Introduction

Processing and imaging of seismic reflection data rely, to a great extent, on stacking procedures for several purposes. Stacking is general performed along user-designed traveltime curves within 2D data sections or surfaces within 3D data volumes. Referred to as *moveouts*, such traveltime depend on one or more parameters, that are estimated by coherence analysis applied directly to the data, so as to produce the maximum stacking energy. The most fundamental of a stacking procedure is velocity analysis, namely to obtain the stacking velocity and zero-offset (ZO) traveltime for reflections within a common-midpoint (CMP) gather (see, e.g., Taner and Koehler, 1969). The standard coherence function is a second-order energy measure, called semblance (Neidell and Taner, 1971). Semblance is computed for windows of N_t samples taken from traces at N_r receivers. Each window follows along the moveout defined by the parameters being estimated, and consists of a few samples before and after the window center. The samples that pertain to the window may need to be interpolated.

Still within the framework of velocity analysis, Biondi and Kostov (1989) and Kirlin (1992) showed that eigen-structure methods for coherence analysis can lead to parameter estimations (in this case velocity spectra) with higher resolution than semblance. One of most commonly used high-resolution methods is MUltiple SIgnal Classification (MUSIC), introduced by Schmidt (1986), which is based in some properties of the eigen-decomposition of the seismic data. Recently, MUSIC has been used in Asgedom et al. (2011) for estimating the common-reflection-surface (CRS) attributes.

The implementation of MUSIC-based velocity spectra is the main focus of this work, for which we propose a number of improvements, namely (a) To reduce its computational complexity, we compute the MUSIC coherence measure based on a single eigenvector, namely the one associated to the largest eigenvalue. Following common practise, that referred to as the *largest eigenvector*. As a consequence, the full eigenvalue decomposition is not required. Moreover, the largest eigenvector can be efficiently computed by means of the power method (Golub, 1996); (b) To obtain further computational savings, we propose a coherence function based on the eigen-decomposition of a matrix whose dimension is lower than the one currently used in the literature and (c) As a byproduct, the use of this lower-dimensional matrix seems to improve the performance of the method when dealing with correlated wavefronts, as indicated by numerical experiments.

Covariance Matrices

As discussed in the introduction, coherence is computed on a window of data centered at some time $\tau_k(i)$, where k corresponds to a given value of the parameter being tested and i designates a given receiver or seismic trace. Here, we assume that the data is sorted in common-midpoint (CMP) gathers with the traveltime $\tau_k(i)$ being the normal moveout (NMO),

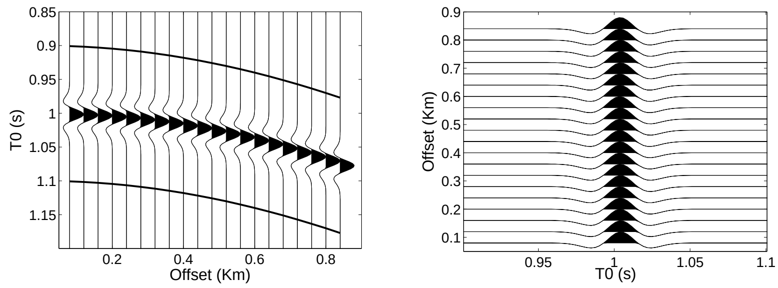
$$\tau_k(i)^2 = \tau_0^2 + \frac{4h_i^2}{v_k^2} \tag{1}$$

in which v_k is the NMO velocity being tested, h_i is the half-offset and τ_0 is the ZO traveltime, supposed fixed. For each τ_0, the windowed data can be written as a matrix $\mathbf{D}(\theta_k)$, with dimension $N_r \times N_t$, where $\theta_k = 1/v_k$. When a window with correct values of τ_0 and v_k is applied, the windowed data matrix will be represented as in Figure 1. In this case, the data will have the form

$$\mathbf{D}(\theta_k) = \mathbf{1}\mathbf{s}^H + \mathbf{N}, \tag{2}$$

where $\mathbf{s}$ is a $N_r \times 1$ vector that contains the samples from the reflected wavelet, $\mathbf{1}$ is a $N_r \times 1$ vector of ones, $\mathbf{N}$ is an $N_r \times N_t$ noise matrix, which may also contain interfering reflections. The superscript H refers to transpose conjugate.

