Reduced Complexity Turbo Equalization for MIMO Channels Using Random Signal Mapping

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***Abstract*— Multiple-input multiple-output (MIMO) wireless systems are known to be robust against fading, providing what is known as diversity gain. However, if traditional techniques to obtain diversity are used, the receiver complexity for a MIMO channel with intersymbol interference (ISI) may be unfeasible. In this paper, we investigate the use of random signal mapping as a source of diversity gain for MIMO channels with ISI. We theoretically demonstrate that the proposed system has much lower complexity than existing solutions and is robust against channel mismatch. Simulation results also show that the new scheme provides good diversity gain.**

I. Introduction

Wireless systems with multiple transmit and/or receive antennas have been the topic of intense research activity lately. The interest in these multiple-input multiple-output (MIMO) systems was spurred by two results. First, in two independent works, Telatar [1], and Foschini and Gans [2] showed that a significant increase in capacity can be obtained in wireless systems when multiple antennas are employed at both the transmitter and the receiver. Besides this increased capacity, multiple antennas can also lead to increased robustness against fading, even without channel knowledge at the transmitter. Indeed, in [3], Tarokh *et al* proposed a transmission scheme known as space-time coding (STC). In STCs, redundancy is introduced into the transmit streams both in space (across transmit antennas) and in time, leading to diversity and coding gains.

STCs for flat fading channels have been extensively analyzed, and they can essentially be divided into space-time trellis codes (STTC) [3]–[5] and space-time block codes (STBC) [6], [7]. However, for high bit rates, the transmit bandwidth may be larger than the channel coherence bandwidth [8], [9]. In this case, the channel becomes frequency selective, leading to intersymbol interference (ISI). During the past few years, several equalizers have been proposed that deal with the ISI problem in MIMO wireless systems [10]–[13].

Recently, a new transmission strategy that achieves spatial diversity based on a random signal mapper (RSM) was proposed in [14]. In RSM, the bit stream is first encoded with a regular error-correcting code. Then, N copies of the encoder output are generated, where N is the number of transmit antennas. Each of these copies goes through a random signal mapper, whose output is then transmitted through one antenna. This simple scheme achieves full diversity. Furthermore, the RSM receiver has much lower complexity than that of STBCs, which is itself less complex than that of STTCs. In addition, transmit and/or receive antennas can be added to an RSM scheme without significant changes to the system, a flexibility not found in STBCs and STTCs. Finally, in many STBCs, adding more transmit antennas incur in a rate penalty, but this penalty is not observed in RSM.

RSM was originally proposed for flat-fading systems. In this paper, we extend RSM to ISI MIMO wireless channels. To that end, we propose the use of a parallel-concatenated turbo code [15], and a receiver employing a turbo equalizer [16]. MIMO systems based on turbo equalizers were first proposed in [10]. However, these are based on STTCs. As we will see, such turbo equalizers are much more complex than that of an RSM receiver. Indeed, the computational complexity in the receiver of [10] grows exponentially with the number of transmit antennas, N, while the complexity of the proposed receiver is not a function of N. Obviously, this represents a significant reduction in complexity. Finally, we will provide simulation results that show that RSM also achieves diversity gain in frequency selective wireless channels.

This paper is organized as follows. In Section II, we present the model of the frequency selective wireless channel, the RSM transmission scheme, and the proposed turbo equalizer. In Section III, we present simulation results that illustrate the system performance under different number of antennas, different block lengths and under imperfect channel knowledge at the receiver. In Section IV, we compare the complexity of the proposed scheme to that of existing alternatives. Finally, we draw some concluding remarks in Section V.

II. Channel Model, Transmitter and Receiver

Consider the transmission of a sequence of equiprobable and independent message bits u_k using N transmit antennas. To that end, we use the RSM transmitter depicted in Fig. 1. The bits first go through a parallel-concatenated turbo code, whose internal interleaver has length I. The output of the encoder then goes through another interleaver of length J, and

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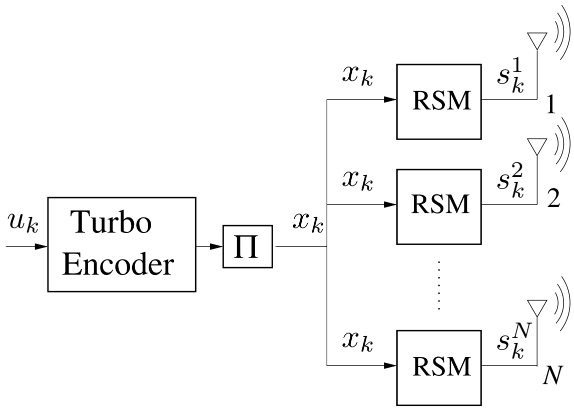

Fig. 1. Block diagram of the proposed transmitter, with N transmit antennas, turbo encoding and random signal mapping. Π represents the interleaver and the m-PSK mapper.

