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Letter

Achievable rates of DSL with crosstalk cancellation

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SUMMARY

Crosstalk is one of the main limiting factors in the data rates achievable by digital subscriber line (DSL) systems, and several algorithms have been proposed to mitigate this impairment. In this paper, we compare the achievable rates of binders under different crosstalk-mitigating techniques. When computing these rates, we also compare two different power constraints: either on the total power in the binder or on the power in each twisted wire pair. We will see that, for the scenarios considered in this paper, the fact that the signals are jointly processed in one or both ends of the DSL link leads to roughly the same performance, which can be far superior to that of systems with no cooperation between the users. Both power constraints also lead to similar achievable rates. Copyright © 2008 John Wiley & Sons, Ltd.

1. INTRODUCTION

Digital subscriber line (DSL) systems exploit the twisted pairs traditionally used for phone services to transmit high-rate data services such as internet access. Twisted pairs from different users are normally deployed in binders, which may contain tens or hundreds of pairs. Due to the proximity of these pairs, their signals are electromagnetically coupled, generating crosstalk between the pairs. Crosstalk is one of the main impairments of DSL system, and one of the main factors limiting the achievable data rates of these systems.

Currently DSL systems use a single wire pair to transmit data between the central office (CO) and the end user. Traditionally, the users are not processed jointly at the CO. In other words, current DSL systems essentially transmit through single-input single-output (SISO) channels. In these cases, little can be done to mitigate crosstalk. One choice is to allocate spectrum to different users so as to minimise the impact of crosstalk, a technique called dynamic spectrum management [1] which is not further discussed here.

It has long been recognised that jointly processing the signals from different users may improve the performance of DSL systems [2, 3]. These works exploit the fact that the CO has access to the signals of all the pairs in a binder to mitigate or cancel crosstalk using signal processing techniques, achieving rates close to the optimum. Note that References [2, 3] assume that the signals are jointly processed only at the CO side and, at the costumer premises (CPE), users have access only to a single pair. Interestingly, it was shown in Reference [3] that, in such cases, due to the characteristics of DSL systems, some suboptimal techniques, e.g. zero forcing, can achieve rates close to the channel capacity.

In connections between COs and cabinets or remote terminals, however, the signals of all pairs in a binder

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can be jointly processed at both ends of the links. Therefore, we can treat this system as a multiple-input multiple-output (MIMO) DSL system, enabling the use of capacity-achieving techniques based on the singular-value decomposition (SVD) of the channel matrix [4]. Although end-users do not have access to all pairs in a binder, we can still use the MIMO-SVD system as an upper bound on the achievable rates of DSL systems.

In this paper, we compute the achievable rates of several binders based on different signal processing algorithms. When computing these rates, we also compare two different power constraints. First, we use the traditional waterfilling constraint, which limits the total power transmitted in the binder. We also use an alternative constraint of more practical interest: a per-line power constraint. This limitation reflects potential regulatory constraints and the fact that the signal in each pair must go through a power amplifier, whose output power is limited by practical considerations.

The rest of this paper is organised as follows. Section 2 introduces the model used through this work, while Section 3 presents the algorithms considered for crosstalk mitigation. The two algorithms used for power allocation are described in Section 4. Section 5 shows comparison results for the proposed algorithms by means of simulations. Conclusions are drawn in Section 6.

2. CHANNEL MODEL AND PROBLEM STATEMENT

In this paper, we consider a DSL system employing perfect frequency-division duplexing (FDD), so that there is no near-end crosstalk. Furthermore, as with all DSL systems, the use of discrete multi-tone (DMT) modulation transforms the time-dispersive channel into multiple parallel channels in the frequency domain, called subchannels or tones, with no intertone-interference. In this case, the channel output at a given tone k, $\mathbf{y}_k$, is given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k \tag{1}$$

where $\mathbf{x}_k$ is a vector containing the signals transmitted from each pair, $\mathbf{H}_k$ is a matrix that models the insertion loss and crosstalk gain between the pairs and $\mathbf{n}_k$ represents the additive white Gaussian noise (AWGN) with covariance matrix $\sigma_k^2 \mathbf{I}_N$.

