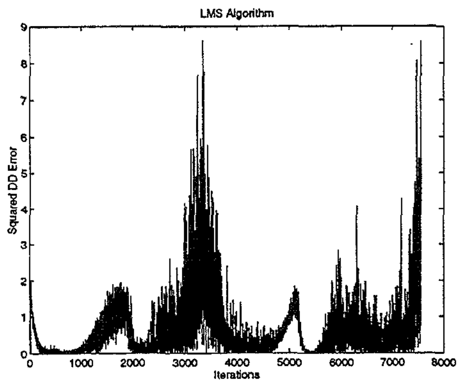

Figure 7. Convergence Behavior of the LMS Algorithm - Non-Stationary Environment

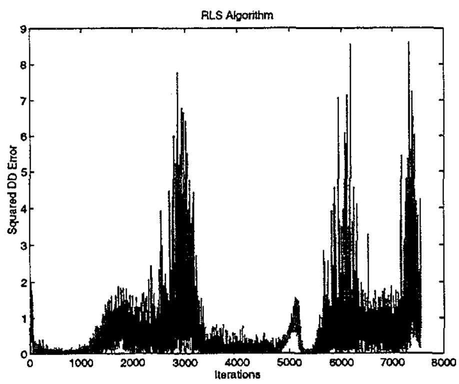

Figure 8. Convergence Behavior of the RLS Algorithm - Non-Stationary Environment

5. CONCLUSION

This paper deals with the non-linear IIR self-learning equalizer proposed in [5]. A novel RLS algorithm has been introduced for this equalizer and compared with the LMS technique. Both algorithms provide a better convergence rate when compared with Bussgang techniques. The tracking capability of the algorithms is confirmed by simulations under a non-stationary environment, since the open-eye condition is recovered several times during the adaptation process. The results concerning the static convergence of the proposed approach has been carried out in [6], where unimodality properties have been derived. Studies concerning dynamic convergence properties are now in course.

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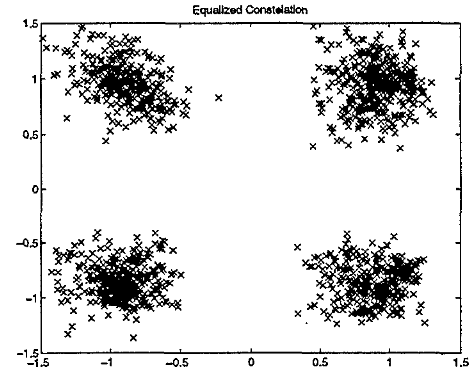

Figure 9. Equalized Constelation - LMS Technique

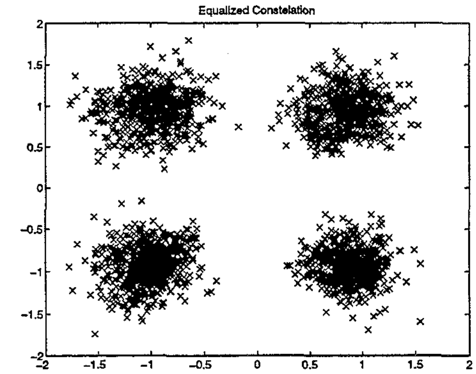

Figure 10. Equalized Constelation - RLS Technique

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