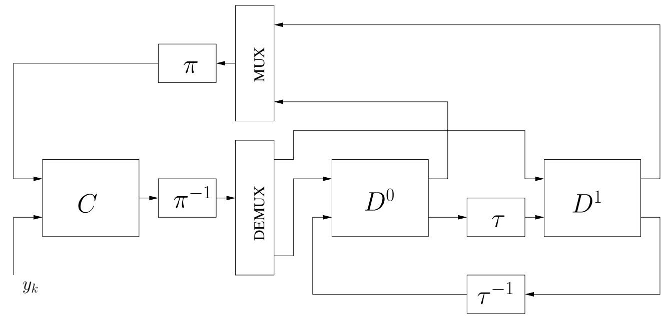

Fig. 2. Receiver block diagram, where D^0 and D^1 represent the constituent decoders, C is the MAP channel equalizer, y_k are the received symbols, τ is the code deinterleaver, and π is the demapper and channel deinterleaver.

is then mapped into an m-PSK symbol. These two operations are represented by Π in Fig. 1, and they result in a block of m-PSK symbols x_k. This block then goes through N random signal mappers, whose outputs are given by $s_k^i = e^{j\phi_k^i} x_k$, for $i = 1, \cdots, N$, where ϕ_k^i are N independent sequences of random phases known also to the receiver. Finally, the sequence s_k^i is transmitted through the i-th antenna with energy E_S/N.

The receiver employs M antennas. The signal received by the j-th receive antenna at time instant k, $y^j(k)$, is given by

$$y^j(k) = \sum_{i=1}^{N} \sum_{d=0}^{D-1} \sqrt{E_S/N}\; s^i(k-d) h_{i,j}(d) + \eta^j(k), \quad (1)$$

where $\eta^j(k)$ is the zero-mean additive white Gaussian noise with variance $N_0/2$ per dimension, and $h_{i,j}(d)$ is the d-th coefficient of the impulse response of the channel between the i-th transmit antenna and the j-th receive antenna. The coefficients $h_{i,j}(d)$ are assumed to be zero-mean independent and identically distributed Rayleigh random variables with variance σ_d^2, where $\sum_{d=0}^{D-1} \sigma_d^2 = 1$. We further assume that the channel coefficients are spatially uncorrelated and remain constant during the transmission of one block, i.e., one codeword of the turbo code.

Note that (1) can be rewritten as

$$y^j(k) = \sum_{d=0}^{D-1} \sqrt{E_S} f_k^j(d) x(k-d) + \eta^j(k), \quad (2)$$

where $f_k^j(d) = \sqrt{1/N} \sum_{i=1}^{N} h_{i,j}(d) e^{j\phi_k^i}$. Thus, using RSM, the received signal at the j-th antenna is actually the output of a single-input single-output (SISO) channel with time-varying taps $f_k^j(d)$. In other words, given that there are M receive antennas, using RSM at the transmitter reduces the MIMO channel to a single-input multiple-output (SIMO) channel. As we will see, this reduction has no negative impact on the diversity gain of the system.

However, reducing the MIMO channel to an equivalent SIMO channel does have a beneficial impact on the receiver complexity. Indeed, we may now employ a maximum a posteriori (MAP) equalizer for a SIMO system, which is a straightforward extension of the MAP equalizer for SISO systems, obtained at the cost of a linear (in M) increase in complexity. Note that, in this case, the resulting computational complexity does not depend on the number of transmit antennas.

Without loss of generality, in this paper we focus on the case of $M = 1$ receive antenna and use the turbo equalizer proposed in [17] and depicted in Fig. 2. This receiver consists of three blocks: a MAP equalizer, which is based on the trellis of the channel given in (2); and two decoders, each corresponding to a constituent encoder of the turbo code. Each of these blocks computes extrinsic information on the transmitted symbols. This information is passed to the other blocks, where it is used as *a priori* probabilities on the transmitted symbols [17].