Several attempts were made to mathematically model the matrix $\mathbf{H}_k$ on each tone [5]. In this paper, we employ what is perhaps the most accurate model in the literature at the time of writing, the one developed by Bin Lee in his doctoral thesis [6]. Based on Bin Lee's model, we compare the rates achieved by systems employing different types of crosstalk-mitigating techniques and different spectrum allocation algorithms. We will also use these algorithms to determine the achievable rates of an actual binder whose channel matrix was measured at a laboratory at Ericsson Research—Broadband and Transport.

3. CROSSTALK-MITIGATING ALGORITHMS

In this section, we describe the three signal processing algorithms used in this paper to mitigate crosstalk. The first one attempts no cancellation whatsoever, and is termed no signal processing (NSP). The second is based on the inversion of the channel matrix, and is called zero-forcing (ZF), since it forces the crosstalk to zero. The third is based on the SVD of $\mathbf{H}_k$.

Note that the techniques used in this paper assume perfect channel knowledge. Furthermore, they can be seen as the multiplication of the transmitted signal by a preprocessing matrix before transmission and/or the multiplication of the received signal by a receive matrix. Thus, all schemes can be represented as in Figure 1. In this figure, N represents the number of pairs in the binder, $\tilde{x}_k^n$ ($\tilde{y}_k^n$) is the information symbol transmitted (received) in the k-th tone of the n-th pair, x_k^n (y_k^n) is the signal actually transmitted (received) in the k-th tone of the n-th pair and $\mathbf{P}_k$ ($\mathbf{R}_k$) represent the transmit (receive) processing matrix. This linear transceiver scheme creates a virtual channel between the transmitted and received information symbols for the pairs, whose output can be written as

$$\tilde{\mathbf{y}}_k = \mathbf{R}_k \mathbf{H}_k \mathbf{P}_k \tilde{\mathbf{x}}_k + \tilde{\mathbf{n}}_k \tag{2}$$

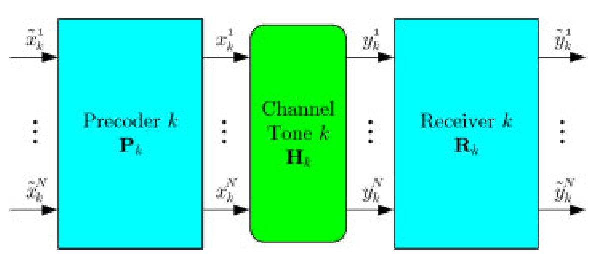

Figure 1. Tone k of a DSL system with a general linear transceiver. This figure is available in color online at www.interscience.wiley.com/journal/ett

where $\tilde{\mathbf{n}}_k$ represents zero-mean additive Gaussian noise with covariance matrix $\mathbf{C}_{\tilde{\mathbf{n}}_k} = \sigma_k^2 \mathbf{R}_k \mathbf{R}_k^H$.

3.1. No signal processing

In this case, we assume that there is no cooperation between the users at either end of the DSL link. In other words, $\tilde{y}_k^n$ depends only on y_k^n, and x_k^n depends only on $\tilde{x}_k^n$. As a consequence, the receiver and preprocessing matrices in Figure 1 must be diagonal, and no attempt can be made to mitigate crosstalk. In this paper, without loss of generality, we assume that

$$\mathbf{P}_k = \mathbf{R}_k = \mathbf{I}_N \; \forall k \tag{3}$$

Note that this approach represents today's main practice in DSL transmission technology.

3.2. Zero forcing

The ZF algorithm assumes that the signals are jointly processed only at the CO and that a single pair is available to each user. In the downstream, the CO is the transmitter, while it is the receiver in the upstream. Thus, for downstream transmission we can only perform ZF precoding, while ZF processing at the receiver is all that is available for upstream transmission. In both cases, following Reference [3], we use the inverse of the channel matrix for processing, so that the equivalent channel $\widetilde{\mathbf{H}}_k = \mathbf{R}_k \mathbf{H}_k \mathbf{P}_k$ in Equation (2) is the identity matrix.