The Spatial Covariance Matrix: Hyperbolic windowing can also be used for eigenstructure-based coherence calculation. Indeed, for each τ_0, different values of θ_k result in different windows, and thus



(a) (b)

***Figure 1** Seismogram before windowing operation (a) and data matrix, $\mathbf{D}(\theta_k)$, after windowing (b).*

in different data matrices. For each θ_k, the covariance matrix to be estimated has the form

$$\hat{\mathbf{R}}(\theta_k) = \frac{1}{N_t}\mathbf{D}(\theta_k)\mathbf{D}^H(\theta_k) \approx \frac{||\mathbf{s}||^2}{N_t}\mathbf{1}\mathbf{1}^H + \sigma^2\mathbf{I}, \tag{3}$$

where the dimension of $\hat{\mathbf{R}}(\theta_k)$ is $N_r \times N_r$, σ^2 is the noise variance and $\mathbf{I}$ is the identity matrix of appropriate dimension. Note that we disregard the cross terms resulting from $\mathbf{D}(\theta_k)\mathbf{D}^H(\theta_k)$, because we assume that the noise is zero-mean and uncorrelated with the signal $\mathbf{s}$. Assuming that the window contains a single reflection, the largest eigenvector of $\hat{\mathbf{R}}(\theta_k)$ will form the so-called signal subspace (see, e.g., Kirlin, 1992). In this case, it can be shown that MUSIC coherence function can be computed as

$$P_S(\theta_k) = \frac{N_r}{N_r - |\mathbf{1}^H\mathbf{v}(\theta_k)|^2}, \tag{4}$$

where $\mathbf{v}(\theta_k)$ is the largest eigenvector of $\hat{\mathbf{R}}(\theta_k)$. The quantity $P_S(\theta_k)$ can be interpreted as a measure of whether $\mathbf{1}$ is the largest eigenvector of $\hat{\mathbf{R}}(\theta_k)$. If this is the case, then $P_S(\theta_k)$ will be large. The benefit of using the signal subspace defined above is that it requires a single eigenvalue and, moreover, leads to a one-dimensional signal subspace, which greatly simplifies the computation of $P_S(\theta_k)$. At the n-th iteration of this method, the estimated eigenvector is

$$\mathbf{v}^{(n)} = \frac{\hat{\mathbf{R}}(\theta_k)\mathbf{v}^{(n-1)}}{||\hat{\mathbf{R}}(\theta_k)\mathbf{v}^{(n-1)}||}. \tag{5}$$

The stop criterion is $||\mathbf{v}^{(n)} - \mathbf{v}^{(n-1)}|| < \xi$, where ξ is a threshold that controls the precision of the algorithm. To reduce the number of iterations, we initialize the power method with the vector $\mathbf{v}^{(0)} = \mathbf{1}$, as we expect that this should be the largest eigenvector.

The Temporal Covariance Matrix: To obtain further computational savings, we propose a coherence function based on the eigen-decomposition of the temporal covariance matrix, estimated as

$$\hat{\mathbf{r}}(\theta_k) = \frac{1}{N_r}\mathbf{D}^H(\theta_k)\mathbf{D}(\theta_k). \tag{6}$$

As the number of samples in the window, N_t, is generally smaller than the number of receivers, N_r, the dimension of $\hat{\mathbf{r}}(\theta_k)$ will be smaller than that of $\hat{\mathbf{R}}(\theta_k)$, leading reduced eigen-decomposition costs. Now, if $\mathbf{v}(\theta_k)$ is the largest eigenvector of $\hat{\mathbf{R}}(\theta_k)$, it can be shown that $(1/N_r)\mathbf{D}^H(\theta_k)\mathbf{v}(\theta_k)$ is the largest eigenvector of $\hat{\mathbf{r}}(\theta_k)$. Thus, the MUSIC coherence function (or *pseudospectrum*) now tests if $\hat{\mathbf{s}} = (1/N_r)\mathbf{D}^H(\theta_k)\mathbf{1}$ is the largest eigenvector of matrix $\hat{\mathbf{r}}(\theta_k)$. Namely,

$$P_T(\theta_k) = \frac{\hat{\mathbf{s}}^H\hat{\mathbf{s}}}{\hat{\mathbf{s}}^H\hat{\mathbf{s}} - |\hat{\mathbf{s}}^H\mathbf{u}(\theta_k)|^2}, \tag{7}$$

where $\mathbf{u}(\theta_k)$ is the largest eigenvector of $\hat{\mathbf{r}}(\theta_k)$. Note that $\mathbf{u}(\theta_k)$ can also be estimated by the power method, now initialized with $\mathbf{u}^{(0)} = \hat{\mathbf{s}}$.

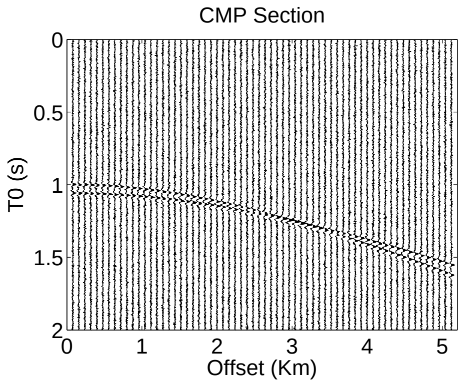


Figure 2 *CMP section used in simulations.*

Numerical Examples

We compare, for simple examples, the use of spatial and temporal covariance matrices to obtain high-resolution NMO velocities. We show MUSIC results computed with spatial and temporal coherency measures, comparing them the ones obtained by semblance as a benchmark. In the simulations, we use the synthetic model of two reflections, with traveltimes generated by the NMO equation. The first reflection has ZO traveltime of 1 s and NMO-velocity of 4000 m/s; the second one has ZO traveltime of 1.06 s and NMO-velocity of 4500 m/s. Both reflections are modeled by a zero-phase Ricker wavelet, with a dominant frequency of 25 Hz and are fully correlated. The CMP section, illustrated in Figure 2, contains 64 receivers. The offset of the first one is 80 m and the distance between them is also 80 m. The sample period is 2 ms. White Gaussian noise was added to get a signal to noise ratio (SNR) of 15 dB.