III. Simulation Results

In this section, we investigate the effect of the number of transmit antennas, decoding delay, and channel estimation in the performance of the proposed system. In the computer simulations we consider that the input bits are first encoded using a standard rate $1/3$ parallel turbo encoder, with one systematic output, and two identical constituent convolutional encoders with generator matrix $G(D) = \left[\frac{1+D+D^2+D^3}{1+D^2+D^3}\right]$. We also consider BPSK modulation ($m = 2$), $M = 1$ receive antenna, ISI length $D = 2$ ($\sigma_0^2 = \sigma_1^2 = 0.5$), 10 iterations of the turbo equalizer, and perfect channel knowledge at the receiver (unless stated otherwise).

A. Number of Transmit Antennas

First we investigate the effect of the number N of transmit antennas in the performance of the proposed system. The interleaver length is set to $I = 97$, which means that the length of the block to be transmitted through the channel is 300 symbols. The channel is considered to be quasi-static during one symbol block. Figure 3 shows the bit error rate (BER) versus E_b/N_0, where E_b is the energy per information bit, for $N = \{1, 2, 4, 8\}$. From the figure we can see that, at a BER of 10^{-3}, the gain of the proposed system when compared with a system with only one transmit antenna is of around 4, 7

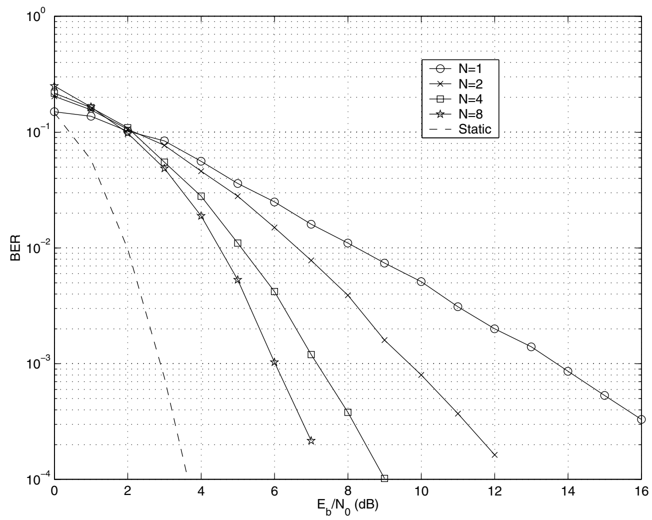

Fig. 3. BER vs. E_b/N_0 for different numbers of transmit antennas ($N = \{1, 2, 4, 8\}$), and a code interleaver of length $I = 97$.

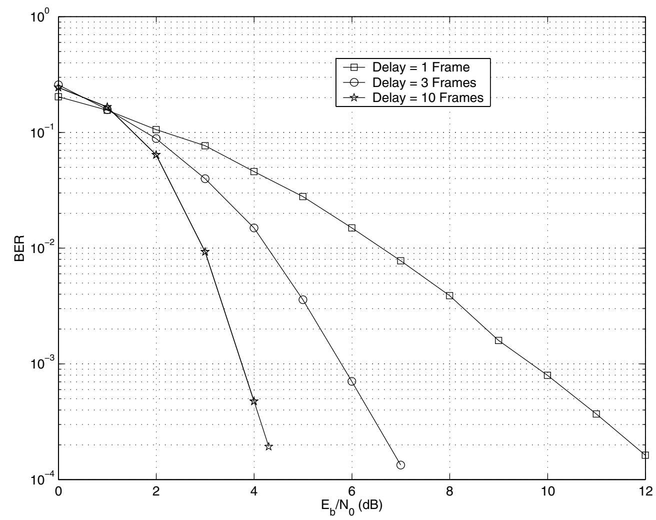

Fig. 4. BER vs. E_b/N_0 for $N = 2$ transmit antennas and different decoding delays: one block ($I = 97$), three blocks ($I = 297$) and ten blocks ($I = 997$).

and 8 dB for the cases of N =2, 4 and 8, respectively. As a reference, we also show the BER for the case of a static channel (without fading) with the same ISI pattern. We can see that, for $N = 8$ the performance approaches that of a static link, which means that the systems was able to overcome most of the degradation introduced by the fading.