A well-known drawback of ZF techniques is power enhancement. In the downstream, the ZF precoder causes an signal-to-noise ratio (SNR) penalty by artificially increasing the power transmitted in each pair. This is because part of the power is used to compensate for the crosstalk and only a fraction of the power in each pair is used to effectively transmit information. The power enhancement is taken into account in the power allocation algorithms presented in Section 4. On the other hand, in the upstream, the ZF receiver causes noise enhancement, since the equivalent noise in Equation (2) now has a covariance matrix given by $\sigma_k^2 \mathbf{H}_k^{-1} \mathbf{H}_k^{-H}$. Both upstream and downstream penalties may lead to a rate loss. However, as we will see, the achievable rates of ZF-based DSL systems are close to the channel capacity, implying that both types of power enhancement have negligible effect.

3.3. Singular-value decomposition

In the fully-coordinated case, both the CO and CPE can jointly process all the DSL pairs. This can happen, for instance, in links between COs and remote terminals. This scenario corresponds to an end-to-end MIMO system. In this fully-coordinated case we can implement the capacity-achieving SVD solution [4], described next.

Let $\mathbf{H}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H$ be the SVD of the channel matrix $\mathbf{H}_k$. Then, if precoding and receiver matrices are given by $\mathbf{V}_k$ and $\mathbf{U}_k^H$, respectively, the equivalent channel is given by $\mathbf{\Sigma}_k$. Since this is a diagonal matrix, we see that crosstalk has been completely eliminated.

It is interesting to notice that the receiver matrix is unitary, so that it does not change the noise statistics, *i.e.* $\mathbf{C}_{\tilde{\mathbf{n}}_k} = \sigma_k^2 \mathbf{I}_N$. The precoding matrix is also unitary, which implies that the total power (sum of the power in all pairs) before and after precoding is the same. In other words, the SVD effectively decouples all the pairs, with no noise enhancement and no increase in the total power. This observation leads to a simple power allocation algorithm, the well-known waterfilling solution described in the next section. On the other hand, the power in a specific pair before and after precoding are different. Thus, if there is a per-line power constraint, the precoding operation must be accounted for. This can be done with the optimal power allocation (OPA) algorithm described in Section 4.

3.4. Summary

The signal processing algorithms described in this section are summarised in Table 1 for downstream transmission and in Table 2 for the upstream. In these tables, the entry $\tilde{g}_k^n$ corresponds to the magnitude-squared of the gain of the equivalent direct channel in Equation (2) for the n-th pair

Table 1. MIMO transceiver structures—downstream.

	SVD	ZF	NSP
$\mathbf{P}_k$	$\mathbf{V}_k$	$\mathbf{H}_k^{-1}$	$\mathbf{I}_N$
$\mathbf{R}_k$	$\mathbf{U}_k^H$	$\mathbf{I}_N$	$\mathbf{I}_N$
$\mathbf{C}_{\tilde{\mathbf{n}}}$	$\sigma_k^2 \mathbf{I}_N$	$\sigma_k^2 \mathbf{I}_N$	$\sigma_k^2 \mathbf{I}_N$
$\tilde{g}_k^n$	$\left\| \left[\mathbf{\Sigma}_k \right]_{n,n} \right\|^2$	1	$\left\| \left[\mathbf{H}_k \right]_{n,n} \right\|^2$

Table 2. MIMO transceiver structures—upstream.