Figures 3(a), 3(b) and 3(c) show velocity spectra calculated with semblance, and the PM-MUSIC pseudospectra from equations (4) and (7), respectively. The window size was 19 samples and velocities were tested from 3000 m/s to 6000 m/s, with increments of 10 m/s. For the spatial covariance matrix, we performed spatial smoothing (Shan et al., 1985), together with forward-backward (FB) averaging (Willians et al., 1988), using 47 subarrays, of 18 receivers each. The results clearly show that both MUSIC algorithms outperform semblance in terms of resolution, resulting in more precise velocity estimates. We also see that MUSIC with temporal correlation presents even better resolution than spatial correlation.

Figures 3(d) and 3(e) show velocity spectra calculated with PM-MUSIC for spatial and temporal covariance matrices, respectively. Clearly, the use of the power method has no impact on the results. However, when comparing Figure 3(d) with Figure 3(b), and Figure 3(e) with Figure 3(c), it can be observed that PM-MUSIC is slightly more accurate when applied to the temporal covariance matrix.

Conclusions

High-resolution eigen-structure algorithms, denoted PM-MUSIC have been proposed for coherence analysis as alternative to conventional semblance. PM-MUSIC was presented in two variants, spatial and temporal. Initial simulations with simple synthetic model, indicated that PM-MUSIC outperforms semblance and that the temporal variant of PM-MUSIC is superior to its spatial counterpart. Moreover, temporal PM-MUSIC is particularly useful when dealing with correlated signals, as we do not need to use spatial smoothing together with forward-backward averaging to make the signals uncorrelated. The complexity order for semblance and PM-MUSIC from equations (4) and (7) velocity spectra are, respectively, $O(N_r)$, $O(N_r^2)$ and $O(N_r)$. We have observed, in synthetic data simulations, that the complexity reduction in PM-MUSIC from the temporal covariance matrix, does not affect its precision, when compared to PM-MUSIC from the spatial covariance one.

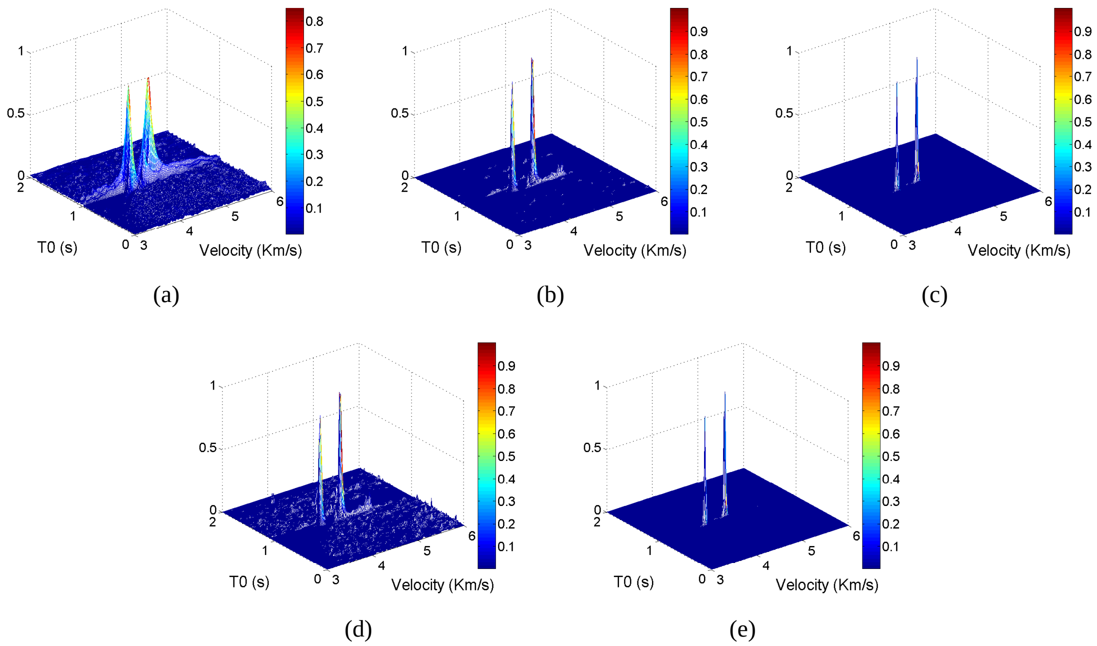


Figure 3 *Velocity spectra from semblance (a), MUSIC from spatial (b) and temporal (c) covariance matrices and PM-MUSIC from spatial (d) and temporal (e) covariance matrices.*

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