B. Decoding Delay

In the previous simulation the decoding delay (codeword length) was equal to the duration of the channel, where duration of the channel is defined as the time during which it can be considered static. Now we allow the codeword to extend over more than one channel duration, so that one transmitted codeword will be affected by more than one channel realization. Again, the channel is assumed to be quasi-static during one block of 300 symbols, and changes independently from frame to frame. Figure 4 shows the BER versus E_b/N_0 for the cases of $N = 2$ transmit antennas and decoding delays of 1, 3 and 10 blocks ($I = 97$, 297 and 997, respectively). For instance, for a delay of 10 blocks of 300 symbols, a codeword transmitted through the channel will be affected by 10 different channel realizations.

From the figure we can see that at a BER of 10^{-3}, increasing the decoding delay to 3 ($I = 297$) and 10 ($I = 997$) blocks results in gains of about 4 and 6 dB, respectively, when compared with the case of $N = 2$ and $I = 97$. Note that the performance for $N = 2$ and $I = 997$ is even better than the performance for $N = 8$ and $I = 97$. Thus, it might be more interesting to explore the gain obtained with a larger decoding delay, than the gain obtained with the increase in the number of transmit antennas. However, note that, strictly speaking, we are not in a block fading environment anymore (since the data blocks extend over more than one channel realization), and some additional diversity gain was already expected. This improvement should come independently of the scheme in use.

C. Imperfect Channel Knowledge

In the previous items we considered that the receiver has perfect knowledge of the channel. In practice, it is necessary to estimate the channel before (or during) equalization and decoding. The Cramér-Rao bound is a lower bound for the mean square error (MSE) of an unbiased estimator. Here, we consider the case where a certain number P of pilots is inserted before a block of S symbols that are going to be transmitted through the channel. In this case, the Cramér-Rao bound is given by [18, Eq. (11)]:

$$MSE[h_{i,j}(d)] \geq \frac{\sigma_n^2}{\sigma_n^2 \rho_h^2 + S\sigma_s^2 + P\sigma_p^2}, \tag{3}$$

where σ_n^2 is the noise variance, $\rho_h^2 = 1/\sigma_d^2$, σ_s^2 is the average symbol energy, and σ_p^2 is the average pilot energy.

Figure 5 shows the BER versus E_b/N_0 for $N = 2, 8$, $S = 300$ ($I = 97$), $P = 10$, and for the cases of perfect channel knowledge at the receiver (PCSI) and for the case where the estimation error is a Gaussian random variable with zero mean and variance given by (3). As we can see, the relative degradation due to imperfect channel knowledge increases with the increase in the number of transmit antenas. However, for the two cases considered, the degradation is still very small in terms of SNR.

IV. Analysis of the Computational Complexity

McEliece and Lin defined in [19] the trellis complexity for a convolutional code, which is directly related to the computational effort required by an algorithm like Viterbi [20] or BCJR [21] to decode one bit, and is a function of the trellis module [19] for that code. Even though the trellis complexity was defined for binary codes, we can apply this concept to trellis-based equalizers for MIMO systems. We concentrate only in the total edge count given a certain

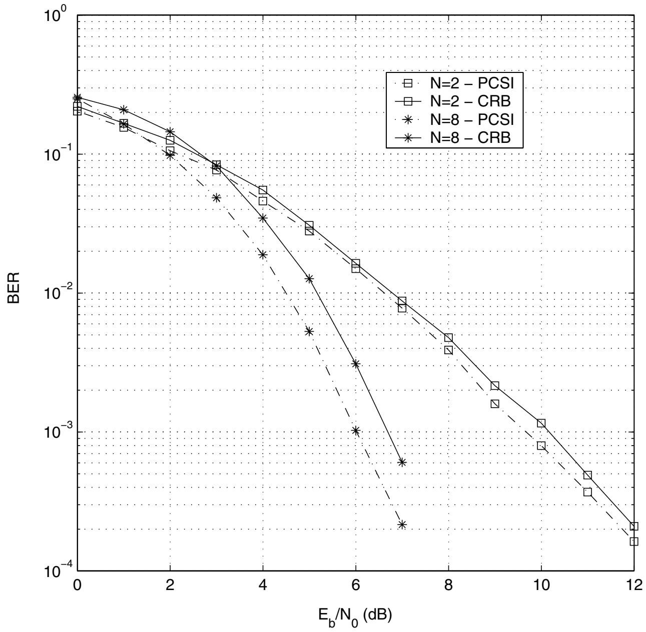

Fig. 5. BER vs. E_b/N_0 for $N = 2, 8$ and $I = 97$, for the cases of perfect channel knowledge (PCSI) and for the Cramér-Rao (CRB) bound considering $P = 10$.