	SVD	ZF	NSP
$\mathbf{P}_k$	$\mathbf{V}_k$	$\mathbf{I}_N$	$\mathbf{I}_N$
$\mathbf{R}_k$	$\mathbf{U}_k^H$	$\mathbf{H}_k^{-1}$	$\mathbf{I}_N$
$\mathbf{C}_{\tilde{\mathbf{n}}}$	$\sigma_k^2 \mathbf{I}_N$	$\sigma_k^2 \mathbf{H}_k^{-1} \mathbf{H}_k^{-H}$	$\sigma_k^2 \mathbf{I}_N$
$\tilde{g}_k^n$	$\left\| \left[\mathbf{\Sigma}_k \right]_{n,n} \right\|^2$	1	$\left\| \left[\mathbf{H}_k \right]_{n,n} \right\|^2$

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and the k-th tone, i.e.

$$\tilde{g}_k^n = \left| \left[\mathbf{R}_k \mathbf{H}_k \mathbf{P}_k \right]_{n,n} \right|^2 \quad (4)$$

4. POWER ALLOCATION ALGORITHMS

Traditionally, OPA is determined under the assumption that the total power to be transmitted is limited, i.e. the sum of the powers in all the pairs in a binder. However, it may be of practical interest to actually consider a power constraint for each line. This is due to the power limitation at the amplifier of each modem, as well as regulatory constraints to avoid excessive interference with other systems. We begin this section with a brief presentation of the waterfilling solution [5], which achieves the channel capacity when only the total transmit power is limited. Then, we present the OPA algorithm [3], which determines the OPA for a precoded system when there is a per-line power constraint.

4.1. Waterfilling

As mentioned in Section 3.3, the use of an SVD-based transceiver yields several parallel and independent AWGN channels. In this case, the noise variance and the total power transmitted in both the equivalent and the actual channels are the same. The problem of power allocation in this case is standard, and its solution is the well-known waterfilling solution [5]: the power allocated to the information symbol $\tilde{x}_k^n$ is given by

$$\tilde{s}_k^n = \left[\lambda - \Gamma \frac{\sigma_k^2}{\tilde{g}_k^n} \right]^+ \quad (5)$$

where

$$[x]^+ \triangleq \begin{cases} x, & \text{if } x \geqslant 0 \\ 0, & \text{if } x < 0 \end{cases} \quad (6)$$

and where Γ stems from the use of the gap approximation [5]. In Equation (5), λ represents the water level, which must be chosen so that the total transmit power is equal to the maximum value allowed, P. In other words, λ is chosen so that

$$\sum_n \sum_k \tilde{s}_k^n = P \quad (7)$$

Finally, it should be pointed out that $\tilde{s}_k^n$ is the power allocated to the symbols at the k-th tone of the n-th pair *before* the precoding matrix (see Figure 1).

4.2. Optimal power allocation with preprocessing

Consider the precoding operation $\mathbf{x}_k = \mathbf{P}_k \tilde{\mathbf{x}}_k$. Assume that the information symbols $\tilde{x}_k^m$ are independent and have power $\tilde{s}_k^m$. Then, taking into account the elements of the preprocessing matrix $p_k^{n,m} \triangleq \left[\mathbf{P}_k \right]_{n,m}$, the output power for each modem can be written as

$$s_k^n = \sum_m \left| p_k^{n,m} \right|^2 \tilde{s}_k^m \quad (8)$$

where s_k^n denotes the power of the precoded symbols x_k^n. The power transmitted in line n is the sum of the power on each tone in this line, and is given by

$$\sum_k s_k^n \quad (9)$$

Using the gap approximation [5] and a diagonalising precoder such as the ZF or the SVD, the maximum bit rate $b_k^n(\tilde{s}_k^n)$ that can be transmitted using power $\tilde{s}_k^n$ at pair n and tone k is given by

$$b_k^n\left(\tilde{s}_k^n\right) = \log_2 \left(1 + \frac{\tilde{g}_k^n \tilde{s}_k^n}{\tilde{\sigma}_k^2 \Gamma} \right) \quad (10)$$

The OPA problem with a per-line power constraint can then be stated as [3]

$$\left\{ \tilde{s}^{*n}_k \right\}_{k=1,\ldots,K}^{n=1,\ldots,N} = \arg \max_{\tilde{s}_k^n,\ \forall k,n} \sum_n \sum_k \log_2 \left(1 + \frac{\tilde{g}_k^n \tilde{s}_k^n}{\tilde{\sigma}_k^2 \Gamma} \right) \quad (11)$$

subject to

$$\begin{aligned} &\sum_k s_k^n \leqslant P, \quad \forall n \\ &\tilde{s}_k^n \geqslant 0, \quad \forall n, k \end{aligned} \quad (12)$$

The solution of the above optimisation problem can be found by using Lagrange multipliers. In Reference [7], Cendrillon proposes the iterative algorithm named OPA, shown in Table 3, to find the optimum solution.

Table 3. Optimal power allocation with precoding.

repeat

$$\forall n, k\colon\ \tilde{s}_k^n = \left[\frac{1}{\sum_m \lambda_m \left|p_k^{m,n}\right|^2} - \Gamma \frac{\tilde{\sigma}_k^2}{\tilde{g}_k^n}\right]^+$$

$$\forall n, k\colon\ \lambda_n = \left[\lambda_n + \mu\left(\sum_k \sum_m \left|p_k^{n,m}\right|^2 \tilde{s}_k^m - P\right)\right]^+$$

until convergence

5. SIMULATIONS

In this section we present simulation results that compare the rates achieved by the different transceivers and power allocation algorithms described in the previous section. In particular, we computed the achievable rates of SVD, ZF and no processing transceivers with OPA, and of an SVD transceiver with total power constraint, both for upstream and downstream channels. Note that the rates achieved with the SVD transceiver with total power constraint and waterfilling gives the capacity of the channel. This result assumes full coordination on both ends. If full coordination is not available, than these rates provide an upper bound on the achievable rates of the system.

5.1. First simulation: results for the Bin Lee's channel model on MIMO-VDSL

In this simulation, the rate as a function of the reach was evaluated for a MIMO-VDSL channel generated using Bin Lee's model. We used a binder with 10 VDSL pairs, with 4096 tones. The wire diameter is 0.5 mm (24-AWG). The target bit error rate (BER) was set to 10^{-7} or less, the coding gain was set to 3.8 dB and the noise margin to 6 dB, which leads to an effective gap of $\Gamma = 9.8 - 3.8 + 6 = 12$ dB. In accordance with the VDSL standards [8, 9], the tone spacing Δf is set to 4.3125 kHz and the DMT symbol rate f_s to 4 kHz. Background noise is composed of white thermal noise with a PSD of -140 dBm/Hz and the available transmitted power for each line is 11.5 dBm for the upstream and 14 dBm for the downstream. Moreover, we use bandplan A [8, 9] to determine the tones used for the upstream and downstream.

Figure 2 depicts the rates achieved by the three processing algorithms implemented (NSP, ZF and SVD) using OPA, as well as an SVD algorithm using total power constraint. The results are plotted as a function of the reach. One can see that the performance when using ZF or SVD is similar, outperforming by far the results obtained without any signal processing, especially for short distances. This is expected,