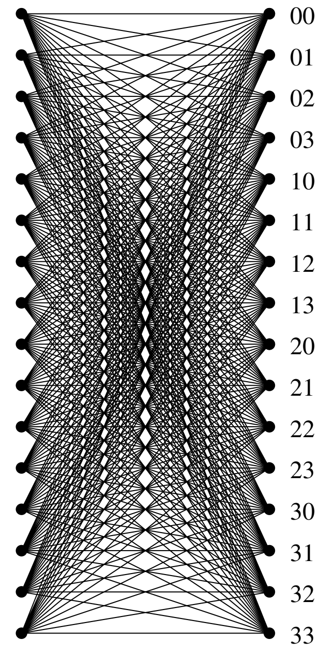

Fig. 6. Trellis used in the MAP equalization of a MIMO channel with $m = 4$, $N = 2$ and $D = 2$, and space-time coding (STC).

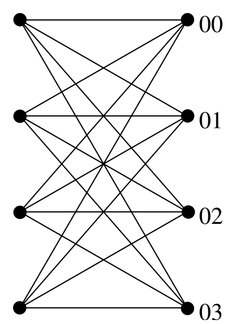

Fig. 7. Trellis used in the MAP equalization, for the case of $m = 4$, $N = 2$ and $D = 2$, and random signal mapping (RSM).

trellis module, not taking into account the number of bits labeling each edge, since this will be the same for every case considered in this paper. For the case of MAP trellis-based equalization of MIMO channels, the total edge count (computational complexity) is proportional to [22]:

$$\Gamma_{STC} = \left(m^N\right)^D. \tag{4}$$

For instance, the trellis used in the MAP equalization of a MIMO system with $m = 4$, $N = 2$ and $D = 2$ is shown in Figure 6, and its complexity is $\Gamma_{STC} = 256$ edges.

Using a standard STBC or STTC, it is necessary to employ an equalizer with complexity proportional to (4). On the other hand, for the RSM scheme the equivalent channel reduces to the SIMO model given by (2), whose total edge count for MAP equalization is proportional only to

$$\Gamma_{RSM} = (m)^D, \tag{5}$$

yielding a savings that increases exponentially with the number of transmit antennas. For instance, for the same case shown in Figure 6 ($m = 4$, $N = 2$ and $D = 2$), the trellis used in the MAP equalization in the case of RSM is shown in Figure 7, and its complexity is of only $\Gamma_{RSM} = 16$ edges.

Table V shows the total edge count for MAP equalization considering space-time coding (STC) and random signal mapping (RSM), varying the number of transmit antennas N, the ISI length D, and 4-PSK modulation. From the Table we can see that the total edge count for the case of STC rapidly explodes, even for relatively small values of N and D. For the case of RSM, the total edge count increases moderately with D, and simply does not change with the increase of the number of transmit antennas N. This is due to the fact that the receiver is designed based on the SIMO model defined by equation (2).

V. Conclusion

We have presented a turbo equalization scheme with reduced complexity for systems with multiple transmit antennas. Diversity is obtained through the use of a parallel-concatenated turbo code and the insertion of a random signal mapper in each transmit antenna branch, avoiding the use of a space-time encoder. This latter modification allows the equivalent channel model to be reduced from a MIMO to a SIMO one, yielding large savings in computational complexity. Computer simulations showed that the proposed scheme is able to provide diversity gain, and is robust against channel mismatch. Moreover, we also investigated the effects of decoding delay and the number of transmit antennas in the system performance. We

TABLE I

TOTAL EDGE COUNT FOR MAP EQUALIZATION, FOR THE CASES OF SPACE-TIME CODING (STC) AND RANDOM SIGNAL MAPPING (RSM), VARYING THE NUMBER OF TRANSMIT ANTENNAS N, THE ISI LENGTH D, AND 4-PSK.

	RSM	**STC**		
D	$N = 2, 4, 8$	$N = 2$	$N = 4$	$N = 8$
2	16	256	65536	$4,30 \times 10^9$
3	64	4096	$1,68 \times 10^7$	$2,82 \times 10^{14}$
4	256	65536	$4,29 \times 10^9$	$1,85 \times 10^{19}$
5	1024	1048576	$1,10 \times 10^{12}$	$1,21 \times 10^{24}$
6	4096	$1,68 \times 10^7$	$2,82 \times 10^{14}$	$7,92 \times 10^{28}$

showed that increasing the delay may have a greater impact than increasing the number of transmit antennas.

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