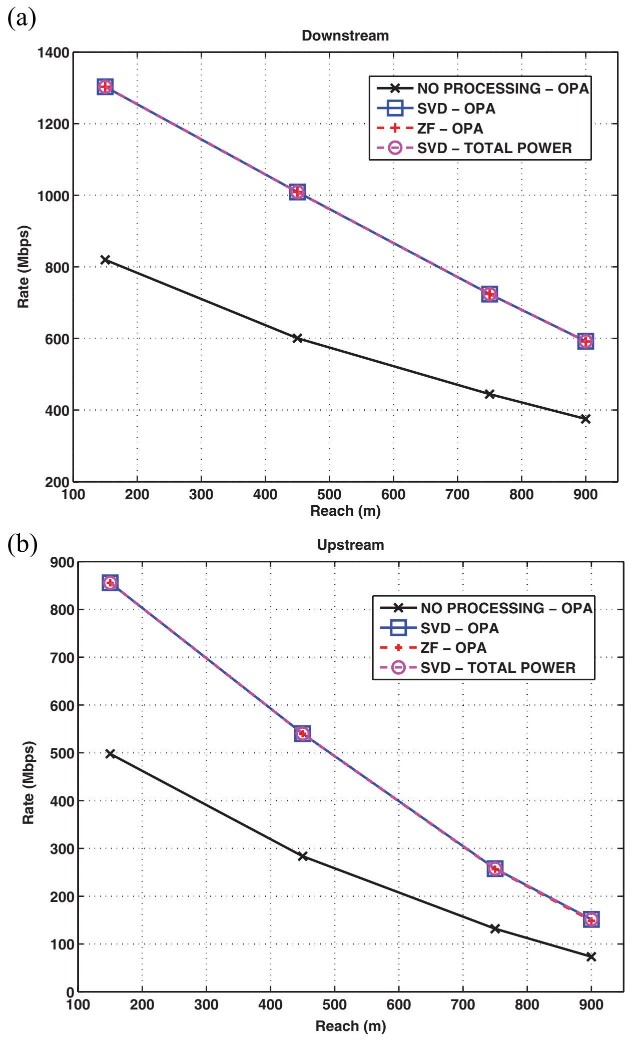

Figure 2. Rate as a function of reach for no processing, SVD and ZF algorithms using OPA and SVD using total power for the VDSL system. (a) Downstream rate. (b) Upstream rate. This figure is available in color online at www.interscience.wiley.com/journal/ett

since for longer loops, the crosstalk level approaches the noise level and the system becomes noise-limited instead of crosstalk-limited.

Also, there is no difference between the SVD results obtained using a per-line power constraint (OPA) and a total binder power constraint. This is because the direct channel and the crosstalk for all the 10 pairs is almost the same, due to the fact that the pairs have the same length, which leads to a similar SNR value in all the cables. In other words, the channels for all pairs are similar, so there is no reason to allocate more power to one line than to another.

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Table 4. Rate *versus* reach for ADSL2+.

	Upstream (Mbps)		Downstream (Mbps)	
Technique	500 m	1500 m	500 m	1500 m
NSP	27.1745	21.1538	379.988	130.381
ZF	27.1747	21.1538	380.456	130.382
SVD	27.1747	21.1538	380.456	130.382

5.2. Second simulation: MIMO-ADSL2+ results for the measured channel

A binder (model EULEV 10x2x0.4 TEH 240 1402/010) with 10 pairs was measured at a laboratory at Ericsson Research—Broadband and Transport, in the frequency range of ADSL2+ for two distances: 500 and 1500 m. We computed the achievable rates for this binder using NSP, ZF and SVD systems. The results are shown in Table 4. As per the ADSL2+ standard [10], Annex A, 512 tones were used, the tone spacing Δf is set to 4.3125 kHz and the DMT symbol rate f_s to 4 kHz. Background noise is composed of white thermal noise with a PSD of -140 dBm/Hz and the available transmitted power for each line is 12.5 dBm for upstream and 19.9 dBm for downstream. The tones used for the upstream and downstream are also defined in the standard. Our computations are based on the gap approximation, using the same parameters as in Section 5.1.

In Table 4, we can notice that using signal processing algorithms like SVD and ZF produces the same data rate as using no crosstalk-mitigating algorithm. The similar performance for all algorithms can be explained by the fact that the channel matrices for all tones are almost diagonal and the crosstalk power is very low in comparison to the noise power, i.e. the performance of the system is limited by the noise.

6. CONCLUSIONS

In this paper, we have computed the capacity as a function of the distance for binders of twisted pairs, as used in DSL systems. We compared the capacity to the achievable rates of signal processing and power allocation algorithms. We have shown that crosstalk-mitigation techniques may yield significant gains over systems that do attempt no cancellation whatsoever. More pronounced gains are obtained in systems that use a wider frequency band, such as VDSL, and for shorter binders. However, our simulation results suggest that the capacity of a system that jointly processes signals at both ends of the communications link is similar to the capacity of a system that performs joint processing only at the CO side. Finally, the results in this paper also indicate that imposing a per-line power restriction incurs almost no rate penalty under the considered scenarios.

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